Equilibria in Campaign Spending Games: Theory and Data

ROBERT S. ERIKSON Columbia University
THOMAS R. PALFREY California Institute of Technology

We present a formal game-theoretic model to explain the simultaneity problem that makes it difficult to obtain unbiased estimates of the effects of both incumbent and challenger spending in U.S. House elections. The model predicts a particular form of correlation between the expected closeness of the race and the level of spending by both candidates, which implies that the simultaneity problem should not be present in close races and should be progressively more severe in the range of safe races that are empirically observed. This is confirmed by comparing simple OLS regression of races that are expected to be close with races that are not, using House incumbent races spanning two decades.

This article presents new estimates of the effects on the vote of spending by incumbents and challengers in U.S. House elections. As first pointed out by Jacobson (1978), estimation by ordinary least squares regression produces strong coefficients for challenger spending but virtually no effect for incumbent spending. Few would argue, however, that the latter does not matter. The obvious reason for the near-zero coefficients for incumbent spending using OLS is a simultaneity bias, because incumbents spend more when they are in electoral trouble. Somewhat less obvious, and less commonly acknowledged, is that the opposite bias can be present for challengers, who spend more when their prospects are good. In short, the OLS estimates are biased because one is not controlling for candidate expectations of the vote, and these expectations drive spending decisions. We address this problem directly by formulating and solving a game-theoretic model of campaign spending and identifying precisely the finer structure of these simultaneity problems. This characterization of the equilibrium of the spending game has direct implications for obtaining unbiased estimates of incumbent and challenger spending effects, without resorting to multiequation systems.

Several statistical solutions have been proposed to estimate the effects of incumbent spending by overcoming the simultaneity bias. The most common solution is two-stage least squares, whereby instrumental variables are used as proxies for observed incumbent spending. Green and Krasno (1988) used lagged incumbent spending as an instrument, whereas Gerber (1998) used such instruments as challenger wealth and state population in an analysis of Senate elections. Both found significant effects for incumbent spending. In an alternative approach, Erikson and Palfrey (1993, 1998) achieved statistical identification by means of restrictions on the variance-covariance matrix. By assuming that the covariances between each spending variable and the vote were caused by only vote-on-spending and spending-on-vote effects, they also found significant effects for incumbent spending, somewhat larger in magnitude than those of Green and Krasno (1988) but still significantly smaller than challenger spending effects. In a third approach, Abramowitz (1991) used OLS but attempted to neutralize the simultaneity bias by using Congressional Quarterly forecasts of election outcomes as a control for expectations. Even with this control, negligible coefficients for incumbent spending were found, and Abramowitz concluded that incumbent spending has little effect on the vote. In sum, due to seemingly unavoidable methodological difficulties, a consensus has not yet emerged regarding both the relative and absolute magnitudes of incumbent and challenger spending effects on congressional election outcomes.

Our new approach to estimating spending effects uses insights from game theory. We apply OLS to a subset of congressional districts for which a game-theoretic model predicts that the simultaneity bias should be minimal or nonexistent. In these districts, new sources of challenger vote support do not necessarily drive up challenger and incumbent spending. Which districts are they? They are districts in which, before taking spending into account, the vote is expected to be close or even slightly in favor of the challenger. When a close race is expected, both spending effects can be reliably estimated by simple OLS.

We proceed as follows. First, we present the theoretical argument for a near-zero simultaneity bias in districts in which the vote is expected to be close. Second, we translate this theoretical argument into a set of statistical hypotheses and derive closed-form expressions for the asymptotic bias of OLS estimates of spending effects. Third, we provide evidence in support of the predicted curvilinear effects of the expected vote on spending. When the expected vote approximates a 50–50 split, the slopes for spending on (expected) vote are near zero, so that the simultaneity bias should be near zero. Fourth, we present the key empirical result. When the vote is expected to be close (and, thus, the simultaneity bias is minimal), the OLS estimates of spending effects are of roughly equal magnitude for incumbents and challengers. We conclude with a discussion of our findings.
THEORY: THE SPENDING GAME

Our empirical results can be understood in the context of an extended version of the simple spending game framework of Erikson and Palfrey (1993). The model is meant to capture the strategic aspect of spending by two competing candidates, $I$ (the incumbent) and $C$ (the challenger). The spending decisions of the two candidates are intertwined because these decisions jointly affect the probability the incumbent will win. Thus, the amount one candidate chooses to spend will depend both on the amount the other candidate is spending and on the expected closeness of the race. This is the spending game. An equilibrium of the game will increase monotonically with the expected closeness of the race.

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The importance of our theoretical analysis is as follows. The formal model implies, among other things, that candidates will indeed maximize their spending when the expected vote is in the neighborhood of 50–50. As a result, for “toss-up” races, the vote-on-spending effects will be approximately zero. For competitive races in the range of a close expected vote but in which incumbents are slightly favored, the vote-on-spending slopes should be negative but small in magnitude. For races expected to be lopsided in favor of the incumbent, the slopes will be negative and large in magnitude, which can create possibly severe simultaneity problems. Because most races fall in the latter category, regressions that pool all races are fundamentally flawed, even if instrumental variables are used, since that method does not adjust for the fact that the simultaneity problem varies with expected closeness in a predictable and nonlinear way.

The formal structure of the game has the following basic features. Each candidate cares about the probability of winning more than 50% of the vote in the election. The probability $I$ wins is determined by district characteristics, short-term forces, candidate characteristics, campaign spending, and chance. For the moment, treat the first three categories of variables as exogenously fixed. We then can summarize the effects of campaign spending and chance by a simple twice continuously differentiable function, $P$, which denotes the incumbent’s probability of winning as a function of spending by each of the candidates. The probability the incumbent wins also depends on what we call the (prespending) anticipated margin of victory of the incumbent, denoted $m$. All else equal, the higher is $m$, the higher is $P$. The anticipated-victory margin incorporates factors that the candidates both treat as exogenous, including national short-term forces and such district characteristics as partisanship, as well as other factors, such as candidates’ health, a recent scandal, and so forth. Each candidate can raise and spend money in the campaign, and the outcome of the election is a function of how much each spends, the prespending anticipated margin of victory of the incumbent, and some random noise.

Raising campaign resources is a costly activity. Promises must be made (Baron 1989a, 1989b), issue positions compromised, fundraisers attended, and so forth. This is formally represented by two twice continuously differentiable, increasing, convex fundraising cost functions, one for each candidate, denoted $K(I)$ and $K(C)$. The final piece of the equation is the value of winning. We normalize this value at 1 for both the incumbent and the challenger. Hence, the payoff functions to the two candidates are given by:

$$U(I, C; m) = P(I, C, m) - K(I);$$
$$U(C(I, C; m) = 1 - P(I, C, m) - K(C).$$

We proceed by studying the properties of the Nash equilibria of this game. A Nash equilibrium is defined as a pair of spending levels for each candidate, $I^*$ and $C^*$, such that each maximizes her payoff (probability of winning minus fundraising costs), given the spending level of the opponent. In particular, we wish to establish existence of a Nash equilibrium of the spending game and then prove that equilibrium spending levels will be higher in closer races.

