Controlled boiling on Enceladus. 2. Model of the liquid-filled cracks

Andrew P. Ingersoll, Miki Nakajima

Abstract
Controlled boiling will occur on Enceladus whenever a long, narrow conduit connects liquid water to the vacuum of space. In a companion paper we focus on the upward flow of the vapor and show how it controls the evaporation rate through backpressure, which arises from friction on the walls. In this paper we focus on the liquid and show how it flows through the conduit up to its level of neutral buoyancy. We find that freezing on the walls in Sections 4 and 5. We present our conclusions in Section 6.

1. Introduction
Postberg et al. (2009, 2011) made a major advance in our study of the Enceladus plumes with observations that a major fraction of the particles, both in the plumes and in the E ring, are made of salt water ice. Their data eliminate or severely constrain non-liquid models and strongly imply that a salt-water reservoir with a large area for the liquid–gas interface and the salinity of the water in the cracks. We discuss the efficiency of convection and the area of the liquid–gas interface in Sections 2 and 3. We discuss freezing of the liquid–gas interface and freezing on the walls in Sections 4 and 5. We present our conclusions in Section 6.

2. Controlled boiling
The implication of controlled boiling is that narrow, liquid-filled cracks with water rising to its level of neutral buoyancy are possi-
ble. This is different from the view of Postberg et al. (2009), who argue that the area of the evaporating surface is orders of magnitude greater than that of the crack itself. This view is expressed in their Fig. S2, which is reproduced as Fig. 1. They favor evaporation from a large gas-filled chamber as shown in the left panel, although in their model the area ratio of the chamber size to the vent size is much greater than that shown in the figure. They say that a small liquid–gas interface, as shown in the right panel, requires implausibly large temperature gradients in the liquid to maintain the heat flux that is required to support the steady-state gas evaporation. The argument is based on their Eq. (S12), which is from experimental and theoretical work on thermal convection between solid horizontal surfaces maintained at different temperatures, the bottom being warmer than the top (e.g., Malkus, 1954; Kraichnan, 1962; Ingersoll, 1966; Grossmann and Lohse, 2001; Ahlers and Xu, 2001). That equation predicts large temperature differences within the fluid, which seem implausible and led Postberg et al. (2009) to propose large vapor chambers. However, we will argue that the water on Enceladus is boiling, and therefore Eq. (S12) does not apply.

The liquid on Enceladus is boiling if it is vented to space, whether there is a large vapor chamber (left panel of Fig. 1) or a narrow crack (right panel). A liquid boils when its saturation vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor pressure is greater than the total atmospheric pressure. If other gases are present initially, they will be carried off with the vapor.

\[ \frac{dT}{dz} = \frac{dP_v}{dz} = -\left( \frac{R_v T^2 \rho_l \kappa}{P_v L_v} \right) = -2.70 \, \text{K m}^{-1} \]  

(1)

We are using numbers from Table 1 for seawater, with salinity \( S = 35 \, \text{g kg}^{-1} \). \( R_v = 8.314/0.018 \, \text{J kg}^{-1} \text{K}^{-1} \text{K}^{-1} \). \( T = 273 \, \text{K} \), \( P_v = 599 \, \text{Pa} \), and \( g = 0.11 \, \text{m s}^{-2} \). The numbers for fresh water differ by only a few percent. If the surface of the liquid were boiling at the seawater freezing point, which is \(-1.92 \, ^\circ\text{C}\), then the liquid 71 cm below the surface would be boiling at \(0 \, ^\circ\text{C}\). Presumably \(0 \, ^\circ\text{C}\) is as warm as it gets in the liquid-filled cracks of Enceladus, because the walls are made of ice. Depending on salinity, all the water 10s of cm down from the liquid–gas interface could be boiling at once.

![Fig. 1. Contrasting views of gas evaporating from a subsurface liquid flowing through a cylindrical conduit to vacuum. This is Fig. S2 in Postberg et al. (2009). They argue that a large evaporating surface (left panel) is required to avoid implausibly large temperature gradients in the liquid (right panel). \( R_{\text{out}} \ll R_{\text{in}} \) and \( R_{\text{in}} \approx R_{\text{out}} \) are the proposed radii of the conduit at the evaporating surface and the vent, respectively.](image)

Table 1 Thermophysical constants for fresh water and seawater. Most of the numbers are from Sharqawy et al. (2010), who give online tables at web.mit.edu/seawater/ for a range of salinities. The expansion coefficient is from the Engineering Toolbox at www.engineeringtoolbox.com/water-thermal-properties-d_162.html.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Units</th>
<th>Fresh water at 0 °C</th>
<th>Seawater at 0 °C</th>
<th>Fresh water at 100 °C</th>
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<tr>
<td>Density</td>
<td>( \rho )</td>
<td>(kg m(^{-3}))</td>
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<tr>
<td>Specific heat</td>
<td>( C_p )</td>
<td>(J kg(^{-1}) K(^{-1}))</td>
<td>4210</td>
<td>3992</td>
<td>4219</td>
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<tr>
<td>Expansion coefficient</td>
<td>( x )</td>
<td>(10(^{-6}) K(^{-1}))</td>
<td>-65.5</td>
<td>26</td>
<td>752</td>
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<tr>
<td>Thermal conductivity</td>
<td>( k )</td>
<td>(W m(^{-1}) K(^{-1}))</td>
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<td>0.570</td>
<td>0.677</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>( \kappa )</td>
<td>(10(^{-6}) m(^{2}) s(^{-1}))</td>
<td>0.136</td>
<td>0.138</td>
<td>0.169</td>
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<tr>
<td>Heat of vaporization</td>
<td>( L_v )</td>
<td>(10(^{3}) J kg(^{-1}))</td>
<td>2.50</td>
<td>2.41</td>
<td>2.26</td>
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<td>Kinematic viscosity</td>
<td>( \nu )</td>
<td>(10(^{-6}) m(^{2}) s(^{-1}))</td>
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<td>( \sigma )</td>
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<tr>
<td>Vapor pressure</td>
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<td>(10(^{2}) Pa)</td>
<td>6.11</td>
<td>5.59</td>
<td>1013</td>
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</table>

