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Excited charmed baryon decays and their implications for fragmentation parameters

John K. Elwood
California Institute of Technology, Pasadena, California 91125
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The production of the excited charmed baryon doublet Λ_c^* via fragmentation is studied. An analysis of the subsequent hadronic decays of the doublet within the framework of heavy hadron chiral perturbation theory produces expressions for both the angular distribution of the decay products and the polarization of the final state heavy baryon in terms of various nonperturbative fragmentation parameters. Future experimental investigation of this system will determine these parameters. In addition, recent experimental results are shown to fix one of the parameters in the heavy hadron chiral Lagrangian. [S0556-2821(96)02409-5]

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I. INTRODUCTION

The production of a heavy quark at high energy via some hard process is a relatively well understood phenomenon, as we may bring the full apparatus of perturbative QCD to bear on the problem. Less well understood is the subsequent fragmentation of the heavy quark to form heavy mesons and baryons. It is the dynamics of this process that we propose to address in this paper. We imagine that a heavy quark with mass $m_Q \gg \Lambda_{\text{QCD}}$ is produced on very short time scales in a hard reaction. It then travels out along the axis of fragmentation and hadronizes on a much longer time scale, at distances of order $1/\Lambda_{\text{QCD}}$. The fractional change in the heavy quark's velocity is therefore of order $(\Lambda_{\text{QCD}}/m_Q)$, and vanishes at leading order in the heavy quark limit. Likewise, the heavy quark spin couples to the light degrees of freedom via the color magnetic moment operator

$$\frac{1}{m_Q}\overline{h_v^{(Q)}}\sigma_{\mu\nu}G^{a\mu\nu}T^a h_v^{(Q)}, \tag{1.1}$$

which again vanishes in the heavy limit. We may therefore view the initial fragmentation process as leaving the heavy quark velocity and spin unchanged. Notice that, in this limit, the dynamics are also blind to the mass of the heavy quark, which therefore acts as a static color source in its interactions with the light degrees of freedom.

This simple result may not apply to the ultimate products of the strong fragmentation process, however, as was pointed out by Falk and Peskin [1]. Specifically, the polarization of the final state heavy baryons and mesons may not be determined solely by the heavy quark spin, but may depend in addition on the spin of the light degrees of freedom involved in the fragmentation process. This is the case when the initial fragmentation products decay to lower energy heavy baryons and mesons on a time scale long enough to allow interaction between the heavy quark spin and that of the light degrees of freedom. We will find that this is indeed the case in the Λ_c^* system.

In this situation, one must know something about the spin of the light degrees of freedom in order to proceed further. The parity invariance of the strong interactions, coupled with heavy quark spin symmetry, demands that formation of light degrees of freedom with spin j depends only on the magnitude of the projection of j onto the axis of fragmentation, and not on its sign. That is, transverse may be preferred to longitudinal, but forward may not be preferred to back. Further, the light system may prefer to invest its angular momentum in orbital channels as opposed to spin channels. These preferences are catalogued by a set of fragmentation parameters: A and ω_1, defined in [1], and B and $\widetilde{\omega}_1$, defined in the following section.

Let us consider a fragmentation process in which light degrees of freedom of spin j are produced. They then associate with the heavy quark spin $s=\frac{1}{2}$ to form a doublet of total spin $J=j\pm\frac{1}{2}$. Two paths now lie open. The doublet (the two members of which have the same decay rate in the heavy quark limit) may decay rapidly enough that heavy quark spin flip processes have no time to occur. Then the doublet states decay coherently, the heavy quark retains its initial polarization in the final states, and the process begins anew with the decay products. On the other hand, heavy quark spin flip processes may have time to occur, in which case the doublet states decay incoherently, and the heavy quark polarization is altered. The two parameters responsible for determining which regime we are in are the total decay rate out of the doublet, Γ, and the mass splitting between the doublet states, Δ. The splitting Δ vanishes in the heavy quark limit, and is of the order of the rate for heavy quark spin flip processes within the doublet. We therefore expect that the situation $\Gamma \gg \Delta$ produces overlapping resonances which decay coherently out of the multiplet, and that the opposite extreme $\Gamma \ll \Delta$ allows for incoherent decays and the influence of the spin of the light degrees of freedom.

II. THE CHARMED BARYON SYSTEM

In the charmed baryon system, the ground state is obtained by putting the light diquark in an antisymmetric $I=S=0$ state with spin-parity $j^P=0^+$. This yields the $J^P=\frac{1}{2}^+$ baryon Λ_c^+, with mass 2285 MeV. Alternatively, the light quarks may form a symmetric $I=S=1$ state with spin-parity $j^P=1^+$. The light spin then couples to that of the heavy quark to produce the symmetric $J^P=(\frac{3}{2}^+,\frac{1}{2}^+)$ doublet $(\Sigma_c^{*(0,+,++)},\Sigma_c^{(0,+,++)})$ with mass (2530 MeV, 2453 MeV). Fragmentation through the $\Sigma_c^{(*)}$ system has already been considered in [1]; we concern ourselves here with the J^P

$=(\frac{3}{2}^-,\frac{1}{2}^-)$ doublet $(\Lambda_{c1}^*,\Lambda_{c1})$ that results when the light diquark is an $I=S=0$ state with a single unit of orbital angular momentum. Allowing the light quarks to have both spin and orbital angular momentum produces a tremendous number of states, none of which have been observed to date. We ignore such states in the analysis that follows.

The fragmentation parameters A,B,ω_1, and $\widetilde{\omega}_1$, may now be defined. A is taken to be the relative probability of producing any of the nine $I=S=1$, $j^P=1^+$ diquark states during fragmentation relative to that of producing the $I=S=0$, $j^P=0^+$ ground state. B is similarly the probability for producing any of the three $I=S=0$, $j^P=1^-$ diquark states relative to ground state production. The parameters ω_1 and $\widetilde{\omega}_1$, on the other hand, encode the orientation of the light diquark angular momentum. The various helicity states of the spin-parity 1^+ and 1^- diquarks are populated with the probabilities

$$P[1]=P[-1]=\frac{\omega_1}{2},\quad P[0]=1-\omega_1\quad \text{for } j^P=1^+, \tag{2.1}$$

and

$$P[1]=P[-1]=\frac{\widetilde{\omega}_1}{2},\quad P[0]=1-\widetilde{\omega}_1\quad \text{for } j^P=1^-. \tag{2.2}$$

The analysis of the excited D system in [1] has already indicated that $\omega_{3/2}$, the analog of ω_1 for the light degrees of freedom in the meson sector, is likely close to zero. One might also anticipate, therefore, that ω_1 would be close to zero. We will concentrate on $\widetilde{\omega}_1$ most heavily in what follows.