To guarantee existence of a Nash equilibrium that is fully characterized by the first-order conditions, we assume that each payoff function is strictly concave in the candidate’s own spending level. This assumption implies properties of both $P$ and $K$. Roughly speaking, it says that in addition to $K$ being convex, $P$ cannot be too deterministic. The concavity assumption is:

$$\frac{\partial^2 P}{\partial I^2} (I, C, m) - K'(I) < 0;$$
$$\frac{\partial^2 P}{\partial C^2} (I, C, m) - K'(C) < 0.$$

To rule out uninteresting boundary solutions, we also assume $K'(0) = K'(C) = 0$. Thus, the optimal level of spending for the incumbent, $I^*$, given some level of spending by $C$, is always unique, is always strictly positive, and is characterized by the solution to:

$$\frac{\partial P}{\partial I} (I^*, C, m) - K'(I^*) = 0.$$

Similarly, for the challenger, we have:

$$1$$ The theoretical analysis also can be applied to open seat races.
$$2$$ Related theoretical work on tournaments as incentive devices and on the effect of commitment on behavior in contests can be found in Dixit 1987, Nalebuff and Stiglitz 1983, Rosen 1986.
Because the game is concave and reaction functions are strictly positive, there exists an equilibrium with strictly positive levels of spending by both candidates. In equilibrium, these conditions are simultaneously satisfied for both, so equilibrium pairs \((I^*, C^*)\) are characterized by the following equations:\(^5\)

\[
\frac{\partial P}{\partial I}(I^*, C^*, m) - K'_c(I^*) = 0; \quad (1)
\]

\[
\frac{\partial P}{\partial C}(I^*, C^*, m) - K'_C(C^*) = 0. \quad (2)
\]

For the empirical section, we are interested in identifying how \(I^*\) and \(C^*\) change with \(m\) and how they vary with \(P\). We next identify intuitive assumptions on the cross-partial derivatives of \(P\), which imply that \(I^*\) and \(C^*\) are decreasing in \(m\) in elections that favor the incumbent, increasing in \(m\) for elections that favor the challenger, and are independent of \(m\) for toss-ups. This also implies that spending covaries with \(P\) in a similar direction. Moreover, we provide a particular specification of \(P()\) that satisfies these assumptions and is consistent with the standard linear regression models used in empirical studies.\(^6\)

Since the first-order conditions must hold for every value of \(m\), we can take the total derivative of the left-hand side of equations 1 and 2 with respect to \(m\); set it equal to zero; and use this to identify conditions \(dI^*/dm < 0\). As we pointed out earlier, and show formally in Appendix A, this will imply that OLS estimates of the \(I^*\) coefficient are biased downward in elections the incumbent is expected to win. This procedure also allows us to obtain sufficient conditions for \(dC^*/dm < 0\), which are slightly different owing to the fact that challengers who expect incumbents to spend less will respond by spending more. Taking these total derivatives produces the following equations:

\[
\frac{dI^*}{dm} = \frac{\partial^2 P}{\partial I \partial C} \frac{dC^*}{dm} + \frac{\partial^2 P}{\partial I^2};
\]

\[
\frac{dC^*}{dm} = -\frac{\partial^2 P}{\partial I \partial C} \frac{dI^*}{dm} + \frac{\partial^2 P}{\partial C^2}. \]

These are linear in the two unknowns, \(dC^*/dm\) and \(dI^*/dm\), and can be solved easily. The solutions are:

\[
\frac{dI^*}{dm} = \frac{\partial^2 P}{\partial I \partial C} \frac{dC^*}{dm} \frac{K'_c + \frac{\partial^2 P}{\partial C^2}}{A};
\]

\[
\frac{dC^*}{dm} = -\frac{\partial^2 P}{\partial I \partial C} \frac{dI^*}{dm} \frac{K'_C + \frac{\partial^2 P}{\partial I^2}}{A};
\]

where

\[
A = \left[ K'_c + \frac{\partial^2 P}{\partial C^2} \right] \left( K'_C + \frac{\partial^2 P}{\partial I^2} \right) - \left( \frac{\partial^2 P}{\partial I \partial C} \right)^2 > 0
\]

by the second-order conditions. Therefore, \(dI^*/dm < 0\) if and only if

\[
\frac{\partial^2 P}{\partial I \partial C} \frac{dC^*}{dm} \left( K'_c + \frac{\partial^2 P}{\partial C^2} \right) - \frac{\partial^2 P}{\partial I^2} < 0, \quad (3)
\]

and \(dC^*/dm < 0\) if and only if

\[
-\frac{\partial^2 P}{\partial I \partial C} \frac{dI^*}{dm} \left( K'_C + \frac{\partial^2 P}{\partial I^2} \right) - \frac{\partial^2 P}{\partial I \partial C} < 0. \quad (4)
\]

We assume that inequalities 3 and 4 hold. We next show that these conditions for \(dI^*/dm < 0\) and \(dC^*/dm < 0\) are satisfied in a wide class of models of \(P\) that are commonly used in empirical work.

First, let the index of incumbent vote margin, \(m\), be simply the baseline (prespending) expected incumbent share of the vote minus .5, normalized at candidate spending levels of \(I = C = 0\). Let the incumbent’s actual margin of victory, \(M\) (which can be negative), equal \(m\), plus some function of challenger spending, \(g_c(C)\), plus some function of challenger spending, \(g_i(I)\), plus a random term, \(\varepsilon\), with cumulative distribution function \(F(\varepsilon)\). Assume \(g_i(I) > 0, g_c(C) < 0; F\) is twice differentiable; and its density function, \(f = F'\), is symmetric and single peaked around 0.\(^7\) This implies \(f'(\varepsilon) > 0\) for \(\varepsilon < 0\), and \(f'(\varepsilon) < 0\) for \(\varepsilon > 0\). Thus, we can write

\[
M = m + g_i(I) + g_c(C) + \varepsilon.
\]

The incumbent wins if and only if \(M > 0\) or if and only if \(\varepsilon > -m - g_i(I) - g_c(C)\), so

\[
P(I, C, m) = 1 - F[-m - g_i(I) - g_c(C)] = F[m + g_i(I) + g_c(C)]. \quad (5)
\]

It is easy to verify that inequalities 3 and 4 for \(dI^*/dm < 0\) and for \(dC^*/dm < 0\) are satisfied when \(f' < 0\) for many \(P\) functions. As an illustration, suppose that \(F\) is normally distributed with variance \(\sigma^2\) and that \(g_i\) and \(g_c\) are linear.\(^8\) Thus, equation 5 reduces to:

\[
P(I, C, m) = \Phi \left[ m + \beta_{\mu} I + \beta_{\nu} C \right].
\]

\(^5\) There may be multiple components of the equilibrium correspondence, but the comparative static properties we derive below are satisfied locally with respect to each equilibrium.

\(^6\) Moreover, equilibrium is unique for this specification, which guarantees unambiguous comparative statics.

\(^7\) The symmetry assumption can be dropped without loss of generality.

\(^8\) This corresponds to most empirical works on the subject, which employ regression models with linear spending effects.
where $\Phi$ is the unit normal cumulative distribution function. Substitution into equilibrium conditions 1 and 2 yields the following two equations as the first-order conditions:

$$
\frac{\beta_{VI}}{\sigma \sqrt{2\pi}} \Phi(\Pi_m^*) = K(I^*); \quad (6)
$$

$$
- \frac{\beta_{VC}}{\sigma \sqrt{2\pi}} \Phi(\Pi_m^*) = K(C^*); \quad (7)
$$

where $\Phi(\Pi_m^*)$ is the unit normal density function evaluated at the expected win margin, adjusted for equilibrium spending effects and incumbent vulnerability, $\Pi_m^* = (m + \beta_{VI}f^* + \beta_{VC}C^*)/\sigma. 9$ The expression $\Pi_m^*$ is the “postspending” expected vote margin for the incumbent, in contrast to $m$, which is the prespending expected vote margin for the incumbent. Thus, it is important to keep in mind that $\Pi_m^*$ is endogenous, whereas $m$ is exogenous.

It follows immediately from equations 6 and 7 that elections expected to be close (measured by $\Pi_m^*$) will involve higher spending levels by both candidates. This is true because $\Phi$ is maximized at $\Pi_m^* = 0$, and fundraising costs increase with spending.