Further values of the expansion coefficient are given in Table 2.

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Boiling is important because it eliminates the thermal boundary layer and the large temperature differences that go with it. In the thermal convection experiments the horizontal boundaries are solid and impermeable. Heat has to get to the wall by thermal conduction, which requires a large temperature gradient close to the wall. The liquid–gas interface of an evaporating liquid might act as an impenetrable wall, although it is not solid. But in a boiling liquid the vapor bubbles arising 10s of cm below the surface carry their latent heat with them through the wall, which is the liquid–gas interface. They stir the fluid and keep the lapse rate at the value given by Eq. (1). Large temperature differences do not arise. There is no need to spread the power over a large evaporating area. There is no need for a large vapor chamber.

3. Thermal boundary layer

To understand the thermal boundary layer in the convection experiments and why it doesn’t occur in a boiling liquid, it is helpful to examine the equation that links the heat flux $q$ to the imposed temperature difference $\Delta T$ and the distance $d$ between the horizontal boundaries. Eq. (S12) of Postberg et al. (2009) is from the last line of Table 1 of Grossmann and Lohse (2001) and is reproduced here:

$$Nu = \frac{qd}{\Delta T} = 0.05 Ra^{1/3} = 0.05 \left( \frac{\nu g \Delta T d^{1/3}}{\nu k} \right)^{1/3}$$

(2)

Here $Nu$ is the Nusselt number and $Ra$ is the Rayleigh number; both are dimensionless, and the equation assumes that $Ra$ is large. The thermophysical constants for water are given in Table 1. Most values for fresh water and seawater at $0 \degree C$ differ by less than 5%. The thermal expansion coefficient varies as $(\Delta T)^{1/4}$ and is independent of $d$. This means that the temperature gradients are confined to the boundary layers at top and bottom, and are zero in the interior of the fluid. High Rayleigh number convection is analogous to electrical charge (heat) flowing through a series of three resistors—the lower boundary layer, the interior fluid, and the upper boundary layer. The fact that the length of the middle resistor (the interior fluid) doesn’t enter means that its resistivity is so low that the entire “voltage” drop (the temperature difference $\Delta T$) is confined to the high-resistivity boundary layers. The traditional explanation (e.g., Krachnan, 1962) is that the eddy size is so large in the interior that the convection is extremely efficient and operates with essentially zero temperature gradient. Equivalently, the convective plumes arise in one thermal boundary layer and move undiluted to the other boundary layer, where the actual mixing takes place.

Since there are two boundary layers, the drop $\Delta T_1$ across each is $\Delta T/2$. If one defines the boundary layer thickness $w$ by the equation $q = k \Delta T_1/w$, then Eq. (2) implies

$$w = \frac{1}{0.05} \left( \frac{\nu k}{\nu g \Delta T} \right)^{1/3} \frac{1}{2^{1/3}}$$

(3)

For $S = 35 \text{ g kg}^{-1}$, $\Delta T_1 = 1$ and 10 K, the thickness $w$ is 3.55 and 1.64 cm, respectively. However, we argue that the bubbles rising to the surface would destroy the upper boundary layer by vigorously stirring the liquid in the upper few 10s of cm. In this region the heat flux is carried to the surface by the latent heat of the vapor rather than by thermal conduction. In the interior the heat is carried by eddies, and the temperature gradients are negligible. Temperature gradients are confined to the lower boundary layer—where the liquid water meets the warm rocks of the interior.

A small modification to the above is needed because the $1/3$ exponent in Eq. (2) is an idealization. Laboratory experiments give values in the range 0.28–0.30, as described by Grossmann and Lohse (2001). A slightly smaller exponent means the heat flux $q$ varies as $d$ raised to a small negative power. This small difference in exponents does not affect our conclusion that Eq. (2) overestimates the temperature differences by a large factor, and that arguments based on such large differences are invalid.

