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Precocious Scaling, Rescaling, and ξ Scaling*

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All effects of target and constituent masses on the Q^2 dependence of lepton-hadron scattering are computable in an expansion in $g(Q^2)$, the effective coupling of asymptotic freedom. To leading order, these are summarized by a "best" scaling variable, ξ, which depends on the particular process. The results relevant to low Q^2 and charm threshold are discussed.

Mass effects in lepton-hadron scattering have been traditionally relegated to "nonleading" terms, which are computable, in principle, order by order in m^2/Q^2. However, approximate scaling has been observed for $Q^2 \sim m_{\text{proton}}{}^2$. Also, it would be desirable to be able to discuss thresholds for heavy particles with new quantum numbers, where Q^2 may be large but still comparable to the new masses. Making only the same basic assumptions as are needed to derive approximate scaling from colored-quark–gluon gauge theories, we note that a reorganization of the operator product expansion allows all such mass effects to be computed, in an expansion in $g(Q^2)$, the effective coupling relevant for Q^2, but *exactly* in m^2/Q^2. The analysis works as long as $g(Q^2)$, a decreasing function of Q^2, is small. Only experiment can determine how low in Q^2 one may go, but this limit is logically independent of the target or constituent mass.

The results can be expressed in terms of a mass-dependent scaling variable ξ which can be interpreted in parton language[1] as the momentum fraction carried by the struck quark. "Scaling" in terms of ξ is a reflection of the smallness of $g(Q^2)$; it does not come simply from dimensional analysis.

The scaling variable ξ is

$$\xi = \frac{Q'^2}{2m_p\nu}\,\frac{2}{1+(1+Q'^2/\nu^2)^{1/2}}, \tag{1}$$

where

$$2Q'^2 = Q^2 + m_F{}^2 - m_I{}^2 + [Q^4 + 2Q^2(m_F{}^2 + m_I{}^2) + (m_F{}^2 - m_I{}^2)^2]^{1/2}.$$

The struck-quark mass is m_I, the produced-quark mass is m_F, the proton mass is m_p, and Q^2 and ν are the usual variables. If the struck quark is light, $m_I \simeq 0$, this formula has a simple interpretation in parton language. ξ is the fraction of the proton momentum carried by the struck quark. The constraint that the produced quark is on its mass shell, $(\xi p + q)^2 = m_F{}^2$, implies Eq. (1) (with $m_I = 0$).

In this paper, we will describe the manipulations necessary to derive the "ξ scaling" discussed above. The details of the derivation (and even some of the results) are quite complicated and will be thoroughly discussed in a subsequent paper. Here we will simply quote the results for a few simple cases of phenomenological interest.

The hadron structure functions measured by lepton currents can be analyzed in terms of the discontinuity of the operator expansion of the product of the appropriate hadronic currents[2]:

$$q_{\mu\nu}W_1+\frac{p_\mu p_\nu}{m_p^2}W_2+\epsilon_{\mu\nu\lambda\sigma}\frac{p_\lambda p_\sigma}{m_p^2}W_3=\frac{1}{\pi}\,\mathrm{Im}\int e^{iqx}dx\langle p|iT[J_\mu(x)J_\nu(0)]|p\rangle=\frac{1}{\pi}\,\mathrm{Im}\sum_n C_n(q)\langle p|O^n|p\rangle. \tag{2}$$

Three important observations about the local operators O^n and their coefficients $C_n(q)$ are the following: (a) The total set of operators is redundant by virtue of the equations of motion. Regardless of how complex these may be, they specify relations between the matrix elements of renormalized operators. So all operators containing any power of $\not{D}$, the gauge-covariant derivative, operating on a quark field may be dropped from the set without compromising its completeness. In the standard analysis, these terms would be ignored because they have twist greater than 2. Instead of ignoring them, we eliminate them by using the equations of motion. This induces a change in the C_n of the remaining O^n which is calculable and vanishes like m^2/Q^2 for momentum transfer much larger than the quark mass m. (b) All remaining derivatives in the O^n have a free Lorentz index, to be contracted with the C_n. If the O^n are organized according to twist (defined as dimension minus spin), as traditionally prescribed, they are traceless. So the tensor structure of the matrix elements is determined:

$$\langle p|O^n|p\rangle=A_n[p^{\mu_1}\dots p^{\mu_n}-\text{traces}].$$

In the standard analysis, only the piece in the matrix element proportional to $p^{\mu_1}\dots p^{\mu_n}$ is discussed. All the rest give contributions which are smaller by factors of m_p^2/Q^2. (c) The O^n can also be organized by their importance in the operator product expansion in orders of the coupling constant $g(Q^2)$. While it would be foolish to estimate $\langle p|O^n|p\rangle$ using perturbation theory,[2] the C_n are computable in an expansion in $g(Q^2)$. Only quark bilinears occur with zeroth-order coefficients. All gluon operators and those with more quarks have C_n's beginning in higher order.

To leading order in $g(Q^2)$, all relevant mass dependence can be collected. Subsequent orders simply involve examining the next important operators according to (c). By that observation, the only operators are quark bilinears. They are twist-2 because all higher twists have been eliminated by (a), which has taken the quark-mass effects of the higher twist bilinears and put them into the computable C_n. All target-mass dependence resides in the trace subtractions and is determined by the tensor structure. We emphasize that these are all the mass effects relevant to the Q^2 dependence. There are indeed other mass dependences in the $\langle p|O^n|p\rangle$ besides the trace subtractions, but they enter the structure functions as Q^2-independent factors in A_n, which are part of the initial structure function measured at some Q_0^2 and not yet computable within this analysis. So the leading (in g) Q^2 dependence comes from the quark-mass dependence of the free-field coefficient functions and from the target masses in the traceless tensors.

The structure functions are obtained from the forward Compton amplitude.[3] The coefficient of $(p\cdot q)^n$ in T_1 is related to the $(n-1)$st moment of W_1, etc. The Q^2 dependence of W_1, to zeroth order in g, is determined by the free-field C_n's, which have a known dependence on the quark masses; by the tensor structure of the proton matrix elements, which is determined by Lorentz invariance and tracelessness; and by the A_n, which at this stage are completely unknown. The A_n, for each quark which appears in the current, are the moments of some function, which we interpret in the parton language as the quark distribution functions. The results for the structure functions will involve known functions of quark masses, the target mass, and Q^2, and unknown functions (one for each quark) of a "scaling variable" ξ, which has a known dependence on quark and target masses. We call this ξ scaling.[4]

Higher-order terms in g are responsible for the well-known logarithmic violations of scaling in asymptotically free theories.[5] Whether these effects are larger than the violations of naive scaling implicit in ξ scaling depends on the process and the Q^2. Including the effect of quark masses does not qualitatively change the logarithmic dependence. The small changes that are induced will be described in a subsequent paper. In the rest of this paper, we will concern ourselves only with lowest-order terms in g.

e or μ scattering off light quarks.—If the quark masses involved are negligible, the appropriate scaling variable is $\xi=2x/[1+(1+4x^2m_p^2/Q)^{1/2}]$.[3] To lowest order in m_p^2/Q^2, $\xi\simeq x/(1+x^2m_p^2/Q^2)$ which is not so different from the Bloom-Gilman variable $x'=x/(1+xm_p^2/Q^2)$. The complete result is

$$x(2\sigma_T - \sigma_L) = 3xW_1 - \left(1 + \frac{4x^2 m_p^2}{Q^2}\right)\frac{\nu W_2}{2m_p} = \frac{x^2}{(1+4x^2m_p^2/Q^2)^{1/2}} F(\xi), \tag{3a}$$

$$x\sigma_L = \left(1 + \frac{4x^2 m_p^2}{Q^2}\right)\frac{\nu W_2}{2m_p} - xW_1$$
$$= \frac{2m_p^2}{Q^2}\frac{x^3}{(1+4x^2m_p^2/Q^2)}\int_\xi^1 d\xi' F(\xi') + \frac{4m_p^4}{Q^4}\frac{x^4}{(1+4x^2m_p^2/Q^2)^{3/2}}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi''). \tag{3b}$$