We also wish to show that spending is higher in closer elections, measured by $m$. Because $m$ indexes incumbent vote margin rather than expected closeness, 10 the comparative statics with respect to $m$ will necessarily depend on whether we are at an equilibrium with $\Pi_m^* > 0$ or $\Pi_m^* < 0$. Suppose that $\Pi_m^* > 0$. Because spending effects enter linearly and the normal density function, $\Phi$, is monotonically decreasing in the absolute value of its argument, inequalities 3 and 4 derived above are both satisfied. Formally:

$$
\frac{\partial^2 P}{\partial \Pi_m \partial \Pi_C} = \frac{\beta_{VI}}{\sigma^2 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0;
$$

$$
\frac{\partial^2 P}{\partial C \partial \Pi_c} = \frac{\beta_{VC}}{\sigma^2 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0;
$$

$$
\frac{\partial^2 P}{\partial \Pi_m \partial \Pi_C} = \frac{\beta_{VI} \beta_{VC}}{\sigma^4 \sqrt{2\pi}} \Phi'(\Pi_m^*) > 0;
$$

$$
\frac{\partial^2 P}{\partial \Pi_m \partial C} = \frac{\beta_{VC}^2}{\sigma^4 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0.
$$

Therefore,

$$
\frac{\partial^2 P}{\partial \Pi_m \partial \Pi_C} K^*_C < 0; \quad - \frac{\partial^2 P}{\partial C \partial \Pi_m} K^*_I < 0;
$$

where $\Phi'(\Pi_m^*)$ is the unit normal density function evaluated at the expected win margin, adjusted for equilibrium spending effects and incumbent vulnerability, $\Pi_m^* = (m + \beta_{VI}f^* + \beta_{VC}C^*)/\sigma. 9$ The expression $\Pi_m^*$ is the “postspending” expected vote margin for the incumbent, in contrast to $m$, which is the prespending expected vote margin for the incumbent. Thus, it is important to keep in mind that $\Pi_m^*$ is endogenous, whereas $m$ is exogenous.

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$$
\frac{\partial^2 P}{\partial \Pi_m \partial \Pi_C} = \frac{\beta_{VI}}{\sigma^2 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0;
$$

$$
\frac{\partial^2 P}{\partial C \partial \Pi_c} = \frac{\beta_{VC}}{\sigma^2 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0;
$$

$$
\frac{\partial^2 P}{\partial \Pi_m \partial \Pi_C} = \frac{\beta_{VI} \beta_{VC}}{\sigma^4 \sqrt{2\pi}} \Phi'(\Pi_m^*) > 0;
$$

$$
\frac{\partial^2 P}{\partial \Pi_m \partial C} = \frac{\beta_{VC}^2}{\sigma^4 \sqrt{2\pi}} \Phi'(\Pi_m^*) < 0.
$$

which, from inequalities 3 and 4, implies that both parties spend more in elections that are expected to be close. That is, $d\Pi_m^*/dm < 0$ and $dC^*/dm < 0$. Also notice that this specification of $P$ has the nice property that if spending effects and marginal fundraising costs are symmetric for incumbents and challengers, then $\Pi_m^*$ is exactly equal to $m$. Even if these effects are not exactly symmetric for the two candidates, continuity implies that $\Pi_m^*$ will closely track $m$, so the prespending index of incumbent strength, $m$, is also an equilibrium postspending index of incumbent expected vote margin. In the closest possible race, in which $\Pi_m^* = 0$, $\Phi$ attains its maximum, so equilibrium spending is maximized for both candidates. For incumbents less vulnerable than this (nearly all cases in practice), equilibrium spending by each candidate is decreasing in the (normalized) equilibrium expected incumbent share, $\Pi_m^*$.

Before proceeding, it is useful to point out that signs on the three cross-partial derivatives—$\partial^2 P/\partial \Pi_m \partial C$, $\partial^2 P/\partial \Pi_m < 0$, and $\partial^2 P/\partial \Pi_C < 0$—have a natural interpretation. Consider the first two of these. Because the density function is single peaked at 0, the effect of spending on the probability of winning is higher the closer the election is expected to be. That is, it is highest at $\Pi_m^* = 0$. Then, for values of $m$ such that the equilibrium probability the incumbent wins is exactly .5, the sign of both cross-partial is 0, since $\Phi'(0) = 0$. (In this case, equilibrium spending levels are locally constant in $m$.) For values of $m$ higher than this, the equilibrium probability the incumbent wins is greater than .5; therefore, as $m$ increases, all else held constant, the election will be less close, so the marginal effect of spending (proportional to $\Phi'(\Pi_m^*)$) is lower, which is precisely what is meant by $\partial^2 P/\partial \Pi_m > 0$. The third cross-partial derivative has a similar interpretation. When $\Pi_m^* > 0$, greater spending by the challenger leads to a closer election, which makes the marginal effect of incumbent spending higher, hence $\partial^2 P/\partial \Pi_C > 0.11$.

A mirror image of this effect arises if $\Pi_m^* < 0$. For this rare category of races, in which incumbents are

\[ 9 \text{ Keep in mind that the equilibrium spending effects themselves are a function of } m. \]
\[ 10 \text{ That is, expected closeness decreases in the absolute value of } m. \]
more likely to lose than to win, equilibrium spending by
each candidate actually increases in the normalized
equilibrium expected incumbent margin of victory, \( \Pi_m^* \). Finally, if cost and spending effects are roughly
the same for the incumbent and the challenger, then
\( \Pi_m^* = 0 \) if and only if \( m = 0 \).

To consider specific candidate cost functions, sup-
pose \( K \) is quadratic, so \( K_I(I) = (1/2a_I)I^2 \) and
\( K_C(C) = (1/2a_C)C^2 \), where \( a_I \) and \( a_C \) can be inter-
preted as fundraising efficiency parameters for the
incumbent and challenger, respectively. Substituting
this into the equilibrium equations above yields
\[
\frac{\alpha_I \beta_{IV}}{\sigma \sqrt{2\pi}} \varphi(\Pi_m^*) = I^*; \quad (8)
\]
\[
-\frac{\alpha_C \beta_{VC}}{\sigma \sqrt{2\pi}} \varphi(\Pi_m^*) = C^*. \quad (9)
\]

This implies that equilibrium candidate spending
should be proportional to the normal density of the
expected incumbent margin of victory. It is also inter-
esting to note another property of the equilibrium. Our
assumptions guarantee that \( C^* \) and \( I^* \) are both strictly
positive, so we can divide equation 8 by equation 9 to
obtain
\[
I^*(m) = \frac{\alpha_I \beta_{IV}}{\alpha_C \beta_{VC}} C^*(m),
\]
which illustrates explicitly why there is a positive cor-
relation between \( I^* \) and \( C^* \) in the normal-quadratic
model that is so widely used for estimation.

The key is that, for districts in the range of a 50–50
expected vote, the vote-on-spending slopes are near
zero; cases with mildly negative slopes balance a similar
number of mildly positive slopes. Therefore, by limiting
our sample to races expected to be close, there should
be little concern about a simultaneity bias to distort the
estimation of spending effects in the OLS estimation of
the vote equation.

METHODOLOGY: SIMULTANEITY BIAIS AND
VARYING PARAMETERS

The following three-equation system is used to explain
the methodological implications of our theoretical
model. The first equation has incumbent share of the
two-party vote as a dependent variable, and the other
two have challenger and incumbent spending, respec-
tively, as the dependent variable. This system was
studied extensively in Erikson and Palfrey (1998), and
the reader is referred there for more details.

\[
V = \beta_V C + I + \gamma Z + \varepsilon_V; \quad (10)
\]

\[
C = \beta_C V_C + \varepsilon_C = \beta_C V + w_C; \quad (11)
\]

\[
I = \beta_I V_I + \varepsilon_I = \beta_I V + w_I; \quad (12)
\]

where \( V = \) incumbent percentage of the (two-party)
vote;

\( C = \) the log of challenger spending (in 1978 dol-
\( I = \) the log of incumbent spending (in 1978 dol-
nars) plus 5,000;
\( Z = a \) set of instruments to predict the incumbent
vote without directly affecting spending;
\( V_C, V_I = \) the challenger and incumbent expectations of
the incumbent vote margin;
\( \beta_V, \beta_I, \beta_C = \) candidates observe an imperfect signal of \( V \); and
\( w_C, w_I = \) \( \beta_{CV} u_C + \varepsilon_C \) and \( \beta_{IV} u_I + \varepsilon_I \).

We assume \( w_C \) and \( w_I \) are uncorrelated with \( \varepsilon_V \), but
we allow that \( w_C \) and \( w_I \) may be correlated with each
other.

Two interesting methodological problems occur in
the estimation of this three-equation system. The first
and most commonly noted is simultaneity. In a nut-
shell, both candidates spend more when races are
close. The second involves “varying parameters.” That
is, the coefficients on the independent variables in the
two spending equations (11 and 12) vary across the
sample in a systematic way. Specifically, our theoretical
model establishes that the equilibrium values of \( \beta_{CV} \)
and \( \beta_{IV} \) covary with expected vote.