To test these ideas, one of us (API) did a kitchen experiment using a digital oil and candy thermometer from Williams-Sonoma. The thermometer has a digital readout in units of $0.1 \degree C$. With an aluminum pot 26 cm in diameter filled with 7 l of water (depth = 13.2 cm) and the gas burner turned on high, the water warmed from 35 $\degree C$ to $70 \degree C$ at a constant rate. The rate that the water started boiling, the temperature at 6 cm depth reached 99.3 $\degree C$ and held steady. The water was turbulent, in a state known as a boiling roll, but the temperatures did not change with time. The temperature 2 cm below the top surface was 99.2 $\degree C$ and the temperature 2 cm above the bottom surface was 99.5 $\degree C$. The difference between the bottom and the top is consistent with Eq. (1). The difference between 99.2 $\degree C$ and 100 $\degree C$ is consistent with the height of the pot above sea level and the sea level pressure on that particular day. In other words, the water at every level was at its own boiling point, determined by the ambient pressure. In contrast, Eq. (2) implies temperature differences $\Delta T_1$ across the top and bottom boundary layers of 23.3 $\degree C$. No such temperature differences were found at any depth, and we conclude that Eq. (2) is irrelevant to the surface of a boiling liquid.

Postberg et al. (2009) invoke Eq. (2) to argue that there are large vapor chambers below the vents. They choose $\Delta T = 1$ K, 10 K, and 100 K, and they compute the heat flux $q$. They regard 100 K as implausibly large, and say that values on the order of a few Kelvins seem more realistic. They use the observed 200 kg s$^{-1}$ gas production rate (Hansen et al., 2011) multiplied by the latent heat of vaporization $Lv = 2.5 \times 10^6$ J kg$^{-1}$ to get 0.5 GW, which they use as the total power, and then they solve for the area that gives the computed $q$. The area is the unknown—the evaporating area in the left panel of Fig. 1. Solving for the area in this way is equivalent to multiplying Eqs. (S3) and (S16) of Postberg et al. (2009). This gives areas of 31, 1.42, and 0.066 km$^2$ corresponding to $\Delta T = 1$, 10, and 100 K, respectively (we have corrected an error in the exponent for the $\Delta T = 10$ K case in their Eq. (S16)). We get the same result directly from Eq. (2) when we use their values for $k$, $\nu$, $g$, $v$, $\kappa$, and $Lv$.

These areas are large, but there are two other factors that make them even larger. First, the power radiated to space is $-4.2$ GW (Spencer et al., 2013), whereas the power associated with the gas production rate is only $\sim 0.5$ GW. Postberg et al. (2009) consider the latter but not the former. The power radiated to space should be included because it is carried upward as vapor and released when the vapor condenses near the tops of the cracks. There it is conducted to the exterior surface and radiated to space. Thus the total power due to evaporation at the liquid–gas interface is 4.7 GW instead of 0.5 GW. Schmidt et al. (2008) assume that the vapor condenses in the vents into icy grains but not on the walls. But since the icy grains constitute only a small fraction of the total mass of the plumes, which are mostly vapor, and since the latent heat carried by the vapor is only a small fraction of the total power

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(Ingersoll and Pankine, 2010), then the power released by condensation on the icy grains is at most only a few percent of the power released by condensation on the walls. If the energy radiated by the icy grains is part of the observed ~4.2 GW radiated to space, then the total power at the liquid-gas interface is still 4.7 GW. Otherwise, if the energy of the icy grains is transformed into kinetic energy and not radiated, then that power should be added to the 4.7 GW in computing the total power at the liquid–gas interface. Thus 4.7 GW is a lower bound, but the upside possibilities are probably negligible.

Second, in their estimate Postberg et al. (2009) assume the thermal expansion coefficient is 200 \( \times 10^{-6} \) K\(^{-1} \), which is appropriate for fresh water at 21.3 °C and for seawater at 12.9 °C but not for water at 0 °C. The water on Enceladus is likely to be near 0 °C because the walls are made of ice. At 0 °C the value of \( \alpha \) for seawater is 26 \( \times 10^{-6} \) K\(^{-1} \), and that for fresh water is 

\[ \text{65.5} \times 10^{-6} \text{ K}^{-1} \]

With 260 \( \times 10^{-6} \) K\(^{-1} \) for \( \alpha \) and 4.7 GW for the power, Eq. (2) gives

\[ 575, 26.3, \text{ and } 1.22 \text{ km}^2 \]

for the areas corresponding to \( \Delta T = 1, 10, \text{ and } 100 \text{ K} \), respectively. If the actual value of \( \Delta T \) were 3 K or less, which is more likely according to Postberg et al. (2009), then the area would be at least 132 km\(^2 \), corresponding to a vapor chamber 264 m wide running the entire 500 km length of the tiger stripes. This is a lower limit. It seems improbably large, and we cite this large number as further evidence that Eq. (2) does not apply to the boiling water on Enceladus.

### 4. Freezing at the liquid–gas interface

In fact, there is a lower limit to the area of the evaporating surface, but it arises from freezing instead of improbably large temperature differences. Freezing and large temperature differences are separate issues. Whether there is a large vapor chamber or not (either side of Fig. 1), water with \( S = 0 \) vented to the exterior would freeze because its value for \( \alpha \) is negative for \( T \leq 4.3 \) °C. Negative \( \alpha \) means that fresh water in this temperature range becomes less dense as it cools. The density is greatest at 4.3 °C. As evaporation proceeds below this value, the column becomes stably stratified with the cold, low-density water on top, and it will freeze. The same is not true for seawater, whose density becomes more dense as it cools—it’s \( \alpha \) is positive. In between there is a salinity lower limit that is necessary to avoid freezing at the liquid–gas interface. The lower limit is not just a simple function of \( \alpha \), as we shall demonstrate.