We claim that so-called precocious scaling, the fact that electroproduction data seem to scale in x' better than in x at low Q^2, is a reflection of ξ scaling. More precisely, Eq. (3a) can be compared with recent Stanford Linear Accelerator Center data.[6] The data show ξ scaling within experimental errors, but there is some evidence for the type of scaling violation which would be expected from logarithmic terms.[7] Furthermore the distribution function $F(\xi)$ determined using Eq. (3a) can be used in (3b) to compute σ_L. The result is too small by an order of magnitude. This shows that the dominant contribution to the breakdown of the Callan-Gross relation comes from terms of order g^2,[8] not from terms of order m_p^2/Q^2. So while ξ scaling is an improvement over naive scaling in x, it is not the whole story. The logarithms are important.

Heavy-quark thresholds in ν scattering.—We predict the nature of the rescaling above a heavy-quark threshold. The simplest and most important case is heavy-quark production off light quarks, m_F large and $m_I \simeq 0$. For simplicity, we consider only $Q^2 \gg m_p^2$. Then $\xi = x(Q^2 + m_F^2)/Q^2 = x + m_F^2/(2ym_pE)$, and W_1, νW_2, νW_3 are functions only of ξ. Indeed, they are the same functions which appear (as functions of x) in neutrino scattering off the same light quark into another light quark, so they can be determined from electroproduction and neutrino-scattering data below the heavy-quark threshold. The quantity ξ and not x is properly regarded as the momentum fraction of the struck quark: Small x does not mean the struck quark is in the sea. Note that x is kinematically limited, $x < Q^2/(Q^2 + m_F^2)$, because $\xi < 1$. [This is just the statement that $(p+q)^2 > m_F^2$.] But because $x > 0$, $\xi > m_F^2/2ym_pE$. Heavy-quark production does not become appreciable until E becomes large enough that ξ is small enough that $W_1(\xi)$, say, is large. Valence quarks are more effective than sea quarks for heavy-quark production because the sea quarks (with smaller mean ξ) are only effective at higher energies. The y distributions are also affected by ξ scaling.[9]

The scaling variable for light-quark production off heavy quarks, m_I large, $m_F \simeq 0$, is $\xi = x$, but the form of the structure functions is more complicated. In practice, this contribution may be negligible because the distribution functions for heavy quarks in the proton are small.

Heavy quarks in e and μ scattering.—Here $m_I = m_F = m$ so, for $Q^2 \gg m_p^2$, we have $\xi = \frac{1}{2}x[1 + (4m^2/Q^2)^{1/2}]$. The complete result is complicated, but one important point can be stated simply. As Q^2 increases for fixed x, ξ decreases. Except at very small ξ, the distribution functions are decreasing functions so the structure functions at fixed x will increase with increasing Q^2. This threshold behavior cannot be mistaken for the logarithmic effects from higher-order terms, which go in the opposite direction.[7]

As far as we know, it is not possible to interpret ξ for $m_I \neq 0$ in the parton language. This is sad because for $m_I = 0$, the parton language is a simple and compelling mnemonic for a turbid field-theoretic argument. We can interpret the success of the parton model by observing that the impulse approximation requires Q^2 to be large enough to treat the partons as approximately free, i.e., $g(Q^2)$ must be small. This says nothing about Q^2 versus m_p or m_F. (Vaguely analogous is the impulse approximation for scattering off hydrogen: $\sqrt{Q^2}$ need only be large compared to 13.6 eV, which is only indirectly related to the mass of the electron.) However, we have not succeeded in generalizing the prescription to include $m_I \neq 0$. So until such further modification, we must regard the parton description as inadequate (independent of the logarithms of Q^2 arising from the breakdown of the impulse approximation).