We next derive the asymptotic bias of the coefficients
in the vote equation and prove that this bias vanishes in
close races. Before presenting the formal argument, we
explain the intuition behind this. The results from the
theory section established that both \( \beta_{CV} \) and \( \beta_{IV} \) are
approximately zero when \( V \) is in the neighborhood of
50%. This is very important, because it means that
incumbent and challenger spending on “close” races in
this range can be treated simply as exogenous, and
unbiased estimation of the vote equation can be done
by OLS, which we prove formally below. Intuitively, it
means that the parameters of equations 11 and 12
vanish. In contrast, for the large majority of districts
used in past cross-sectional studies, \( V \) is significantly
greater than 50, and \( \beta_{CV} \) and \( \beta_{IV} \) are negative in this
region, with their magnitude varying with \( V \) in a
nonlinear way. This implies a negative bias in the OLS
estimates of \( \beta_{CV} \) and \( \beta_{IV} \), so OLS will inflate chal-
lenge spending effects and deflate incumbent spending
effects.

To derive formally the asymptotic bias of the coefficients
in the vote-on-vote coefficients, we fix \( Z \) at some level, so that
\( \beta_V \) and \( \beta_C \) can be treated as approximately constant.
To simplify the exposition, suppose that, conditional on
\( Z, I \) and \( C \) are uncorrelated.12 Standard analysis then
yields (see Appendix A for derivation):

\[
\text{plim} \frac{\beta_{IV}}{\sigma^2_v} = \beta_{IV} + \frac{\beta_{IV} \sigma^2_v}{(1 - \beta_V \beta_{IV} - \beta_{CV} \beta_{VC}) \sigma^2_V} \neq \beta_{IV}; \quad (13)
\]

unless \( \beta_{IV} = 0 \), and

\[
\text{plim} \frac{\beta_{VC}}{\sigma^2_v} = \beta_{VC} + \frac{\beta_{VC} \sigma^2_v}{(1 - \beta_V \beta_{IV} - \beta_{CV} \beta_{VC}) \sigma^2_V} \neq \beta_{VC}; \quad (14)
\]

12 The general case is treated in Appendix A.
unless $\beta_{CV} = 0$. The second terms in equations 13 and 14 represent the asymptotic bias of the OLS estimate of the spending-on-vote coefficients, and $\sigma^2$ denotes variance.

It follows immediately that if $\beta_{IV} = \beta_{CV} = 0$, then the asymptotic bias disappears for both challenger and incumbent spending effects, and OLS estimates of $\beta_{IV}$ and $\beta_{VC}$ are consistent. This is indeed our main theoretical point, since the spending game model implies that $\beta_{IV} = \beta_{CV} = 0$ when the expected vote is 50–50. Therefore, $\beta_{IV}$ and $\beta_{VC}$ can be estimated without simultaneity bias by using close races. Furthermore, as we show in Appendix A, this is true even if $I$ and $C$ are highly correlated.

Even when races are not expected to be close, it is not difficult to sign the bias terms. It is easy to show that $(1 - \beta_{IV}^2 - \beta_{CV}^2) > 0$, so the sign of the two bias terms in equations 13 and 14 are given by the signs of $\beta_{IV}$ and $\beta_{CV}$, respectively. If the incumbent is expected to have a safe margin of victory, then the vote effects on spending ($\beta_{IV}$ and $\beta_{CV}$) are negative (safer incumbents induce less spending), so the estimates of both spending effects on the vote are biased in the negative direction. Hence, the bias deflates the positive effect of incumbent spending on the incumbent vote and inflates the negative effect of challenger spending on the incumbent vote.

Interestingly, the two biases are not exactly symmetrical, even if the vote-on-spending effects are equal for incumbents and challengers. The reason is that, empirically, challenger spending has more variance than incumbent spending ($\sigma^2_V < \sigma^2_C$), so the negative simultaneity bias deflates the incumbent spending effect to a greater extent than it inflates the challenger spending effect. One final point remains. The parameters $\beta_{IV}$ and $\beta_{CI}$ are only approximately constant because, for instance, $\beta_{IV}$ will locally vary as $V$ responds as a function of both $w_v$ and $\epsilon_v$, even with $Z$ held fixed. But if the variability in $\beta_{IV}$ and $\beta_{CI}$ is symmetrical in the neighborhood of 50% (so that the effect of 50 + $\epsilon$ percent of the vote equals the effect of 50 – $\epsilon$ percent), then nothing of consequence changes. Should this symmetry not hold, the point at which the asymptotic bias is exactly equal to 0 could be slightly greater than or slightly less than 50–50.

THE DATA

The units of our analysis are 1,792 contested House races, 1974–80 and 1984–90, involving a veteran (i.e., nonfreshman) incumbent who won a contested race from the same district in the previous election. We generated a reduced-form equation whereby we predict the expected prespending incumbent vote ($Z$) from a collection of exogenous variables: lagged incumbent vote; district presidential vote for the incumbent’s party (1976 in the 1970s, 1988 in the 1980s); and effects for year, incumbent party, a dummy for southern states, and the relevant interaction terms. Because this reduced-form equation is lengthy and tangential to our central point, the details of its construction are presented in Appendix B. It accounts for 58% of the variance in the incumbent vote. We exploit this equation to obtain an expected incumbent vote, which is not identical to the expected vote that candidates themselves observe, but we consider it a good measure of the candidates’ expected vote that is uncontaminated by spending effects.

This measure of the expected vote ($Z$) serves as an instrumental variable for a two-stage least-squares analysis of the effects of the (expected) vote on the two spending variables. If the vote-on-spending effects were linear, then we would simply regress spending on the predicted vote. Here, our prediction is decidedly nonlinear, following our theoretical argument. Accordingly, our task is to estimate the nonlinear slope contingent on the value of the expected vote.

The Effect of the Vote on Spending

Do we obtain our prediction of a curvilinear effect of the vote on spending, that is, zero when the vote is 50–50? Figure 1 is a scatter plot of the logarithm of real challenger spending in all veteran-contested House elections between 1972 and 1990 against incumbent share of the vote. The fit with the simple normal-quadratic model proposed above is surprisingly good. To illustrate, we have superimposed a fitted nonlinear regression line, using locally weighted least squares, or LOWESS (Cleveland 1979). This has a shape that resembles the bell curve signature of a normal distribution, with a maximum near to 50% of the incumbent vote, as predicted. A similar graph in Figure 2 for incumbent spending as a function of vote shows a similar fit.

As clear as the patterns of figures 1 and 2 appear to be, technically they are not quite right. Just as we should not estimate the effects of spending on the vote without considering the simultaneity bias from the influence of anticipated vote on spending, we should not depict the (anticipated) vote-on-spending effect without considering the reverse effects of spending on the vote. Accordingly, we turn to the two-stage least-squares approach to estimate the nonlinear slopes of spending predicted from the proxy for the vote, that is, our prediction of the vote in the reduced-form equation ($Z$). We simply show the nonlinear slopes of spending contingent on the expected vote rather than the actual vote. This task approximates the presentation of figures 1 and 2; by using the instrumental

13 By continuity, the asymptotic bias is approximately 0 for close races in the neighborhood of 50–50.
14 Note that if the expected incumbent vote were less than 50%, then the direction of the bias would reverse, and spending effects would be inflated.
15 The 1982 races were excluded because we used lagged incumbent vote to construct $Z$, and the 1980 incumbent vote does not match the 1982 incumbent vote due to redistricting.
16 Year, incumbent party, and region (South/non-South) were combined in all possible interactions, in effect generating separate intercepts for 8 x 2 x 2 = 32 groups, such as “nonsouthern 1974 Republicans.”
17 The fit is done using a bandwidth of .20.
variable of the expected vote, we avoid any distortion due to causal feedback.