To the best of our knowledge, there are no published papers that seriously consider freezing of the evaporating liquid. Schmidt et al. (2008) study the flow of gas escaping through channels of variable cross section from a reservoir at the triple point of water, but they do not consider freezing at the interface. Brilliantov et al. (2008) provide a quantitative analysis of two models published by other groups. Brilliantov et al. conclude that neither model matches the data, but they do not discuss freezing of the liquid–gas interface. Postberg et al. (2009) mention the problem of freezing at the interface (Supplementary Information, p. 12), and they assume that having a large gas–liquid interface “a few to tens of square kilometers” in area would solve it. However, they do not justify this assumption. They mention that the highest salinity water will establish at the bottom of the liquid, but they do not pursue the implications for freezing. Ingersoll and Pankine (2010) study the flow of gas that enters the channel either from the bottom or the sides, but they do not make a distinction between liquid or solid sources. All of these models would work if the sources of vapor were sublimating ice, but they do not necessarily work for liquid sources. Here we argue that salinity is crucial for liquid sources, and we find that there is a critical value of salinity, 16.2 g kg\(^{-1} \), below which an evaporating liquid will freeze, regardless of how large its surface area is.

To prevent freezing, the liquid near the surface has to be in contact with a much more massive heat reservoir—either the global ocean or the rocky mantle. This is because the latent heat of the vapor has to come from the internal heat of the liquid, and the former, gram for gram, is much larger than the latter. “In contact” means convection currents circulating in the cracks, with warm, relatively low-density fluid rising to the surface and relatively cold, high-density fluid sinking to the bottom. Salinity plays an important role in generating the high-density fluid.

In a salty liquid, evaporation increases the density of the liquid in two ways. One is by decreasing the temperature, which increases the density if the thermal expansion coefficient \( \alpha \) is positive at the freezing point and above—that is, if the salinity is greater than 24.7 g kg\(^{-1} \). The other way is by increasing the salinity by removal of fresh water. To quantify this, consider a liquid that loses heat per unit mass \( dH \) by evaporation. The heat comes from internal energy of the liquid, so its temperature change is

\[ dT = \text{d}H/c_p, \]

and this causes a fractional change in density \( -\alpha \cdot dT \). The heat goes into latent heat of the vapor, leaving the salt behind, so the fractional change of liquid fresh water is \(-dH/L_v\), which causes an increase in salinity \( dS = dH/L_v \). This causes a fractional change of density \( \rho S \), where

\[ \beta = \frac{(1/\rho)(d\rho/dS)}{dS} \]

is the salinity contraction coefficient. Adding these two effects gives

\[ \frac{d\rho}{\rho} = \left( \alpha + \frac{\beta S c_p}{L_v} \right) \frac{dH}{c_p} = \frac{dH}{c_p} - \frac{dH}{c_p} \frac{dS}{\rho} = \frac{dH}{c_p} \frac{dS}{\rho} = \frac{dH}{c_p} \frac{\rho S}{\rho} \]

Here \( dH \) is the loss of heat, and \( dT \) is the corresponding change of temperature, so \( dT < 0 \) when \( dH > 0 \). Similarly, \( dS > 0 \) when \( dH > 0 \). Therefore density increases when \( \dot{S} > 0 \). The salinity change \( dS \) and the temperature change \( dT \) are not independent, since both are proportional to \( dH \). They are related to each by

\[ c_p dT = -L_v dS/S, \]

which states that the reduction in internal energy is equal to the energy of evaporation. Therefore we can write \( d\rho/\rho \) in terms of \( dT \) or \( dS \); the two forms are entirely equivalent. Eq. (4) serves as a definition of \( \alpha_S \), the effective thermal expansion coefficient. Sample values are given in Table 2. Eq. (4) also serves as a definition of \( \beta \), the effective salinity contraction coefficient, where \( \beta = \frac{\rho S}{\rho S} \).

Here the value of \( \beta \) is 0.796 when \( S \) is measured in g g\(^{-1} \) and 0.796 \( \times 10^{-3} \) when \( S \) is measured in kg kg\(^{-1} \). The fact that \( \beta = 0.796 \) instead of 1.0 is because addition of salt increases both the mass and the volume of the liquid.

The second term in the definition of \( \alpha_S \) appears only when salt water is cooled by evaporation. Cooling by sensible heat transfer and cooling by radiation do not have this term. The value of \( \alpha_S \) is zero at \( S = 16.2 \) g kg\(^{-1} \), which is about half the salinity of seawater but in the 5–20 g kg\(^{-1} \) range of the high-salinity particles in the plumes (Postberg et al., 2009). Water with salinity less than 16.2 g kg\(^{-1} \) will become less dense during evaporative cooling, and will stay on the surface and freeze. Water with salinity greater than this value will become more dense during evaporative cooling, allowing it to sink. But to avoid freezing, the crack has to be wide enough to allow a circulating flow that brings warm water (e.g., at \( T = 0 \) °C, the melting point of the icy walls) up to the surface to replace the cold water (e.g., at \( T < 0 \) °C but above the freezing point of the salt solution) that was formed during the evaporation process. Thus there are two lower bounds, the crack width and the salinity, that must be satisfied simultaneously to prevent freezing.