The masses of the Λ_{c1}^* and Λ_{c1} are naively expected to be split by $\sim(\Lambda_{\rm QCD}^2/m_c)\simeq 30$ MeV, in fortuitously close agreement with the recently measured values $M_{\Lambda_{c1}^*}=2625$ MeV and $M_{\Lambda_{c1}}=2593$ MeV [2]. Decay of the Λ_{c1}^* to Λ_{c1} via pion emission is thus kinematically forbidden, and the corresponding electromagnetic transition is very slow compared with strong decays out of the doublet. Indeed, the dominant decay mode of both Λ_{c1}^* and Λ_{c1} is to Λ_c via pion emission. As both $(\Lambda_{c1}^*,\Lambda_{c1})$ and Λ_c are $I=0$ states, single pion emission is forbidden by isospin conservation, and the dominant modes are $\Lambda_{c1}^*\to\Lambda_c\pi\pi$ and $\Lambda_{c1}\to\Lambda_c\pi\pi$. The mass differences $(M_{\Lambda_{c1}^*}-M_{\Lambda_c})=340$ MeV and $(M_{\Lambda_{c1}}-M_{\Lambda_c})=308$ MeV are very close to threshold, and the pions produced will be soft. We therefore expect the decays to be accurately described by heavy hadron chiral perturbation theory.

The CLEO Collaboration recently measured the Λ_{c1} width to be $\Gamma_{\Lambda_{c1}}=3.9^{+1.4+2.0}_{-1.2-1.0}$ MeV, and placed a new upper bound on the Λ_{c1}^* width: $\Gamma_{\Lambda_{c1}^*}<1.9$ MeV [2]. It is an interesting breakdown of the naive heavy quark approximation that these rates are significantly different. The explanation is that, at leading order in the heavy hadron chiral Lagrangian, Λ_{c1}^* is connected to Λ_c only via an intermediate Σ_c^*, whereas Λ_{c1} is connected via an intermediate Σ_c. Kinematics allows the Σ_c, but not the Σ_c^*, to go on shell. The Λ_{c1} thus enjoys a resonant amplification of its decay rate. We also note that the rates above place us securely in the regime $\Gamma\ll\Delta$, so that we anticipate interaction of the heavy quark spin with the light degrees of freedom in decays to the Λ_c. This will allow us to shed some light on the parameter $\widetilde{\omega}_1$. In the following section, we provide a brief review of heavy hadron chiral perturbation theory before tackling the $(\Lambda_{c1}^*,\Lambda_{c1})$ decays.

III. HEAVY HADRON CHIRAL PERTURBATION THEORY

Heavy hadron chiral perturbation theory incorporates aspects of both ordinary chiral perturbation theory and the heavy quark effective theory, and describes the low energy interactions between hadrons containing a heavy quark and the light pseudo Goldstone bosons. It has been discussed previously in a number of papers [3].

For definiteness we consider the charmed baryon system. Members of the ground state $J^P=\frac{1}{2}^+$ antitriplet are destroyed by the velocity dependent Dirac fields $\mathcal{T}_i(v)$, where

$$\mathcal{T}_1=\Xi_c^0,\quad \mathcal{T}_2=-\Xi_c^+,\quad \mathcal{T}_3=\Lambda_c^+. \tag{3.1}$$

The symmetric $J^P=\frac{1}{2}^+$ states are destroyed by the Dirac fields $S^{ij}(v)$ with components

$$S^{11}=\Sigma_c^{++},\quad S^{12}=\sqrt{\tfrac{1}{2}}\Sigma_c^+,\quad S^{22}=\Sigma_c^0,\quad S^{13}=\sqrt{\tfrac{1}{2}}\Xi_c^{+\prime},$$
$$S^{23}=\sqrt{\tfrac{1}{2}}\Xi_c^{0\prime},\quad S^{33}=\Omega_c^0, \tag{3.2}$$

and their symmetric $J^P=\frac{3}{2}^+$ counterparts by the corresponding Rarita-Schwinger fields $S_\mu^{*ij}(v)$. Finally, we define Dirac and Rarita-Schwinger fields $R_i(v)$ and $R_{\mu i}^*(v)$ to annihilate the $J^P=\frac{1}{2}^-$ and $J^P=\frac{3}{2}^-$ excited antitriplet states respectively. In our analysis the components of interest will be $R_3=\Lambda_{c1}$ and $R_{\mu 3}^*=\Lambda_{c1,\mu}^*$.

As the heavy quark mass goes to infinity, the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ members of the sextet and excited antitriplet multiplets become degenerate. It is then useful to combine them to form the superfields $\mathcal{R}_{\mu i}$ and $\mathcal{S}_\mu^{ij}$, defined by

$$\mathcal{R}_{\mu i}=\sqrt{\tfrac{1}{3}}(\gamma_\mu+v_\mu)\gamma^5 R_i+R_{\mu i}^*, \tag{3.3}$$

$$\mathcal{S}_\mu^{ij}=\sqrt{\tfrac{1}{3}}(\gamma_\mu+v_\mu)\gamma^5 S^{ij}+S_\mu^{*ij}. \tag{3.4}$$

If we are to discuss decay by π emission, we must also incorporate the pseudo-Goldstone boson octet into our Lagrangian. The Goldstone bosons are a product of the spontaneous breakdown of the chiral flavor symmetry $SU(3)_L\times SU(3)_R$ to $SU(3)_V$, its diagonal subgroup. They appear in the octet

$$M=\sum_a \pi^a T^a$$
$$=\sqrt{\tfrac{1}{2}}\begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \overline{K^0} & -2\eta/\sqrt{6}\end{pmatrix}, \tag{3.5}$$

and are conveniently incorporated into the Lagrangian via the dimensionless fields $\Sigma \equiv e^{2iM/f}$ and $\xi \equiv e^{iM/f}$, where $f=f_\pi = 93$ MeV, the pion decay constant, at lowest order in chiral perturbation theory.

