Explanation of the Discontinuity in the Spin-Relaxation Time of $^3$He-$A_1$

In the superfluid $A_1$ phase of $^3$He only one spin population is condensed. The broken (relative) spin-orbit-gauge symmetry implies a magnetic-induced superflow or magnetic fountain effect.\(^1\) By using this effect Lu, Jiang, and Kojima\(^3\) (LJK) measured the spin-relaxation time $\tau$, i.e., the time needed for the nonequilibrium magnetization ($M \neq \chi H$) to come to its equilibrium value ($M = \chi H$) due to internal spin relaxation. The most striking result of LJK was a discontinuity of $\tau$ as a function of temperature at $T \approx T_m \equiv (T_{c1} + T_{c2})/2$. [$T_{c1}$ and $T_{c2}$ are the $A_1$-$A$ and $A_1$-normal ($N$) transition temperatures, respectively (Fig. 1).]

The goal of this Comment is to show that the discontinuity may be caused by the change in the interface boundary present in the apparatus used to measure $\tau$. The basic setup is given in Refs. 2 and 3. The $A_1$ phase is contained by a rigid and insulating wall at $x=0$ (Fig. 1). Because of the decreasing magnetic field for $x \to \infty$ the $A_1$ phase is in contact with the $A$ phase or $N$ phase at $x=L$ ($L$ is of order centimeters) depending on the temperature (Fig. 1). The change from the $A$ to $N$ phase also occurs at a temperature approximately equal to $T_m$. This is the key to explaining the measured discontinuity.

Magnetization may be transported away with a typical time $\tau$, rather than relaxing with $\tau$. This is analogous to the equilibration of temperature in an inhomogeneously heated body.\(^4\) If the $N$ phase is a sufficiently good magnetic insulator, the measured time is indeed the intrinsic time $\tau$. The $A$ phase (and also the $A_1$ phase) can transport magnetization without dissipation: They are ideal magnetic conductors. In this case the transport time is limited only by the need for dissipative currents due to the boundary conditions at $x=L$ and $x=0$. This is because a dissipation-free (reactive) magnetization current is coupled to a heat current. The latter current must be compensated by a dissipative heat current because no heat may be transported. The dissipative heat current, which is also coupled to a dissipative magnetization current, is one limiting factor in the transport time $\tau_c$ [second term of Eq. (1) below]. Furthermore, the normal-fluid velocity $v^p$ is essentially clamped because of the small flow-tube diameter (fourth-sound geometry). At $x=0$ the insulation imposes no mass, heat, and magnetization current. The boundary conditions at $x=L$ are more complicated: They may be obtained by a generalized theory of superfluid-normal interfaces.\(^5\) With these boundary conditions the calculation of the transport time $\tau_c$ is analogous to the normal-fluid thermal case.\(^4\) The result takes essentially the form

$$\tau_c = AL + BL^2 + CLg(L),$$

with $1/g = \tanh(L/\lambda) + \text{const.}$ ($\lambda$ is the shock length of order centimeters and the constant is an unknown effective surface coefficient.) The first term in Eq. (1) is due to the jump in magnetization at the interface, proportional to the spin current (magnetic Kapitza resistance). It scales with volume/surface $\propto L$. The second term is caused by the spin diffusion in $^3$He-$A_1$. The bulk dissipation gives the $L^2$ dependence ($B \sim \text{sec/cm}^{2}$). The third term has to do with the exponential variation of each hydrodynamic variable near the interface ($\sigma$ modes).\(^6\) A rough estimate gives $C \sim \text{sec/cm}$.

I conclude that the jump in the measured relaxation time is caused by the change from $A$ to $N$ phase at $x=L$. The measured time is given by the transport time $\tau_c$, Eq. (1), so long as the $A$ phase is coupled. For the $A_1$-$N$ interface, a transport is also possible, but different from the one above. The corresponding transport time becomes longer. In the latter case the results may depend on the boundary conditions at the right-hand end of the apparatus. One might expect that the corresponding time is sufficiently bigger than the intrinsic relaxation time $\tau$.

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