Figure 3 displays the estimated vote-on-spending effects, using $Z$ as our proxy for the actual vote. The slopes shown are LOWESS estimates—nonlinear curves built from locally linear approximations over different bands of the expected vote. Figure 3 replicates the patterns shown in figures 1 and 2. We observe the nonlinearity in the slopes, corresponding to the kind of nonlinearity predicted by the theoretical model presented previously. For lopsided races in the 55–75% range, the spending-on-vote slopes are steeply negative. As the expected incumbent vote approaches 50%, the spending-on-vote slopes flatten considerably. We also see evidence of the predicted sign reversal when the incumbent’s expected vote drops below 50%, a fate that befell 17 incumbents.

By overlaying the spending functions for incumbents and challengers in the same graph, Figure 3 highlights asymmetries between their spending equations. The slope is steeper for challengers than for incumbents. The largest spending gaps are in safe districts, which is consistent with the theoretical argument that in safer seats there is more incentive for contributors to invest in incumbents rather than challengers (Baron 1989a, 1989b; Snyder 1990). Note that in very competitive districts the gap between incumbent and challenger spending virtually disappears. Incumbents evidently are usually better able to raise money than are challengers but not due only to their incumbency. Rather, an incumbent’s advantage over a challenger in raising cash appears to increase with his likelihood of winning. To the extent that the expectation of victory induces lower costs of fundraising, the effect of spending on the vote becomes all the more crucial. A spending advantage induced by an anticipated victory further increases the advantage of likely winners if indeed candidate spending appreciably affects the vote.

For the purpose of estimating spending effects, it is less important to account for the exact spending functions than to note that the functions shown in figures 1 and 2 are maximized when the vote is 50–50. We now see that our game-theoretic model of optimal spending decisions by candidates is not merely an exercise in formal theory; it matches up with the data surprisingly well. Thus, it provides a rigorous and empirically sound theoretical foundation for what common sense has long suggested should be a basic law of campaign spending: All else equal, both candidates will spend more when the election is expected to be close.

These results clearly indicate the nature of the measurement problem in estimating the effects of challenger and incumbent spending on the vote. Since the (expected) incumbent vote in contested House elections is almost always in the range of 55–75%, simultaneity should bias OLS estimates of incumbent spending effects downward and estimates of challenger spending effects upward (more negative for the incumbent vote).18 Any simple one-equation model will

18 Approximately 80% of the cases (1,491 of 1,792) have an expected vote in the 55–75% range.
generally overestimate the negative effect of challenger spending and underestimate the positive effect of incumbent spending on the incumbent share of the vote.

We are now in a position to correct this situation. By isolating districts where the vote is expected to be close, we select districts where the additional shocks to the expected vote should have little effect on spending. For these districts, incumbent spending and challenger spending can be considered exogenous, which we can estimate via OLS.

**The Effects of Spending on the Vote**

Table 1 presents OLS equations that predict the incumbent vote from the two logged spending variables and the expected vote, with cases grouped according to expected vote. In column 1 is the equation for the 40 very close races with an expected incumbent vote below 52%. For these contests, the means for both the actual and predicted incumbent vote are virtually 50%. In theory, if the effects of incumbent and challenger spending are identical, we should find virtually identical coefficients for the two spending variables, except for their sign. This is what column 1 shows. More-over, even with only 40 cases, both spending coefficients are statistically significant.20

Column 2 expands the analysis to the 77 cases in which the expected incumbent vote was 52–55%. As illustrated in Figure 3, this is the beginning of the range of the expected vote at which the slopes for its effect on spending are negative. Here, the statistical bias should create only a mild diminution of the incumbent spending coefficient. Column 2 shows another statistically significant coefficient for incumbent spending, but less than that for challenger spending, as predicted.21

When the incumbent is expected to win by about 55% or more, the vote-on-spending slopes are steeply negative, so the bias is quite severe. As column 3 of Table 1 reveals, for districts in which the incumbent is expected to receive 55–58% of the vote, the OLS estimate of incumbent spending is negligible and decidedly nonsignificant. This pattern is repeated in further categories of three-point increments of the expected incumbent vote. Overall, when that vote exceeds 58%, the simultaneity problem is so severe that

20 These results hold when candidate quality is controlled by means of a dummy variable, challenger prior office, added to the right-hand side; they also hold (with an increased standard error, as expected), when Z is replaced by its many component variables. See Appendix B for details.

21 The difference is significant at the $p = .11$ level.
the coefficient on incumbent spending actually reverses sign. The estimates are presented in column 4. The pooled results for all cases are given in column 5, which produces the well-known implausible result that challenger spending matters, but incumbent spending does not.

Our OLS estimates convincingly demonstrate that incumbent spending matters in close races. It may be tempting also to accept the OLS estimates for less competitive races, which would suggest that, unlike vulnerable incumbents, safe incumbents cannot spend effectively. It would be a mistake, however, to treat the coefficients in Table 1, columns 4 and 5, as anything short of seriously biased, given the formal results discussed earlier. Of course, there is no plausible reason why marginal incumbents (often marginal because of their own ineptitude) would spend effectively, whereas safe incumbents (coded safe in part due to their track record) would spend ineffectively.

As we move from competitive to safe seats, the OLS coefficients for incumbent spending decline so far as to be virtually zero. Meanwhile, the OLS coefficients for challenger spending show no corresponding increase. If simultaneity bias for noncompetitive races accounts for the deflated estimates for incumbent spending, why do we not see an inflation in coefficients for challenger spending? There are at least three contributing factors that can be identified.

First, for challengers and perhaps for incumbents as well, it is plausible that spending effects (votes gained per logged dollar) decline when the seat is perceived as safe. When the outcome appears foregone, voters may pay less attention, and candidates may spend a smaller portion of their budget on actual vote-winning activities. Furthermore, the generally inexperienced challengers for safe seats may spend inefficiently. All this suggests offsetting processes that affect OLS estimates of challenger spending effects as one moves from competitive to safe seats. The bias exaggerates challenger spending effects that are getting weaker and deflates incumbent spending effects that may also be getting weaker.

A second reason OLS estimates of challenger spending effects appear insensitive to simultaneity bias may be that there actually is less such bias for challengers. Being less informed than incumbents, inexperienced challengers may estimate the expected vote with greater noise. In particular, they may be less able to gauge the unmeasured causes of the vote, such as their own vote appeal, the contribution of the previous challenger to the incumbent's previous margin, and any shift in the incumbent's attractiveness to voters.22

A third reason, already noted, is the fundamental asymmetry of the simultaneity bias for challengers and incumbents: The vote-on-spending effects and spending-on-vote effects have opposite signs for incumbents but the same sign for challengers. For favored incum-

22 Some empirical support for this argument is reported in Erikson and Palfrey 1998.
bents, the cycle is negative: New spending increases the incumbent's vote, which dampens the value of spending further. For underdog challengers, the cycle is positive: New spending increases their vote, which amplifies the value of spending further. One statistical consequence is that (everything else being equal), in equilibrium, challenger spending will have greater variance than incumbent spending, a pattern clearly observed in the data. As can be inferred from equations 13 and 14, higher spending variance attenuates the bias in estimated spending effects. See Appendix A for further details.

Putting the three parts of the explanation together, as one moves from competitive to safe districts, the OLS estimate of incumbent spending collapses due to the increased bias plus a possible decline in spending effectiveness when the seat is safe. Meanwhile, the OLS estimate of the challenger spending effect stays roughly constant, as the lesser bias may decline only modestly and is offset by the decline in spending effectiveness.23

The CQ Expected Vote

One alternative way of measuring perceived electoral closeness deserves our attention. We supplement our measurement of electoral expectations by incorporating Congressional Quarterly's measure of the expected vote. During October of every campaign over the two decades of this analysis, CQ rated each of the 435 House races on a scale from "safe Democratic" to "safe Republican."24

We fold this scale based on incumbent party affiliation to form a scale of expected vulnerability. Three categories emerge (the numbers in parentheses are usable cases): close, coded by CQ as "favoring the challenger" or "too close to call" (N = 83); leaning, coded by CQ as "leaning" or "likely" for the incumbent (N = 340); and safe, coded by CQ as "safe" for the incumbent (N = 1,369).

A district's rating on this scale reflects not only the exogenous variables that make up our measure of the expected vote but also the more intangible sources of the current vote (challenger strength, and so on) that candidates may observe during the campaign but are not part of our equation. To the extent that CQ raters observe the same intangible sources of the vote that candidates observe when they make their spending decisions, the simultaneity problem becomes minimized.