The goal is to combine these fields to produce a Lorentz invariant, parity even, heavy quark spin symmetric, and light chiral invariant Lagrangian. To this end, we now assemble various transformation properties of the fields. Under parity, P, the superfields transform as

$$P\mathcal{R}_\mu(\vec{r},t)P^{-1} = \gamma_0 \mathcal{R}^\mu(-\vec{r},t), \tag{3.6}$$

$$P\mathcal{S}_\mu(\vec{r},t)P^{-1} = -\gamma_0 \mathcal{S}^\mu(-\vec{r},t), \tag{3.7}$$

$$P\mathcal{T}(\vec{r},t)P^{-1} = \gamma_0 \mathcal{T}(-\vec{r},t). \tag{3.8}$$

They also obey the constraints

$$v^\mu \mathcal{R}_\mu = v^\mu \mathcal{S}_\mu = 0; \quad \not{v}\mathcal{R}_\mu = \mathcal{R}_\mu;$$
$$\not{v}\mathcal{S}_\mu = \mathcal{S}_\mu; \quad \not{v}\mathcal{T} = \mathcal{T}. \tag{3.9}$$

The Rarita-Schwinger components obey the additional constraints

$$\gamma^\mu \mathcal{R}^*_{\mu i} = \gamma^\mu \mathcal{S}^{*ij}_\mu = 0. \tag{3.10}$$

We are also interested in how the various fields transform under chiral SU(3). The Σ and ξ fields obey

$$\Sigma \to L\Sigma R^\dagger, \tag{3.11}$$

$$\xi \to L\xi U^\dagger(x) = U(x)\xi R^\dagger, \tag{3.12}$$

where L and R are global SU(3) matrices, and $U(x)$ is a local member of $\mathrm{SU}(3)_V$. If we further define the vector and axial vector fields

$$V^\mu = \tfrac{1}{2}[\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger], \tag{3.13}$$

$$A^\mu = \tfrac{i}{2}[\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger], \tag{3.14}$$

we find that, under chiral SU(3),

$$V^\mu \to UV^\mu U^\dagger + U(\partial^\mu U^\dagger), \tag{3.15}$$

$$A^\mu \to UA^\mu U^\dagger. \tag{3.16}$$

The only constraint imposed on the heavy fields is that they transform according to the appropriate sextet or antitriplet representation under transformations of the $\mathrm{SU}(3)_V$ subgroup.

There remains one final symmetry to aid us in constructing our Lagrangian, and that is symmetry under reparametrization of the heavy field velocity. The momentum of a heavy hadron is written $p = Mv + k$, where k is termed the residual momentum of the hadron. If we make the following shifts in v and k:

$$v \to v + \epsilon/M; \quad k \to k - \epsilon, \tag{3.17}$$

with $v\cdot\epsilon = 0$, then $p \to p$ and $v^2 \to v^2 + O(1/M^2)$. Therefore, if we are working only to leading order in the $(1/M)$ expansion, we demand that our Lagrangian be invariant under such a transformation. The corresponding shifts induced in the fields are [4]

$$\delta\mathcal{R}_\mu = \frac{\not{\epsilon}}{2M}\mathcal{R}_\mu - \frac{\epsilon^\nu \mathcal{R}_\nu}{M} v_\mu, \tag{3.18}$$

$$\delta\mathcal{S}_\mu = \frac{\not{\epsilon}}{2M}\mathcal{S}_\mu - \frac{\epsilon^\nu \mathcal{S}_\nu}{M} v_\mu, \tag{3.19}$$

$$\delta\mathcal{T} = \frac{\not{\epsilon}}{2M}\mathcal{T}. \tag{3.20}$$

Invariance of the Lagrangian under these shifts further restricts the terms that may appear, and leaves us with the following form for the most general Lorentz invariant, parity even, heavy quark spin symmetric, and light chiral invariant Lagrangian:

$$\begin{aligned}
\mathcal{L}_v^{(0)} = \{ & \overline{\mathcal{R}}^i_\mu(-iv\cdot\mathcal{D} + \Delta M_{\mathcal{R}})\mathcal{R}^\mu_i \\
& + \overline{\mathcal{S}}^\mu_{ij}(-iv\cdot\mathcal{D} + \Delta M_{\mathcal{S}})\mathcal{S}^{ij}_\mu + \overline{\mathcal{T}}^i iv\cdot\mathcal{D}\mathcal{T}_i \\
& + ig_1 \epsilon_{\mu\nu\sigma\lambda}\overline{\mathcal{S}}^\mu_{ik} v^\nu (A^\sigma)^i_j (\mathcal{S}^\lambda)^{jk} \\
& + ig_2 \epsilon_{\mu\nu\sigma\lambda}\overline{\mathcal{R}}^{\mu i} v^\nu (A^\sigma)^i_j (\mathcal{R}^\lambda)_j \\
& + h_1[\epsilon_{ijk}\overline{\mathcal{T}}^i (A^\mu)^j_l \mathcal{S}^{kl}_\mu + \epsilon^{ijk}\overline{\mathcal{S}}^\mu_{kl}(A_\mu)^l_j \mathcal{T}_i] \\
& + h_2[\epsilon_{ijk}\overline{\mathcal{R}}^{\mu i} v\cdot A^j_l \mathcal{S}^{kl}_\mu + \epsilon^{ijk}\overline{\mathcal{S}}^\mu_{kl} v\cdot A^l_j \mathcal{R}_{\mu i}]\},
\end{aligned} \tag{3.21}$$

where $\Delta M_{\mathcal{R}} = M_{\mathcal{R}} - M_{\mathcal{T}}$ is the mass splitting between the excited and ground state antitriplets, and $\Delta M_{\mathcal{S}} = M_{\mathcal{S}} - M_{\mathcal{T}}$ is the corresponding splitting between the sextet and the ground state antitriplet.