We estimate these lower bounds with two highly simplified models—one in which the circulating current is laminar and the other in which it is turbulent. We assume 4.7 GW is spread over a crack of width \( \delta \) and length equal to the 500 km length of the tiger stripes. We assume a single linear crack for simplicity. Other
geometries (e.g., cylindrical vents) are possible but they should not affect our qualitative results as long as their smallest horizontal dimensions are of order $\delta$. Thus the evaporative heat flux $q$ is

$$\begin{align*}
q & = 4.7 \times 10^9 \\
& \times 5 \times 10^3 \delta = 9400 \left( \frac{1 \text{ m}}{\delta} \right) \text{W m}^{-2} = \rho \mathcal{C}_p \overline{\omega}
\end{align*}$$

(5)

The last step says that the heat flux is carried in the circulating currents as internal energy, where $u$ is the upward velocity, the overbar is the horizontal average, and $\theta$ is the temperature anomaly— the departure from the horizontal mean.

In the laminar model, we balance the buoyancy force with the viscous force arising from the no-slip boundary condition on the walls of the crack. The hydrostatic pressure gradient is balanced by gravity, and we assume the residual vertical pressure gradient is negligible. The residual vertical force balance is

$$z_a g \theta + \frac{\partial \mathcal{U}}{\partial x} \frac{\partial^2 \mathcal{U}}{\partial y^2} = 0$$

(6)

$$\theta = \theta_0 \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{2 \pi y}{\lambda} \right), \quad u = u_0 \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{2 \pi y}{\lambda} \right)$$

(7)

The horizontal coordinates perpendicular and parallel to the crack are $x$ and $y$, respectively. The sine functions have warm updrafts that span the crack and alternate along the length of the crack with cold downdrafts that also span the crack. The two parts of the term $\frac{\partial}{\partial y} (\rho \mathcal{U} / \partial y)$ are the viscous accelerations arising from no-slip boundary conditions on the walls and friction between the alternating currents, respectively. The width of the crack is $\lambda$, and the wavelength of the updraft–downdraft pattern is $\lambda$, which would be $2 \pi$ if the updrafts and downdrafts had square footprints in the horizontal plane. This is a circulating current; the value of $u_0$ is much larger than the mean upward velocity due to evaporation at the liquid–gas interface.

The term $z_a g \theta$ is the buoyancy—the force per unit mass. An alternate expression for the buoyancy is $-g \rho / \rho$, and they are equivalent according to Eq. (4). The buoyancy consists of two roughly equal terms, a term due to thermal expansion and a term due to change of salinity. The two terms are closely related. They are formed by the same process, and they are conserved on fluid parcels until mixing or interaction with the walls disrupts them. By writing Eq. (6) in terms of $\theta$, we are assuming that the relative contributions of the two terms remain constant throughout the depth of the fluid. Since salinity doesn’t diffuse through the ice, that part of the buoyancy stays with the parcel. But in Eq. (7) we are assuming that the temperature anomaly $\theta$ itself remains constant throughout the depth of the fluid. These are important assumptions, and we deal with them in the third and fourth paragraphs below.

Eq. (6) gives the relation between $\theta$ and $u$, but it does not give its magnitudes. These come from the boundary condition, which says that the circulating currents are carrying the heat flux $q$. We are assuming that the boiling liquid near the liquid–gas interface is an efficient heat exchanger—all the warm upwelling fluid is transferred into dense downwelling fluid by latent heat uptake and fresh water removal. This assumption is based on the observation that boiling stirs the fluid and prevents any thermally insulating, low-density layers from developing at the interface. The usual $x \propto (\Delta T)^{1/3}$ scaling, which follows from Eq. (2), does not apply because that equation does not apply to a boiling liquid.

Applying the heat flux boundary condition to Eqs. (6) and (7), we obtain

$$q = \rho \mathcal{C}_p \overline{\omega} = \frac{1}{4} \rho \mathcal{C}_p u_0 \theta_0 = \frac{z_a g \theta \rho \mathcal{C}_p}{4 \pi^2 \mathcal{V} (1 + 4 \delta^2 / x^2)} (\theta_0)^2$$

(8)

The factor $1/4$ in the third expression comes from averaging $(\sin \theta)$ over one cycle in $x$ and one cycle in $y$ and taking the product. The fourth expression comes from evaluating the second derivative in the viscous acceleration in (6) and using the result, a relation between $\theta_0$ and $u_0$ to eliminate the latter in the expression for the flux. Eq. (8) allows us to estimate the peak temperature difference $2 \theta_0$ between the warm updraft and the cold downdraft. Using the tables referenced in Tables 1 and 2 for salinities 5 of 21 and 28 g kg$^{-1}$, and with $q$ given in Eq. (5), we obtain

$$\theta_{0|\lambda=21} = \left[ \frac{2 \pi \delta (1 + 4 \delta^2 / x^2)}{2 z_a g \theta \rho \mathcal{C}_p} \right]^{1/2} = 0.270 \text{ °C} \left[ 1 + \frac{4 \delta^2}{x^2} \right]^{1/2} \left( \frac{1 \text{ m}}{\delta} \right)^{3/2}$$