This argument is not new (Abramowitz 1991). Following logic similar to that in the previous paragraph, Abramowitz decided to include the CQ expected vote category in OLS equations predicting the incumbent vote. Even with CQ ratings on the right-hand side of the equation, the coefficient for incumbent spending was insignificant. Abramowitz concluded that the estimates of small incumbent spending effects were not an artifact of simultaneity bias, as commonly supposed.

One limitation of CQ ratings as a control for intangible sources of the vote is that they chop the cases coarsely into only three usable categories, and 78% fall in the safe category. Obviously, different degrees of "safeness" are not distinguished. In contrast, the CQ-coded close races are very close on average, so the simultaneity problem issue 41 (the 41st week), and (for the years of our analysis) can always be found for the CQ issue for the date ranging from October 9 to October 15.

23 As side evidence for declining spending effectiveness in general as seat safety increases, consider the OLS regression of the incumbent vote on the difference between I and C for different levels of Z. Since the IC difference expands as a function of the anticipated vote margin, the positive bias in this estimate should increase with Z, the instrument. Instead, the coefficient for the IC difference declines with Z.

24 The CQ predictions are found in Congressional Quarterly's biennial election supplements. Each election year, Congressional Quarterly publishes a supplement of more than 100 pages, handicapping the congressional elections. The supplement is almost always published in issue 41 (the 41st week), and (for the years of our analysis) can always be found for the CQ issue for the date ranging from October 9 to October 15.
but with the addition of two CQ variables. Here, CQ variables the log of spending for incumbents, the log of American Political Science Review Vol. 94, No. 3

Simply including these variables in the categorization helps predict the vote independent of measure is of little importance for predicting the served, we find a nonsignificant coefficient for incumbent spending.25 In races that lean to the incumbent, column 2, the coefficient for incumbent spending is of lesser magnitude than that for challenger spending.26 Yet, the incumbent spending coefficient is statistically significant, even with the evident simultaneity bias.

Let us focus on the crucial set of races deemed by informed observers at the time as too close to call. These are the pivotal districts that were seen to be most in play during the actual election campaign. The incumbent spending coefficient for this set of cases is, if anything, slightly stronger than that for challengers because of a drastic reduction if not elimination of the simultaneity bias. In races that CQ rates too close to call, the simultaneity bias is minimal in the first place.

Note that the coefficients for both incumbent and challenger spending in CQ’s close races are lower than in races predicted to be close by the Z index. This is expected, because the CQ rating incorporates the effects of early spending. In effect, Table 2 shows the influence of unanticipated late spending by both candidates, as measured by the proxies of incumbent and challenger spending levels over the duration of the campaign. Put another way, the coefficients for spending were muted due to CQ’s anticipation of their effects. Therefore, if anything, Table 2 underestimates the magnitudes of spending effects.

CONCLUSION

This article has examined a simple theoretical model of the spending game in an election between competing candidates for public office. That model strongly suggests a nonlinear relationship between expected incumbent share of the vote and spending by both candidates. It also illuminates the nature of the simultaneity bias introduced when OLS regressions use cross-sectional data pooled across both safe and competitive districts. The key insight is that, in equilibrium, total spending is continuously increasing in the expected closeness of the race. Because total spending reaches a maximum when expected incumbent share of the vote is 50%, the slope of spending with respect to incumbent vulnerability in this range is necessarily zero, so a sample

is minimal. The mean incumbent vote in the CQ-defined close races is 49.5%.

In the analysis that follows, we estimate the vote equation separately for the three categories (close, leaning, and safe districts), using as right-hand-side variables the log of spending for incumbents, the log of spending for challengers, and our measure of the expected vote. We also show the results when we pool the cases, with CQ groupings on the right-hand side as dummy variables. Table 2 displays the results. (See Appendix B for the auxiliary analysis, which gives the results with challenger prior office added to the right-hand side and with Z replaced by its many component parts.) With the control for CQ ratings, the spending coefficients are slightly lower than in Table 1, because the ratings are created just a few weeks before the election and inevitably absorb some of the effects of early spending during the campaign.

The fourth column of Table 2 shows the results with all cases pooled, comparable to column 5 of Table 1 but with the addition of two CQ variables. Here, CQ categorization helps predict the vote independent of our expected-vote measure; just as Abramowitz observed, we find a nonsignificant coefficient for incumbent spending. Simply including these variables in the regression does not redress the theoretical problem that equilibrium spending varies systematically with expected closeness. Hence, we consider the first three columns, which show the regressions separately by degree of CQ “safeness.”

In column 1 are the OLS results for races CQ defined as close. For this group, our expected vote measure is of little importance for predicting the outcome. Crucially, incumbent spending is statistically significant and, for the first time among our sets of equations, larger in absolute magnitude than the coefficient for challenger spending.25

This result contrasts with that given in column 3, at the other end of the safety spectrum. For the vast bulk of contests deemed safe for the incumbent, the coefficient for incumbent spending is nonsignificant, as if incumbent spending did not matter. In these races the simultaneity distortion is severe.

In races that lean to the incumbent, column 2, the coefficient for incumbent spending is of lesser magnitude than that for challenger spending.26 Yet, the incumbent spending coefficient is statistically significant, even with the evident simultaneity bias.

25 The difference in magnitude is not statistically significant, however. Using conventional significance levels, one cannot reject the null hypothesis that incumbent and challenger spending effects are equal.

26 The null hypothesis of equal spending effects for incumbents and challengers is rejected at the .01 significance level.
that includes only close races (expected incumbent vote share in the neighborhood of 50% will be immune from the kind of simultaneity bias that plagues OLS regression on the full sample. This facilitates a clean estimate of incumbent and challenger spending effects.

We tested the model with veteran incumbent U.S. House races from 1974 to 1990. For close races, not only does incumbent spending pass the threshold of statistical analysis but also our estimate of the size of this effect is statistically indistinguishable from the effect of challenger spending. Thus, in close races, a given amount of spending wins about as large a share of the vote for an incumbent as for a challenger. We also show clearly how the simultaneity problem grows progressively more severe as the closeness of races diminishes. Two different measures of expected closeness are employed, and both yield identical conclusions. The first combines prespending long-term partisan strength in the district, short-term national forces, and lagged incumbent vote. The second uses the pre-election CQ categorization of close, incumbent-leaning, and safe races.

Both the theory and empirical findings complement the results reported in Erikson and Palfrey (1998), which presents a three-equation model of spending and makes two qualitative findings: (1) Incumbent and challenger spending is an increasing function of race closeness, and (2) the estimated effects of incumbent spending decline with seniority. Point 1 is a central prediction of our theoretical model, and point 2 is consistent with the fact that junior incumbents are typically more vulnerable than senior incumbents. In our vulnerable incumbent subsample (expected vote <.52), nearly half (48%) were either sophomores or juniors, compared to only slightly more than one-third (36%) in the remaining races. As for the magnitude of spending effects, the estimates for veteran races in Erikson and Palfrey (1998) (and elsewhere in the literature) are smaller in magnitude than those reported here for close races. There are two possible reasons. First, because of pooling over all races, close and safe, the estimates elsewhere could be biased, even with carefully designed methods to correct for simultaneity. We showed here, both theoretically and empirically, that the degree of bias will vary with incumbent vulnerability in predictable ways, and this is not accounted for in those methods. Second, it may be that spending effects are indeed high in a tight race, due to its salience to the voters and extensive media coverage. This interesting possibility would require a new approach to sort out differences in spending effects as a function of incumbent vulnerability.

To summarize, the results presented here have potentially important consequences for understanding the connection between money and the incumbency advantage. Political scientists have hesitated to endorse this view, in part because it is difficult to estimate spending effects. Our findings have implications too complex to explore here, but they should finally put to rest any lingering doubt about the significance (and similarity) of incumbent and challenger spending effects in close races.