In defining the velocity dependent heavy fields which appear above, a common mass must be scaled out of all heavy fields

$$H = e^{-iMv\cdot x}H_v, \tag{3.22}$$

despite the different masses of the various heavy baryons. In the above analysis we have chosen $M = M_{\Lambda_c}$.

It is also instructive at this point to examine the term proportional to h_2, which allows single π transitions between the excited antitriplet and sextet states. This term induces only S-wave transitions, although naive angular momentum and parity arguments would allow D-wave transitions as well. The D-wave transitions are induced by a higher dimension operator which is therefore suppressed by further powers of M and does not appear at leading order in the heavy hadron Lagrangian. This absence of D-wave transitions simplifies the way in which the π distributions depend on $\tilde{\omega}_1$ in the $\Lambda_{c1}^{(*)}$ decay process. Finally, we comment quickly on the errors induced by keeping only leading order terms. The relevant expansion parameter in our analyses is (p_π/M), so that we expect our results to be valid to $\sim(200/2285) \simeq 10\%$.

IV. THE PARAMETER h_2

The term proportional to h_2 in the leading order Lagrangian is responsible for the tree-level decay $\Lambda_{c1}\to\Sigma_c\pi$, the rate for which is easily calculated to be

$$\Gamma(\Lambda_{c1}\to\Sigma_c\pi)=\frac{|h_2|^2}{4\pi f^2}\frac{M_{\Sigma_c}}{M_{\Lambda_{c1}}}(M_{\Lambda_{c1}}-M_{\Sigma_c})^2 \times\sqrt{(M_{\Lambda_{c1}}-M_{\Sigma_c})^2-m_\pi^2}, \qquad (4.1)$$

as was done previously in [4]. The Σ_c may then decay to $\Lambda_c\pi$ through the term proportional to h_1, producing a decay rate $\Gamma(\Lambda_{c1}\to\Lambda_c\pi\pi)$ that scales like the combination $|h_1|^2|h_2|^2$. A quick calculation allows us to express $|h_1|^2$ in terms of the partial width $\Gamma(\Sigma_c\to\Lambda_c\pi)$,

$$\Gamma(\Sigma_c\to\Lambda_c\pi)=\frac{|h_1|^2}{12\pi f^2}\frac{M_{\Lambda_c}}{M_{\Sigma_c}}[(M_{\Sigma_c}-M_{\Lambda_c})^2-m_\pi^2]^{3/2}, \qquad (4.2)$$

which is by far the dominant contribution to Γ_{Σ_c}. We may therefore view $\Gamma(\Lambda_{c1}\to\Lambda_c\pi\pi)$ as a function of h_2 and Γ_{Σ_c}. This decay is dominated by the pole region where Σ_c is close to being on shell, and its rate coincides with that for $\Lambda_{c1}\to\Sigma_c\pi$ as $\Gamma_{\Sigma_c}\to 0$. In this narrow width approximation, we obtain

$$\Gamma(\Lambda_{c1}\to\Lambda_c\pi^+\pi^-)=4.6|h_2|^2\ \text{MeV}. \qquad (4.3)$$

The result is modified slightly if we allow the Σ_c to have a finite width. The Σ_c is not expected to have a width greater than a few MeV. Setting $\Gamma_{\Sigma_c}=2$ MeV, we find

$$\Gamma(\Lambda_{c1}\to\Lambda_c\pi^+\pi^-)=4.2|h_2|^2\ \text{MeV}. \qquad (4.4)$$

Comparison with the CLEO measurement [2]

$$\Gamma(\Lambda_{c1}\to\Lambda_c\pi^+\pi^-)=3.9^{+1.4+2.0}_{-1.2-1.0}\ \text{MeV} \qquad (4.5)$$

then yields a central value of $|h_2|\simeq 0.9$ in the narrow width approximation, or $|h_2|\simeq 1.0$ with $\Gamma_{\Sigma_c}=2$ MeV.

V. PRODUCTION AND DECAY OF Λ_{c1} AND Λ_{c1}^*

The probabilities for fragmentation to the Λ_{c1} and Λ_{c1}^* states of various helicities may be expressed in terms of the parameters $\widetilde{\omega}_1$ and B once the initial polarization of the heavy quark is given. For simplicity, we assume that the initial charm quark is completely left-hand polarized in the analysis that follows. With this assumption, the relative populations of the Λ_{c1}^* and Λ_{c1} states are

$$P[\Lambda_{c1}^*]=\frac{B}{1+A+B}\left[\frac{\widetilde{\omega}_1}{2},\frac{2}{3}(1-\widetilde{\omega}_1),\frac{\widetilde{\omega}_1}{6},0\right], \qquad (5.1)$$

$$P[\Lambda_{c1}]=\frac{B}{1+A+B}\left[\frac{1}{3}(1-\widetilde{\omega}_1),\frac{1}{3}\widetilde{\omega}_1\right], \qquad (5.2)$$

where the helicity states for Λ_{c1}^* read $-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2}$ from left to right, and those for Λ_{c1} read $-\frac{1}{2},\frac{1}{2}$.

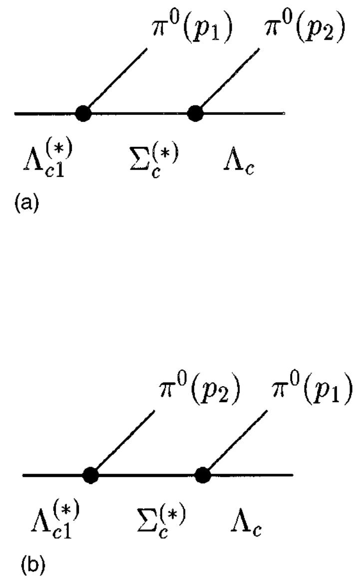

FIG. 1. Feynman diagrams contributing to $\Lambda_{c1}^{(*)}\to\Lambda_{c1}\pi\pi$ at leading order in the heavy hadron chiral Lagrangian.