(9a)

$$\theta_{0|\lambda=28} = \left[ \frac{2 \pi \delta (1 + 4 \delta^2 / x^2)}{2 z_a g \theta \rho \mathcal{C}_p} \right]^{1/2} = 0.173 \text{ °C} \left[ 1 + \frac{4 \delta^2}{x^2} \right]^{1/2} \left( \frac{1 \text{ m}}{\delta} \right)^{3/2}$$

(9b)

Eqs. (9a) and (9b) give the peak temperature anomalies—positive for the upwelling fluid and negative for the downwelling fluid—that would balance the heat flux. The required temperature anomalies are proportional to $\delta^{-3/2}$, because both the viscous force and the heat flux are larger for a narrower crack. On the other hand, for $\delta \geq 1 \text{ m}$ the temperature anomalies are less than the difference between the freezing point of fresh water and the freezing point of salt water as given in Table 2 for the two salinities. Thus the surface will not freeze if the upwelling water is 0 °C. This is the most likely temperature if the water is warm at the bottom of the crack and the walls are made of fresh water ice. For given width $\delta$, having the upward and downward currents be widely separated, such that $4 \delta^2 \gg \delta^2$, gives the minimum $\theta_0$ and therefore the most favorable condition for the circulating currents.

A separate issue is whether the buoyancy acceleration $z_a g \theta$, given in Eq. (6), is constant throughout the depth of the fluid. There are two ways it might vary. The first is through thermobaricity, which involves the increase of $x$ with pressure and hence with depth. In Earth’s oceans this leads to a downward acceleration of cold, sinking parcels, whose negative buoyancy increases as they sink (e.g., Ingersoll, 2005). This will happen on Enceladus as well, although the lower gravity means that the pressure difference from top to bottom is less than on Earth, and the effect is smaller. Nevertheless, thermobaricity will have a positive effect on the convection currents, increasing their strength and reducing the required temperature differences given in Eq. (9). Because the effect is positive, it cannot destroy the circulation, and we focus instead on processes that have a negative effect.

Horizontal conduction through the icy walls, from the updraft column to the downdraft column, could reduce the thermal contrast and destroy the buoyancy. In Eq. (7) we have assumed that peak temperatures in the liquid vary with horizontal distance along the crack as $\theta_0 \sin(2 \pi y / \lambda)$. With this as a boundary condition,
steady-state diffusion within the ice leads to a solution of LaPlace’s equation for the temperature distribution of the form \( \theta(x) = \exp(-2\pi x/\lambda) \sin(2\pi y/\lambda) \), where \( \lambda \) is the horizontal distance into the ice from the wall. The heat flux \( F \) into the ice at the wall is therefore \( k_{\text{ice}}(2\pi x/\lambda)\sin(2\pi y/\lambda) \), where \( k_{\text{ice}} = 2.20 \text{ W m}^{-1} \text{ K}^{-1} \) is the thermal conductivity of ice obtained from the Engineering Toolbox referenced in Table 1. Since there are two walls, the heat lost (power per unit volume) by a water parcel moving upward at velocity \( u_b \) is 2\( F/\delta \), which is equal to \( -\rho C_p u_b \partial \theta_b / \partial z = 2F/\delta \), or

\[
\frac{1}{2} \frac{\partial \theta_b}{\partial z} = -\frac{F}{2\delta} = \frac{k_{\text{ice}} \theta_b}{2\delta^2} \frac{2\pi}{\lambda} = -0.01 \text{ km}^{-1} \left( \frac{1 \text{ m}}{\lambda} \right) \left( \frac{1 \text{ m}}{\delta} \right)^{3/2} \quad (\text{Eq. 10})
\]

In obtaining this result we used Eq. (5) for \( q \delta \). It is a conservative result because we used the larger of the two values for \( \theta_b \) in Eq. (9), 0.270 °C. Also, we ignored the dominant contribution of salinity to the buoyancy, as shown in the next paragraph. Finally, we have some margin in Eq. (9) because those temperature differences are comfortably smaller than the differences between the freezing points of salt water and 0 °C, as given in Table 2. Eq. (10) says that if \( \lambda = 10 \text{ m} \) and \( \delta = 1 \text{ m} \), such that updrafts and downdrafts each occupy a 5 m × 1 m footprint, the fluid loses 1% of its thermal buoyancy when it travels upward by 1 km. The loss is inversely proportional to \( \lambda \) because less heat is conducted through the ice when the distance is large. Despite considerable uncertainty in Eq. (10), it seems likely that the buoyancy that drives the circulating current can exist over a column 25 km deep (McKinnon, 2015). This circulating current and the salinity are what keep the water at the liquid–gas interface from freezing.