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[1]R. P. Feynman, in *High-Energy Collisions*, edited by C. N. Yang *et al*. (Gordon and Breach, New York,

1970); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[2]What is needed is the order-of-magnitude estimate on the matrix elements given by dimensional analysis for operators renormalized on the scale of m_p. In one instance where these have been measured, i.e., the separate quark and gluon pieces of the energy-momentum tensor, such estimates are accurate. In the presence of large quark masses, $m_q \gg m_p$, such estimates are true to all orders in perturbation theory for the O^n excluding $\not{p}$'s as in (a) above.

[3]N. Christ, B. Hasslacher, and A. H. Mueller, Phys. Rev. D 6, 3543 (1972).

[4]Others have suggested scaling variables which resemble our ξ in some kinematic regions: See F. Gürsey and S. Orfanidis, Nuovo Cimento 11A, 225 (1972); O. W. Greenberg and D. Bhaumik, Phys. Rev. D 4, 2048 (1971); G. Domokos, Phys. Rev. D 4, 3708 (1971); M. Breidenbach and J. Kuti, unpublished; H. J. Schnitzer, Phys. Rev. D 4, 1429 (1971).

[5]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973), and Phys. Rev. D 8, 3633 (1973), and 9, 980 (1974); H. Georgi and H. D. Politzer, Phys. Rev. D 9, 416 (1974).

[6]E. M. Riordan *et al.*, SLAC Report No. SLAC-PUB-1634, 1975 (unpublished).

[7]A. De Rújula, H. Georgi, and H. D. Politzer, Phys. Rev. D 10, 2141 (1974).

[8]A. Zee, F. Wilczek, and S. B. Treiman, Phys. Rev. D 10, 2881 (1974).

[9]Implications of these ideas for various models and detailed comparisons with data have been worked out by M. Barnett, to be published.

Comparison of the Yields of Inelastic Electron and Positron Scattering from Hydrogen and Deuterium at 15 (GeV/c)^2 *

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We have measured the ratio, Y^+/Y^-, of positron to electron inelastic scattering yields from hydrogen and deuterium at Q^2, the square of the four-momentum transfer, between 2.4 and 14.9 (GeV/c)^2. The ratios are consistent with $Y^+/Y^-=1$ to within errors of a few percent.

We report the results of a measurement of Y^+/Y^-, the ratio of the yield for inelastic positron scattering to that for inelastic electron scattering from hydrogen and deuterium,[1] which was carried out as part of a larger program of measurements of electron-proton and electron-deuteron scattering cross sections, using the Stanford Linear Accelerator Center (SLAC) spectrometer facility. This ratio is sensitive to the mechanism of the hadron-lepton interaction. For example, if in addition to the usually assumed one-photon exchange process there is also two-photon exchange, the interference between the two occurs with a different sign for electrons and positrons, and the ratio of the cross sections goes like $1+4\times\mathrm{Re}(A_2/A_1)$, where A_1 and A_2 are the amplitudes for one- and two-photon exchange, respectively. Also, the existence of a direct, nonelectromagnetic interaction between electrons and hadrons, as was suggested[2] to explain certain features of the early e^+e^- storage-ring results,[3] would lead, in some models, to a ratio appreciably different from unity.

Previously, measurements had been made for *elastic* scattering for Q^2 up to 5 (GeV/c)^2, and the ratios of cross sections were consistent with unity.[4] Some measurements have also been made for inelastic scattering using incident muons, but at lower Q^2, with similar results.[5]

To make the present measurements, positrons were produced by the electron beam in a radiator[6] one-third of the way down the SLAC linear accelerator and accelerated to a final energy of 13.9 GeV in the remaining two-thirds of the machine. In separate runs, a similar beam of electrons was also produced from the same radiator, as well as the usual electron beam accelerated directly from the electron gun. While the ordinary electron beam was of much higher intensity and thus yielded improved statistical accuracy, we took data with both types of electron beam to look for systematic effects due to differences in intensity and to possible differences in transverse phase space and energy spectrum. In fact, we found no significant differences in the data from the two types of electron beams; therefore, we averaged these yields to obtain the final results.