APPENDIX A

Consider our three-equation system with Z fixed at some level:

\[ V = \beta_1 I + \beta_{1C} C + \epsilon_v \]  
\[ I = \beta_{1V} V + w_i \]  
\[ C = \beta_{1C} V + w_c. \]

Since \( \beta_{1V} \) and \( \beta_{1C} \), themselves vary as a function of expected vote, we fix Z (the instrument for expected vote) in the analysis below, so \( \beta_{1V} \) and \( \beta_{1C} \) can be treated as constant. Our interest is in the bias in the OLS estimates of \( \beta_{1V} \) and \( \beta_{1C} \), the spending effects on vote. The analysis below derives the asymptotic bias of the OLS estimate of \( \beta_{1V} \). Calculation of the asymptotic bias of \( \beta_{1C} \) is done in a similar way.

To start, by successive algebraic substitution, we solve the simultaneous equations for \( I, \) \( V, \) and \( C \) and obtain the following exact expression for the OLS estimate of \( \beta_{1V} \). The lowercase symbols indicate sample values of \( V, I, \) and \( C \); observations in the sample are indexed by \( j \); and \( b_{1C} \) is the sample bivariate regression coefficient of \( I \) on \( C \).

\[ \beta_{1V}^{OLS} = \frac{\sum (i - b_{1C}) (v - b_{1C} c)}{\sum (i - b_{1C})^2} \]

\[ OLS = \beta_{1V} + \frac{\sum (i - b_{1C}) \epsilon_v}{\sum (i - b_{1C})^2}. \]

Applying the rules of probability limit gives

\[ \operatorname{plim} \beta_{1V}^{OLS} = \beta_{1V} + \frac{\operatorname{cov}(I - \gamma C, \epsilon_v)}{\operatorname{var}(I - \gamma C)}, \]

where \( \gamma \) is the expected value of \( b_{1C} \). Some more algebra yields the following expressions:

\[ \operatorname{cov}(I, \epsilon_v) = \frac{\beta_{1V} \sigma_{1V}^2}{1 - \beta_{1V}^2 \beta_{1C} \beta_{1C}^2}, \]

\[ \operatorname{cov}(C, \epsilon_v) = \frac{\beta_{1C} \sigma_{1C}^2}{1 - \beta_{1V} \beta_{1C} - \beta_{1V} \beta_{1C}^2}, \] and

\[ \operatorname{var}(I - \gamma C) = \sigma_I^2 - \gamma \sigma_C^2, \]

which gives

\[ \operatorname{plim} \beta_{1V}^{OLS} = \beta_{1V} + \frac{(\beta_{1V} - \gamma \beta_{1C}) \sigma_v^2}{(1 - \beta_{1V} \beta_{1C} - \beta_{1V} \beta_{1C}^2)(\sigma_I^2 - \gamma \sigma_C^2)} = \beta_{1V} + \Delta \Phi, \]

where

\[ \Phi = \beta_{1V} + \Delta \Phi. \]

27 Recall that \( \beta_{1V} \) and \( \beta_{1C} \) vary as a function of \( V \), so, for any fixed \( Z \), they are only approximately constant.
\[ \Delta = \frac{\sigma_{\gamma}^2}{(1 - \beta_{\gamma}^2 \beta_{\gamma} - \beta_{CV} \beta_{VC})(\sigma_{\gamma}^2 - \gamma^2 \sigma_{\gamma}^2)} \]

and

\[ \Phi = \beta_{\gamma} - \gamma \beta_{CV}. \]

So the asymptotic bias of the OLS estimate of the incumbent spending effect on vote is simply \( \Delta \Phi \). Because \( \Delta \) is always positive, the sign of the bias is determined by the sign of \( \Phi \), which is governed by the signs of \( \beta_{\gamma} \), \( \beta_{CV} \), and \( \gamma \).

It follows immediately that if \( \beta_{\gamma} = \beta_{CV} = 0 \), then the asymptotic bias disappears. This is indeed our main theoretical point, since our model implies that \( \beta_{\gamma} = \beta_{CV} = 0 \) when the expected vote is 50–50.\(^{28}\) Therefore, spending effects \( \beta_{\gamma} \) and \( \beta_{VC} \) can be estimated consistently by restricting attention to close races. This key result also would hold in more general models if one were to introduce additional parameters to index the degree of the simultaneity problem, as in Erikson and Palfrey (1998).

We next turn to signing the bias for the remainder of races in our sample, those in which the incumbent is expected to win with a safe margin. For these races, our model shows that \( \beta_{\gamma} \) and \( \beta_{CV} \) are both negative. Therefore, unless \( \gamma \) is very large (and positive), but safe races, by the game-theoretic model.\(^{29}\) The bias could not necessarily be signed were \( \gamma \) very large, which implies the intuitive result that OLS causes a downward bias in the estimated incumbent spending effect.

By similar argument, we can obtain the following expression for the bias in challenger spending effects:

\[ \text{plim} \left( \beta_{CV}^{OL} \right) = \beta_{CV} + \left( \beta_{CV} - \delta \beta_{\gamma} \right) \left( \sigma_{\gamma}^2 - \frac{\gamma^2 \sigma_{\gamma}^2}{(1 - \beta_{\gamma}^2 \beta_{\gamma} - \beta_{CV} \beta_{VC})(\sigma_{\gamma}^2 - \delta^2 \sigma_{\gamma}^2)} \right), \]

where \( \delta \) is the expected value of \( \beta_{CV} \), and the asymptotic bias is determined by the sign of \( \beta_{CV} - \delta \beta_{\gamma} \). As with the incumbent, if \( \beta_{\gamma} = \beta_{CV} = 0 \), then OLS estimates are consistent. Otherwise, the bias is negative (which overestimates the effect of challenger spending, since \( \beta_{CV} < 0 \)) unless \( \delta \) is very large and positive, in which case the negative bias becomes negligible or possibly reverses sign. Empirically, it turns out that \( \delta \) is much larger than \( \gamma \), which helps explain why the data for safe races indicate a greater problem of simultaneity bias in \( \beta_{CV}^{OL} \) compared to \( \beta_{CV}^{OLS} \). By definition, \( \delta = (\sigma_{\gamma}^2/\sigma_{\gamma}^2) \gamma \), and empirically \( \sigma_{\gamma}^2 > \sigma_{\gamma}^2 \) (even with \( \gamma \) held constant, as we have been assuming throughout). The latter fact has implications for the numerator and denominators of both bias terms. This is evident from inspection of equations 13 and 14.

The difference in magnitude between \( \sigma_{\gamma}^2 \) and \( \sigma_{\gamma}^2 \) is partly driven by fine details of the simultaneous system. From equations 15–17 we can decompose \( \sigma_{\gamma}^2 \) and \( \sigma_{\gamma}^2 \) as follows:

\[ \sigma_{\gamma}^2 = \beta_{\gamma}^2 \sigma_{\gamma}^2 + \sigma_{\gamma}^2 + 2 \beta_{\gamma} \sigma_{\gamma} \gamma, \]

\[ \sigma_{\gamma}^2 = \beta_{CV} \sigma_{\gamma}^2 + \sigma_{\gamma}^2 + 2 \beta_{CV} \sigma_{\gamma} \gamma. \]

Due to the opposite signs of the spending-on-vote effects, the terms \( \sigma_{\gamma} \gamma \) and \( \sigma_{\gamma} \gamma \) have opposite signs, with \( \sigma_{\gamma} \gamma > 0 \) and \( \sigma_{\gamma} \gamma < 0 \). As a result, \( 2 \beta_{\gamma} \sigma_{\gamma} \gamma > 0 \) and \( 2 \beta_{CV} \sigma_{\gamma} \gamma > 0 \), thus expanding \( \sigma_{\gamma}^2 \) while contracting \( \sigma_{\gamma}^2 \), even if everything else is symmetric between incumbents and challengers.

APPENDIX B

Our presentation of results in the text is drawn from regressions in which the control variables are summarized in terms of the predicted vote, or \( Z \) in the terminology of our statistical discussion. The measure \( Z \) is derived from a reduced-form regression that predicts the vote from its exogenous sources—the lagged incumbent vote, the district presidential vote, and dummy variables representing year, incumbent party, region (South/non-South), and their interactions. The regression equation that produces \( Z \) is shown in Table B-1.