Before presenting our turbulent model, we give the values of other key quantities that follow from the values of \( \theta_b \) in Eq. (9). We suppress the dependence on crack width \( \delta \) and assume \( \delta = 1 \text{ m} \), but only for these two paragraphs. The fractional density difference \( d\rho/\rho = -\gamma \theta_b - \beta S_\theta \) is 5.08 × 10^{-6} for \( S = 21 \text{ g kg}^{-1} \) and 7.87 × 10^{-6} cm s^{-1} for \( S = 28 \text{ g kg}^{-1} \). Here \( S_\theta \) is the salinity anomaly—the counterpart of \( \theta_b \) in Eq. (6)—one-half the peak-to-peak amplitude of the salinity variation in the horizontal plane. For \( S = 21 \text{ g kg}^{-1} \), the individual contributions \( -\gamma \theta_b \) and \( \beta S_\theta \) to \( d\rho/\rho \) are \( -2.40 \times 10^{-6} \) and \( 7.48 \times 10^{-6} \), respectively. For \( S = 28 \text{ g kg}^{-1} \), the individual contributions \( -\gamma \theta_b \) and \( \beta S_\theta \) are \( 1.52 \times 10^{-6} \) and \( 6.35 \times 10^{-6} \), respectively. For both salinities \( S_\theta \) is the dominant term, and for \( S = 21 \text{ g kg}^{-1} \) it has to overcome the \( \gamma \theta_b \) term, which is negative. Since \( \beta = 0.796 \times 10^{-3} \text{ (g kg}^{-1} \text{)}^{-1} \), the fractional variation \( S_\theta /\delta S \) are 0.448 × 10^{-3} for \( S = 21 \text{ g kg}^{-1} \) and 0.285 × 10^{-3} for \( S = 28 \text{ g kg}^{-1} \).

The vertical velocity \( u_0 \) is given in Eq. (8) as \( 4q/\rho \rho C_p/\delta \), which is 3.55 cm s^{-1} for \( S = 21 \text{ g kg}^{-1} \) and 5.55 cm s^{-1} for \( S = 28 \text{ g kg}^{-1} \). If we assume the upward–flowing current turns when it reaches the liquid–gas interface and flows horizontally parallel to the crack for a distance \( D \), one can solve for the depth \( h \) from which it gives up its energy \( C_p \theta_0 \) before returning to the heat source below. Consider a rectangular box of width \( \delta \), horizontal distance \( D \), and depth \( h \). The difference in power entering and leaving at the two ends is \( \rho \rho C_p u_0 \delta \), and the power leaving the top is \( q \delta \). Equating these two quantities and using Eq. (8) gives \( hD = \delta \). Thus \( h \) could be the depth of the boiling zone and \( D \) could a distance along the crack twice as great as the crack width. The important point is that the circulating current has time to give up its heat and gain salinity while passing through the boiling zone.

For the turbulent model we balance the buoyancy forces with drag forces on the walls. Each of the currents—upward and downward—is treated as a plane Poiseuille flow in a channel of width \( \delta \). Orlandi et al. (2015) give a summary of experiments and numerical models that relate the stress \( \tau \) on each wall to the Reynolds number \( Re_b \) of the bulk flow—the mean velocity \( u_b \) of one of the currents:

\[
\tau = C_f \rho u_b^2 \quad C_f = 0.0725 \text{ Re}^{-1/4} \quad \text{Re}_b = \delta u_b / \nu \quad (11)
\]

The friction factor \( C_f \) is a weak function of the bulk velocity through \( Re_b \). Then Eq. (6) becomes

\[
\alpha_\theta \delta u_b = 2\tau / \rho = 2C_f u_b^2 \quad (12)
\]

The factor \( \delta \) arises from integrating the buoyancy force across the current, which spans the width of the crack. The factors \( 2\tau \) and \( C_f \) arise because the current is feeling the friction force on two sides, \( \theta_b \) and \( u_b \) are the average for the current. Each current is carrying heat flux \( \rho C_p \theta_b u_b \) over the channel, so that is also the average heat flux for the whole channel. The \( 1/4 \) power in Eq. (11) leads to some strange-looking exponents, and the equivalent to Eq. (9) becomes

\[
\theta_b \mid_{\delta=21} = \left( \frac{9400}{\rho C_p} \right)^{7/11} \left( \frac{\nu}{\gamma} \right)^{1/11} \left( \frac{0.0725}{\alpha_\theta g/2} \right)^{4/11} \left( \frac{1}{\delta} \right)^{12/11} \quad (13a)
\]

\[
= \left( 0.278 \text{ °C} \right) \left( \frac{1}{\delta} \right)^{12/11} \quad (13a)
\]

\[
\theta_b \mid_{\delta=28} = \left( \frac{9400}{\rho C_p} \right)^{7/11} \left( \frac{\nu}{\gamma} \right)^{1/11} \left( \frac{0.0725}{\alpha_\theta g/2} \right)^{4/11} \left( \frac{1}{\delta} \right)^{12/11} \quad (13b)
\]

\[
= \left( 0.201 \text{ °C} \right) \left( \frac{1}{\delta} \right)^{12/11} \quad (13b)
\]

Eq. (13a) is for \( S = 21 \text{ g kg}^{-1} \) and Eq. (13b) is for \( S = 28 \text{ g kg}^{-1} \). Again we are assuming the currents alternate along the length of the crack.