We now wish to calculate the double-pion distributions in the decays of these states to the ground state Λ_c. The differential decay rate may be written

$$\frac{d\Gamma}{d\Omega_1 d\Omega_2}=\frac{|M_{fi}|^2}{8M_{\Lambda_{c1}^{(*)}}M_{\Lambda_c}(2\pi)^5}\sqrt{(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)} \times\delta(M_{\Lambda_{c1}^{(*)}}-E_1-E_2-M_{\Lambda_c})dE_1dE_2, \qquad (5.3)$$

where Ω_1 and Ω_2 contain the angular variables for the two pions and E_1 and E_2 are their energies. A glance at the expression above indicates that we are conserving three momentum, but not energy. The explanation is simply that, in the infinite mass limit, the charm baryon recoils to conserve momentum, but carries off a negligible amount of energy in the process.

Let us first address the case of Λ_{c1}^* and Λ_{c1} decay to $\Lambda_c\pi^0\pi^0$. The relevant Feynman diagrams which arise from the Lagrangian (3.21) are shown in Fig. 1. In calculating the decays between Λ_{c1}^* and Λ_{c1} states of definite helicity, we find two distinct angular patterns, depending only on the change in the component of spin along the fragmentation axis, ΔS_z, between the initial and final state heavy hadrons:

$$F_1(\Omega_1,\Omega_2)=\frac{3}{32\pi^2}[\cos^2\theta_1+\cos^2\theta_2+\alpha\ \cos\theta_1\cos\theta_2], \qquad (5.4)$$

$$F_2(\Omega_1,\Omega_2)=\frac{3}{64\pi^2}[\sin^2\theta_1+\sin^2\theta_2 +\alpha\ \sin\theta_1\sin\theta_2\cos(\phi_2-\phi_1)], \qquad (5.5)$$

where θ_1 and θ_2 are the angles between the two pion momenta and the fragmentation axis, and ϕ_1 and ϕ_2 are the azimuthal angles of the pion momenta about this axis. These

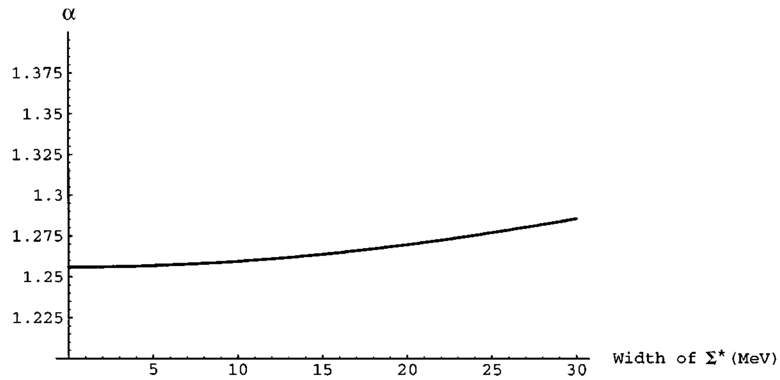

FIG. 2. The variation of the coefficient α as a function of the width of Σ_c^*.

angles are defined in the rest frame of the decaying $\Lambda_{c1}^{(*)}$. The number α arises from interference between the two graphs depicted in Fig. 1, and is defined in (5.6) below. Its dependence on the width $\Gamma_{\Sigma_c^*}$ is plotted in Fig. 2. To the order we are working, $\alpha = 1.3$ for any reasonable value of $\Gamma_{\Sigma_c^*}$:

$$\alpha \equiv \alpha_1/\alpha_2;$$

$$\alpha_1 = \int_{m_\pi}^{M_{\Lambda_{c1}^*}-M_{\Lambda_c}} dE_1 \int dE_2\, \delta(M_{\Lambda_{c1}^*}-M_{\Lambda_c}-E_1-E_2)$$
$$\times\left(\frac{2E_1E_2(E_1^2-m_\pi^2)(E_2^2-m_\pi^2)[(M_{\Sigma_c^*}-M_{\Lambda_c}-E_1)(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)+(\Gamma_{\Sigma_c^*}/2)^2]}{[(M_{\Sigma_c^*}-M_{\Lambda_c}-E1)(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)+(\Gamma_{\Sigma_c^*}/2)^2]^2+(\Gamma_{\Sigma_c^*}/2)^2(E_1-E_2)^2}\right);$$

$$\alpha_2 = \int_{m_\pi}^{M_{\Lambda_{c1}^*}-M_{\Lambda_c}} dE_1 \int dE_2\, \delta(M_{\Lambda_{c1}^*}-M_{\Lambda_c}-E_1-E_2)\left(\frac{E_1^2(E_2^2-m_\pi^2)^{3/2}(E_1^2-m_\pi^2)^{1/2}}{(M_{\Sigma_c^*}-M_{\Lambda_c}-E_2)^2+(\Gamma_{\Sigma_c^*}/2)^2}\right). \tag{5.6}$$

The normalized differential rates $(1/\Gamma)(d\Gamma/d\Omega_1 d\Omega_2)$ for the various decays are then given in terms of F_1 and F_2 by

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\left\{\left[\Lambda_{c1}^*\left(+\frac{1}{2}\right)\to\Lambda_c\left(+\frac{1}{2}\right)\right],\left[\Lambda_{c1}^*\left(-\frac{1}{2}\right)\to\Lambda_c\left(-\frac{1}{2}\right)\right]\right\}=F_1(\Omega_1,\Omega_2), \tag{5.7}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\left\{\left[\Lambda_{c1}^*\left(+\frac{3}{2}\right)\to\Lambda_c\left(+\frac{1}{2}\right)\right],\left[\Lambda_{c1}^*\left(+\frac{1}{2}\right)\to\Lambda_c\left(-\frac{1}{2}\right)\right],\left[\Lambda_{c1}^*\left(-\frac{1}{2}\right)\right.\right.$$
$$\left.\left.\to\Lambda_c\left(+\frac{1}{2}\right)\right],\left[\Lambda_{c1}^*\left(-\frac{3}{2}\right)\to\Lambda_c\left(-\frac{1}{2}\right)\right]\right\}=F_2(\Omega_1,\Omega_2). \tag{5.8}$$

The decays $\Lambda_{c1}^*(\pm\frac{3}{2})\to\Lambda_c(\mp\frac{1}{2})$ are forbidden. A similar calculation for Λ_{c1} decays yields