In the text, the key regressions (tables 1 and 2) predict the vote, when it is expected to be close, from the two spending variables plus \( Z \). Other specifications are possible. For the record, tables B-2 and B-3 present estimates from our key equations using regressions in which there is no control and in which \( Z \) is replaced by its 34 component parts. To provide complete information, and comparability with past findings, tables B-2 and B-3 also show the insignificant contribution of a dummy variable, whether the challenger held prior elective office. This dummy typically produces significant effects in regressions that use the full cross-section of races (Jacobson 1978).

The contribution (or lack of it) of the prior office dummy variable is noteworthy. Although it shows the expected negative effect on incumbent vote when all cases are combined, its effect is statistically insignificant when the regressions are conducted separately on each safeness category. Furthermore, the inclusion or exclusion of this dummy has no effect on our estimated coefficients on incumbent and challenger spending.

In theory, the separate components should predict the vote no better than their summary measure, \( Z \). This is confirmed by the deterioration of the explained variance when the components are substituted for \( Z \). Also as expected, we observe that the coefficients for incumbent and challenger spending are unaffected by the replacement of \( Z \) by its components. But also as expected, because substituting the components for \( Z \) uses additional statistical degrees of freedom, this substitution expands the standard errors of the coefficients. (Tables B-2 and B-3 present standard errors rather than \( t \)-values in order to highlight that the declines in significance are due to inflated standard errors rather than deflated regression coefficients.)

For the key regressions, the spending coefficients are insensitive to the control for \( Z \), and they even show about the same values when there are no control variables whatsoever. In fact, except for the set of incumbent-leaning districts, \( Z \) and its components make little contribution to the equations at all. This is as one would expect. When \( Z \) is held constant at values below 52% or between 52% and 55%, its range is sufficiently restricted for the presence of controls to be of little practical consequence. In effect, \( Z \) is already controlled. Similarly, the control matters little for districts that \( CQ \) rates as too close to call. The reason here is that \( CQ \) takes into account variables omitted by \( Z \). Once a race is rated a toss-up, it matters not whether the verdict results from

\(^{28}\) By continuity, the asymptotic bias is approximately 0 for close races in the neighborhood of 50–50.

\(^{29}\) For the rare races in which the challenger is expected to win handily, the bias will have the perverse opposite sign, and incumbent spending effects would be overestimated.
TABLE B-1. Coefficients from Regression Equation Generating Z, the Predicted Vote

\[ \text{Incumbent Vote}_t = \alpha_{\text{yrp}} + 0.63 (0.02) \times \text{Incumbent Vote}_{t-1} + 0.22 (0.02) \times \text{Presidential Vote}, \]

where \( \alpha_{\text{yrp}} \) is the following matrix of year-region-party dummies

<table>
<thead>
<tr>
<th>Year</th>
<th>Nonsouthern Republicans</th>
<th>Southern Republicans</th>
<th>Nonsouthern Democrats</th>
<th>Southern Democrats</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>2.25 (1.94)</td>
<td>1.49 (2.40)</td>
<td>17.64 (1.91)</td>
<td>18.72 (2.90)</td>
</tr>
<tr>
<td>1976</td>
<td>13.97 (1.93)</td>
<td>11.85 (2.33)</td>
<td>10.98 (1.92)</td>
<td>8.80 (1.92)</td>
</tr>
<tr>
<td>1978</td>
<td>14.69 (1.95)</td>
<td>14.89 (2.59)</td>
<td>10.69 (1.99)</td>
<td>12.29 (2.24)</td>
</tr>
<tr>
<td>1980</td>
<td>14.49 (1.95)</td>
<td>12.11 (3.19)</td>
<td>9.63 (1.89)</td>
<td>7.03 (2.31)</td>
</tr>
<tr>
<td>1984</td>
<td>15.68 (1.95)</td>
<td>14.81 (2.72)</td>
<td>10.05 (1.96)</td>
<td>6.12 (2.31)</td>
</tr>
<tr>
<td>1986</td>
<td>11.46 (1.91)</td>
<td>10.23 (2.53)</td>
<td>16.91 (1.89)</td>
<td>21.21 (2.28)</td>
</tr>
<tr>
<td>1988</td>
<td>12.84 (1.91)</td>
<td>12.40 (2.35)</td>
<td>13.22 (1.90)</td>
<td>14.16 (2.31)</td>
</tr>
<tr>
<td>1990</td>
<td>6.11 (1.93)</td>
<td>5.85 (2.20)</td>
<td>10.29 (1.89)</td>
<td>14.05 (2.24)</td>
</tr>
</tbody>
</table>

Note: Standard errors are given in parentheses. \( N = 1,792 \). Coefficients for year-party-region combinations represent the constant or intercept particular to the specific set of cases.

TABLE B-2. OLS Regression of Incumbent Vote on Spending by Expected Incumbent Vote, Controlling for Z and Its Components

<table>
<thead>
<tr>
<th>Z &lt; 52</th>
<th>52 &lt; Z &lt; 55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Z, Components, Prior Office</td>
<td>Control Z, Components, Prior Office</td>
</tr>
<tr>
<td>Control Z, Prior Office</td>
<td>Control Z, Prior Office</td>
</tr>
<tr>
<td>No Control</td>
<td>Control Z</td>
</tr>
<tr>
<td>Constant</td>
<td>50.55 (15.94)</td>
</tr>
<tr>
<td>Log (I)</td>
<td>4.04 (1.65)</td>
</tr>
<tr>
<td>Log (C)</td>
<td>-4.11 (1.16)</td>
</tr>
<tr>
<td>Exp. vote (Z)</td>
<td>-0.07 (0.48)</td>
</tr>
<tr>
<td>IV(_{t-1})</td>
<td>0.02 (0.51)</td>
</tr>
<tr>
<td>PV</td>
<td>-0.32 (0.28)</td>
</tr>
<tr>
<td>C's prior office</td>
<td>-1.10 (1.77)</td>
</tr>
<tr>
<td>Year-party-region dummies</td>
<td>not significant</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>.22</td>
</tr>
<tr>
<td>SEE</td>
<td>5.16</td>
</tr>
</tbody>
</table>

Note: \( IV_{t-1} \) = lagged incumbent vote, \( PV \) = incumbent party presidential vote (1976 or 1988). Year-party-region dummies represent combinations of the year, incumbent party, and South/non-South. Constants are not reported when year-party-region dummies are used, since the choice of base category is arbitrary. Standard errors are given in parentheses.
### TABLE B-3. OLS Regression of Incumbent Vote on Spending by CQ Categories of Competitiveness, Controlling for Z and Its Components

<table>
<thead>
<tr>
<th>CQ Rated Close</th>
<th>CQ Rated Leaning Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control Z, Prior Office, Components,</td>
</tr>
<tr>
<td></td>
<td>Control Z, Prior Office, Components,</td>
</tr>
<tr>
<td></td>
<td>Control Z, Prior Office, Components,</td>
</tr>
<tr>
<td>Constant</td>
<td>44.89</td>
</tr>
<tr>
<td></td>
<td>(9.99)</td>
</tr>
<tr>
<td>Log (I)</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>(1.11)</td>
</tr>
<tr>
<td>Log (C)</td>
<td>-2.47</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
</tr>
<tr>
<td>Exp. vote (Z)</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
</tr>
<tr>
<td>IVt-1</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
</tr>
<tr>
<td>PV</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
</tr>
<tr>
<td>C’s prior office</td>
<td>-0.34</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
</tr>
<tr>
<td>Year-party-region dummies</td>
<td>not significant</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>SEE</td>
<td>4.78</td>
</tr>
<tr>
<td></td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>4.50</td>
</tr>
<tr>
<td>N</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>83</td>
</tr>
</tbody>
</table>

Note: $IV_{t-1}$ = lagged incumbent vote. $PV$ = incumbent party presidential vote (1976 or 1988). Year-party-region dummies represent combinations of the year, incumbent party, and South/non-South. Constants are not reported when year-party-region dummies are used, since the choice of base category is arbitrary. Standard errors are given in parentheses.

Measurable variables composing $Z$ or intangible sources of candidate strength and weakness known to $CQ$ and other observers but not measured as part of $Z$.

**REFERENCES**