The laminar and turbulent models give almost the same numerical results, as seen by comparing Eqs. (9) and (13). This is because the velocity is a few cm s^{-1} and the corresponding Reynolds number is \( \sim 10^4 \), which puts the flow in the transition regime (Orlandi et al., 2015) between the laminar and turbulent limits. For the two cases the conclusion is the same: For crack widths \( \delta \) of 1 m or more, the required temperature differences are less than the differences between the melting point of fresh water and that of salt water ice. The implication is that the water in the cracks will not freeze if the salinities are greater than about 20 g kg^{-1} and the crack widths are greater than about 1 m. The salinities can’t be much less than 20 g kg^{-1}, however, because at 16.2 g kg^{-1} the effective thermal expansion coefficient \( \chi \) becomes negative and the cold water will stay on the surface and freeze.

5. Freezing on the walls

Freezing of the liquid as it flows upward through the crack releases latent heat of fusion, and the rate of freezing is controlled by the rate at which this heat is conducted through the ice to the exterior surface, where it is radiated to space. The question is whether the water loses a large or small fraction of its mass as it flows from bottom to top. To answer this question we use the same parameterized form of heat transfer into the ice as in our model of the vapor in Part 1. If the exterior surface were at constant temperature \( T_w \) and the walls of the crack were at constant temperature \( T_{ew} \), the temperature in the ice would be

\[
T(x, z_d) = T_s + (T_w - T_{ew}) \frac{2}{\pi} \tan^{-1} \left( \frac{x z_d}{\delta} \right) \quad (14)
\]

Here \( x \) is horizontal distance away from the wall and \( z_d \) is depth increasing downward from the exterior surface. The solution looks much like Fig. 2 of Abramov and Spencer (2009). According to Eq. (14) the heat flux into the wall is
We have argued that liquid water below the surface of Enceladus is boiling if it is vented to space and is acting as the source of the plumes. To avoid freezing at the liquid–gas interface, the liquid must have a salinity of at least 20 g kg$^{-1}$, which is roughly half the salinity of seawater. To supply the observed power, which is emitted both as latent heat and as radiation, the evaporating area must be comparable to a crack 1 m wide extending for the 500 km length of the tiger stripes. There is no need for large vapor chambers.

In reaching these conclusions, we have assumed that the water at the evaporating surface cannot be warmer than 0 °C, since the walls of the water-filled cracks are presumably made of ice. The salinity constraint would disappear if the liquid water were resting in depressions in hot rocks inside Enceladus, but then one would have to explain how the icy shell can rest on the hot rocks and not melt. More likely, the rock–ice interface and any liquid in between are all close to 0 °C. Then the depression of the freezing point due to salinity, combined with convection currents in the liquid-filled cracks, are what keeps the liquid–gas interface from freezing.

6. Discussion and conclusions

The convection currents have to operate within a narrow range of temperatures, 0 °C at the bottom and −1 °C or −2 °C at the top, depending on the salinity. Driving convection at these temperatures is complicated because the coefficient of thermal expansion is small, but this is mitigated by the increase of salinity that occurs when salt water evaporates. The remaining salt water can sink, and this brings up slightly warmer water at 0 °C, which prevents freezing if the water is salty enough. We proposed two conceptual models of the convection currents—one laminar and the other turbulent—but we did not present a full solution of the Navier–Stokes equations. Nor could we, since the flow is turbulent and the boundary conditions are uncertain. Despite the uncertainties, we offer the models as constraints—lower bounds—on the salinity of the liquid and the widths of the cracks.

Our lower bound on the salinity is consistent with the estimate of Postberg et al. (2009), who find particles with salinity in the range 5–20 g kg$^{-1}$. They argue that the salinity of the Enceladus ocean may be higher than these values. We suggest that Enceladus might be unique among the icy satellites with liquid water oceans, in that its salinity is high enough to prevent freezing of the liquid–gas interface. Other satellite oceans might not be so lucky and therefore don’t have plumes.

Controlled boiling might help to explain the particle size distribution of the Enceladus plumes. Boiling involves bubbles breaking at the surface, and this generates spray. The subject is of interest to oceanographers (e.g., de Leeuw et al., 2011; Veron, 2015) because spray droplets can linger in the atmosphere for days, becoming cloud condensation nuclei and altering atmospheric chemistry and the radiation budget. There are two mechanisms for launching droplets. Film droplets form when the thin upper surface of a bubble cavity collapses; several droplets are formed in a vertical column with radii in the range ~0.01 μm and 1–2 μm. Jet droplets form when the bubble cavity collapses; several droplets are formed in a vertical column with radii in the range ~1–50 μm (de Leeuw et al., 2011; Veron, 2015). These sizes overlap with the particles in the Enceladus plumes (Postberg et al., 2011), and a detailed comparison would be worthwhile.

Liquid water rising in meter-sized cracks and getting within ~2 km of the exterior surface may help to explain the low-temperature radiation in km-wide swaths straddling the tiger stripes. This radiation has been difficult to measure, and probably accounts for the uncertainty of estimates of the total internal power. The high-temperature component is around 4.2 GW (Spencer et al., 2006, 2013; Goguen et al., 2013), and is relatively...
easy to measure. Estimates of the total power, including the low-temperature component, have ranged up to 15.8 GW (Howett et al., 2011; Spencer and Nimmo, 2013). This power is hard to separate from re-radiated sunlight. Presumably this ambiguity will diminish now that the South Pole is in darkness, but the transition will not be instantaneous and the stored solar energy may continue to come out. Continued measurements of the South Pole would certainly be worthwhile.

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