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\left\{\left[\Lambda_{c1}\left(+\frac{1}{2}\right)\to\Lambda_c\left(+\frac{1}{2}\right)\right],\left[\Lambda_{c1}\left(-\frac{1}{2}\right)\to\Lambda_c\left(-\frac{1}{2}\right)\right]\right\}=G_1(\Omega_1,\Omega_2), \tag{5.9}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_1 d\Omega_2}\left\{\left[\Lambda_{c1}\left(+\frac{1}{2}\right)\to\Lambda_c\left(-\frac{1}{2}\right)\right],\left[\Lambda_{c1}\left(-\frac{1}{2}\right)\to\Lambda_c\left(+\frac{1}{2}\right)\right]\right\}=G_2(\Omega_1,\Omega_2), \tag{5.10}$$

where

$$G_1 = \frac{3}{32\pi^2}[\cos^2\theta_1+\cos^2\theta_2+\beta\,\cos\theta_1\cos\theta_2], \tag{5.11}$$

$$G_2 = \frac{3}{64\pi^2}[\sin^2\theta_1+\sin^2\theta_2+\beta\,\sin\theta_1\sin\theta_2\cos(\phi_2-\phi_1)]. \tag{5.12}$$

The ratio β is defined analogously to α in (5.6), but with the substitutions $M_{\Lambda_{c1}^*}\to M_{\Lambda_{c1}}$, $M_{\Sigma_c^*}\to M_{\Sigma_c}$, and $\Gamma_{\Sigma_c^*}\to\Gamma_{\Sigma_c}$, that is, by removing all stars in (5.6). Its dependence on Γ_{Σ_c} is shown in Fig. 3. That β is much smaller than α is easily understood. Both α and β arise from the interference between Feynman graphs, but in the case of Λ_{c1} decay, the intermediate Σ_c may go

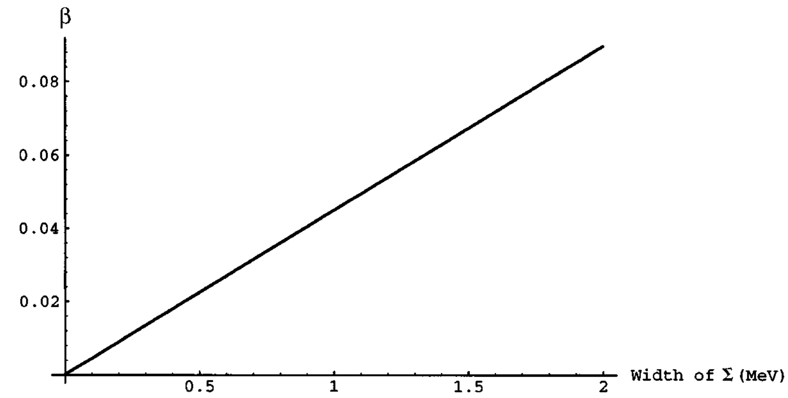

FIG. 3. The variation of the coefficient β as a function of the width of Σ_c.

on shell, and in fact, the rate is dominated by this region of phase space. The Λ_{c1} decay is thus essentially a two-step process, and interference effects are therefore relatively unimportant. The steep dependence of β on the intermediate state width does not significantly limit our predictions since it is numerically small.

We now take into account the initial populations of the various helicity states, as displayed in (5.1) and (5.2), and allow them to decay incoherently in light of the relation $\Gamma_{\Lambda_{c1}^{(*)}} \ll (M_{\Lambda_{c1}^*} - M_{\Lambda_{c1}})$. This produces, after summing final state helicities, the following double pion distributions for decay through Λ_{c1}^* and Λ_{c1} states separately:

$$\frac{1}{\Gamma}\frac{d\Gamma(\Lambda_{c1}^*\ \text{only})}{d\Omega_1 d\Omega_2}=\frac{3}{32\pi^2}\Bigg\{\Bigg[\frac{1}{3}+\frac{1}{2}(\cos^2\theta_1+\cos^2\theta_2)+\frac{2\alpha}{3}\cos\theta_1\cos\theta_2+\frac{\alpha}{6}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)\Bigg] + \tilde{\omega}_1\Bigg[\frac{1}{2}-\frac{3}{4}(\cos^2\theta_1+\cos^2\theta_2)-\frac{\alpha}{2}\cos\theta_1\cos\theta_2+\frac{\alpha}{4}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)\Bigg]\Bigg\}, \tag{5.13}$$

$$\frac{1}{\Gamma}\frac{d\Gamma(\Lambda_{c1}\ \text{only})}{d\Omega_1 d\Omega_2}=\frac{1}{32\pi^2}\{2+\beta[\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)+\cos\theta_1\cos\theta_2]\}. \tag{5.14}$$

Combining both Λ_{c1}^* and Λ_{c1} decays incoherently yields

$$\frac{1}{\Gamma}\frac{d\Gamma(\text{combined})}{d\Omega_1 d\Omega_2}=\frac{1}{32\pi^2}\Bigg\{\Bigg[\frac{4}{3}+\cos^2\theta_1+\cos^2\theta_2+\left(\frac{4\alpha}{3}+\frac{\beta}{3}\right)\cos\theta_1\cos\theta_2+\left(\frac{\alpha}{3}+\frac{\beta}{3}\right)\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)\Bigg] + \tilde{\omega}_1\Bigg[1-\frac{3}{2}(\cos^2\theta_1+\cos^2\theta_2)-\alpha\cos\theta_1\cos\theta_2+\frac{\alpha}{2}\sqrt{(1-\cos^2\theta_1)(1-\cos^2\theta_2)}\cos(\phi_2-\phi_1)\Bigg]\Bigg\}. \tag{5.15}$$

Note from Fig. 3 that β approaches zero as the width Γ_{Σ_c} vanishes. This means that the double pion distribution (5.14) resulting from Λ_{c1} decay becomes isotropic in this limit. This is easily understood as follows. As Γ_{Σ_c} approaches zero, Λ_{c1} decay is entirely dominated by production of a real intermediate Σ_c as discussed above, a process which may occur only via S-wave pion emission. The subsequent single pion decay of the Σ_c is also isotropic if Λ_c helicities are summed over, as previously observed in [1].

Integration of the combined distribution over azimuthal angles produces

$$\frac{1}{\Gamma}\frac{d\Gamma(\text{combined})}{d\cos\theta_1 d\cos\theta_2}=\frac{1}{8}\Bigg\{\Bigg[\frac{4}{3}+\cos^2\theta_1+\cos^2\theta_2+\left(\frac{4\alpha}{3}+\frac{\beta}{3}\right)\cos\theta_1\cos\theta_2\Bigg]+\tilde{\omega}_1\Bigg[1-\frac{3}{2}(\cos^2\theta_1+\cos^2\theta_2)-\alpha\cos\theta_1\cos\theta_2\Bigg]\Bigg\}, \tag{5.16}$$

which is plotted for a variety of $\tilde{\omega}_1$ values in Figs. 4–6.

Alternatively, we may prefer to integrate over pion angles and observe instead the polarization of the final Λ_c. We then find the population ratios

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})}=\frac{2-\tilde{\omega}_1}{4+\tilde{\omega}_1}, \tag{5.17}$$

for fragmentation through Λ_{c1}^* alone,

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})}=\frac{2-\tilde{\omega}_1}{1+\tilde{\omega}_1}, \tag{5.18}$$

for fragmentation through Λ_{c1} alone, and

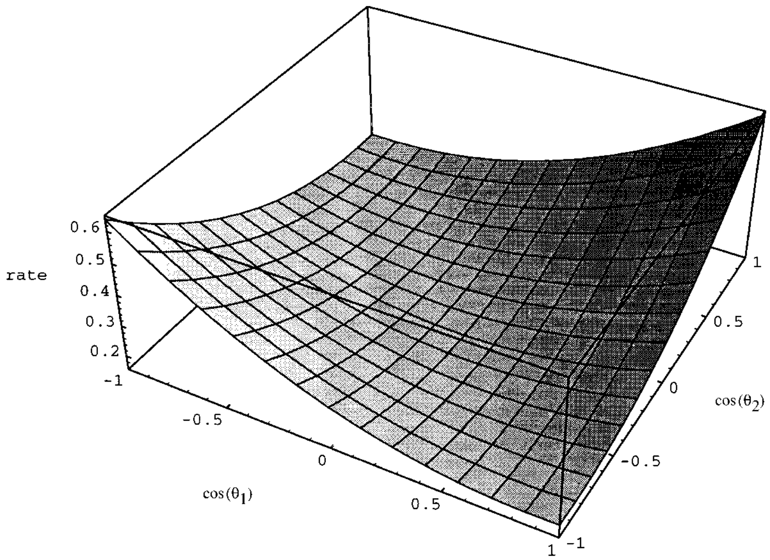

FIG. 4. Normalized differential decay rate for the case α=1.3, β=0.08, and $\tilde{\omega}_1=0$.

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})}=\frac{4-\tilde{\omega}_1}{5+2\tilde{\omega}_1}, \tag{5.19}$$

for the incoherent combination of the two. To be consistent, however, we must include also the effects of initial fragmentation to (Σ_c^*,Σ_c) and Λ_c. This analysis was already carried out in [1], and including such effects leaves us with

$$\frac{\Lambda_c(+\frac{1}{2})}{\Lambda_c(-\frac{1}{2})}=\frac{2A(2-\omega_1)+2B(2-\tilde{\omega}_1)}{A(5+2\omega_1)+B(5+2\tilde{\omega}_1)+9}. \tag{5.20}$$

We may define the polarization of the final state Λ_c in terms of the relative production probabilities for $\Lambda_c(+\frac{1}{2})$ and $\Lambda_c(-\frac{1}{2})$ as

$$\mathscr{P}=\frac{\mathrm{Prob}[\Lambda_c(-\frac{1}{2})]-\mathrm{Prob}[\Lambda_c(+\frac{1}{2})]}{\mathrm{Prob}[\Lambda_c(-\frac{1}{2})]+\mathrm{Prob}[\Lambda_c(+\frac{1}{2})]}. \tag{5.21}$$

For the case of a completely left-handed initial heavy quark, we find

$$\mathscr{P}=\frac{A(1+4\omega_1)+B(1+4\tilde{\omega}_1)+9}{9(A+B+1)}. \tag{5.22}$$

This function may never fall below $\frac{1}{9}$, so that the initial polarization information may never be entirely obliterated by the fragmentation process. As a first guess as to what polarization we may actually expect to measure, we may use the value $\omega_1=0$, suggested by experimental study of the charmed meson system [1], and $A=0.45$, the default Lund value [5,9]. If we further assume that the light degrees of freedom fragment to $j^P=1^+$ and $j^P=1^-$ states indiscriminately so that $A=B$, we find that $\mathscr{P}$ ranges from 0.58 to 0.79 as $\tilde{\omega}_1$ ranges from 0 to 1. For a heavy quark with initial polarization **P**, the above results for P are simply multiplied by **P**. It is not unreasonable, therefore, to expect a significant fraction of the initial heavy quark's polarization to be observable in the final state Λ_c.

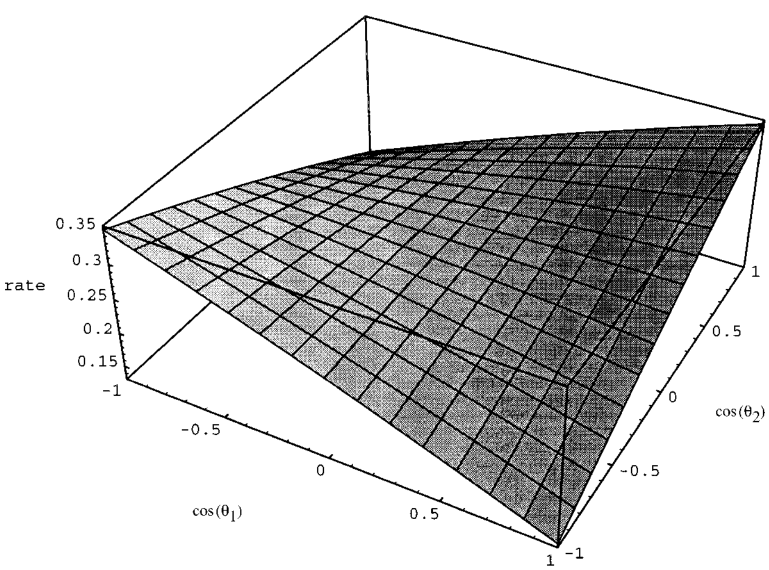

FIG. 5. Normalized differential decay rate for the case α=1.3, β=0.08, and $\tilde{\omega}_1=0.7$.

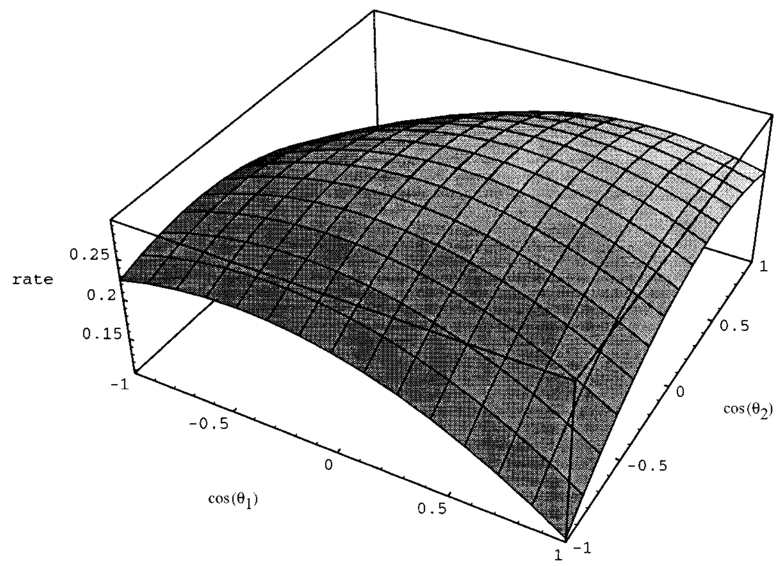

FIG. 6. Normalized differential decay rate for the case $\alpha=1.3$, $\beta=0.08$, and $\tilde{\omega}_1=1$.

The parameters A and B are also of phenomenological interest. Accurate association of Λ_c with final state pions should measure the number of zero, one, and two pion events in the ratio:

$$\Lambda_c:\Lambda_c\pi:\Lambda_c\pi\pi=1:A:B. \tag{5.23}$$

Information on A and B may also be obtained by measuring the relative number of fragmentation events containing Σ_c as opposed to those containing Σ_c^*. Direct fragmentation to (Σ_c^*,Σ_c) produces them in the ratio $\Sigma_c^*:\Sigma_c=2:1$. This ratio will be diminished, however, by Λ_{c1} that decay to real Σ_c on their way to Λ_c. The decays of Λ_{c1}^* are kinematically forbidden from producing such an enhancement in the Σ_c^* population. In the narrow width approximation for Σ_c, we find

$$\frac{\text{events with } \Sigma_c^*}{\text{events with } \Sigma_c}=\frac{2}{\left[1+\dfrac{B}{A}\right]}. \tag{5.24}$$

An accurate measurement of such departure from naive spin counting could provide information on this interesting ratio, (B/A), and would be especially useful for checking the predictions of various fragmentation models.

A few remarks are in order concerning the decays to $\Lambda_c\pi^+\pi^-$. This case is slightly more complicated than the $\pi^0\pi^0$ case because the propagator connecting Λ_{c1}^* to Λ_c may be either $\Sigma_c^{(*)0}$ or $\Sigma_c^{(*)++}$. This fact, coupled with the different Σ_c masses,

$$\begin{aligned} M[\Sigma_c^{++}]&=2453.1\pm0.6\ \text{MeV},\\ M[\Sigma_c^{+}]&=2453.8\pm0.9\ \text{MeV},\\ M[\Sigma_c^{0}]&=2452.4\pm0.7\ \text{MeV}, \end{aligned} \tag{5.25}$$

produces distributions in Λ_{c1} decay that are not symmetric with respect to the π^+ and π^- momenta. Indeed, if we boldly accepted the central values of the sigma masses above, we would proceed to calculate an enhancement in the coefficient of $\cos^2\theta_{\pi-}$ by approximately 10% with respect to that of $\cos^2\theta_{\pi+}$ in (5.4) above, and a similar enhancement for the coefficient of $\sin^2\theta_{\pi-}$ relative to that of $\sin^2\theta_{\pi+}$ in (5.5). In light of the errors listed in (5.25) and the order to which we are working, however, such a conclusion would be inappropriate. The $\pi^+\pi^-$ distributions are, within the accuracy of this calculation, indistinguishable from those of the neutral pions.

VI. CONCLUDING REMARKS

In this paper, we have studied fragmentation through the $(\Lambda_{c1}^*,\Lambda_{c1})$ system, and have calculated the resultant double pion decay distributions in the well satisfied limit $\Gamma(\Lambda_{c1}^{(*)})\ll(M_{\Lambda_{c1}^*}-M_{\Lambda_{c1}})$. In so doing, we have introduced the fragmentation parameters $\tilde{\omega}_1$ and B, and have shown how $\tilde{\omega}_1$ may be extracted from pion angular data. We have also found that the final state Λ_c particles produced in the fragmentation process should retain a significant fraction of the initial heavy quark's polarization, allowing a test of the standard model's predictions for heavy quark polarization in such hard processes.

Experimental determinations of the ω parameters are extremely important in testing various ideas about fragmentation. Chen and Wise [6] have estimated $\omega_{3/2}$ using the $m_c/m_b\to0$ limit of a *perturbative* QCD calculation of $b\to B_c^{**}$ done by Chen [7], and have found that $\omega_{3/2}=29/114$. That this admittedly oversimplified approach gives reasonable agreement with the experimentally suggested $\omega_{3/2}<0.24$ [1] is of significant interest. Yuan [8] has augmented this analysis with a calculation of the dependence of $\omega_{3/2}$ on the longitudinal and transverse momentum fractions of the meson. Furthermore, fragmentation models such as the Lund model make predictions for parameters related to A [5,9]. Similar predictions will be possible for the remaining fragmentation parameters discussed in this paper, in either a limiting case of QCD, or in a model such as Lund, and the experimental extraction of these parameters will therefore provide nontrivial constraints on such methods. Determination of $\widetilde{\omega}_1$ may in fact soon be possible at CLEO [10].

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