[4]The notation for the Poincaré algebra, including the required complex extension, is given by Heine [V. Heine, Group Theory in Quantum Mechanics (Pergamon Press, New York, 1964), p. 351.

W-SPIN AND B-SPIN SUBGROUPS OF SU(12)

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Calculations of many physical processes according to the SU(12) theories[1] can be facilitated by the use of appropriate subgroups of SU(12). Direct calculations are difficult because of the complexity of the group and the present lack of available Clebsch-Gordan coefficients. Tables of SU(6) Clebsch-Gordan coefficients do exist,[2] however, and have been used for calculations of nonrelativistic processes. In this paper we present SU(6) subgroups which differ from the conventional SU(6) by the replacement of ordinary spin by different SU(2) subgroups of SU(12). These new subgroups can be used to extend SU(6) calculations into the relativistic domain. The use of these subgroups is analogous to that of the U-spin and V-spin subgroups of SU(3).

We consider theories based on the transformation properties of a 12-component Dirac quark. Although we use the quark model explicitly, the argument is valid for any theory in which the SU(12) transformation properties are the same as those given in a quark model. The SU(12) generators are represented by 143 combinations of Dirac matrices and SU(3) generators which define their operation on the fundamental representation $(\underline{12})$. We choose a representation for the Dirac matrices in which γ_0 and $\vec{\sigma}$ are Hermitean and γ_5 is anti-Hermitean.

The conventional SU(6) group is a subgroup of SU(12) whose 35 generators are combinations of SU(3) generators and the three Dirac matrices σ_x, σ_y, and σ_z. We denote this subgroup by $SU(6)_\sigma$. An alternative SU(6) subgroup can be constructed by using any set of three Dirac matrices which constitute an SU(2) Lie algebra.

W spin.—Consider the three SU(12) generators defined by their operation on the 12-dimensional quark representation as follows:

$$W_z(\underline{12}) = \sigma_z/2, \tag{1a}$$

$$W_x(\underline{12}) = \gamma_0\sigma_x/2, \tag{1b}$$

$$W_y(\underline{12}) = \gamma_0\sigma_y/2. \tag{1c}$$

The operation of these generators on the $(\underline{12^*})$ antiquark representation differs from the relations (1) in a reversal of the phases of the generators W_x and W_y:

$$W_z(\underline{12^*}) = \sigma_z/2, \tag{2a}$$

$$W_x(\underline{12^*}) = -\gamma_0\sigma_x/2, \tag{2b}$$

$$W_y(\underline{12^*}) = -\gamma_0\sigma_y/2. \tag{2c}$$

The "W-spin" operators satisfy SU(2) commutation rules and represent a "conserved spin" in any theory invariant under SU(12). W-spin operators are particularly useful because they commute with the generators of Lorentz transformations in the z direction.[3] Thus a particle moving with arbitrary momentum in the z direction has the same W spin as the corresponding particle state at rest. We define the subgroup $SU(6)_W$ of SU(12) by combining the W-spin and SU(3) generators. This subgroup and W-spin conservation can be used not only in the nonrelativistic limit, but also in all "one-dimensional" relativistic processes. These include all cases in which all momenta are in the z direction in some coordinate system; e.g., two-body decays of resonances, forward and backward scattering, photoproduction in the forward direction, radiative decays into a particle and a photon, weak two-body decays, and weak decays into a particle and a lepton pair.

The W-spin and SU(6) classification of quarks, antiquarks, mesons, baryons, and photons moving in the z direction is obtained from Eqs. (1) and (2), and can be determined from states at rest. For quarks and antiquarks at rest we set $\gamma_0 = +1$ in Eqs. (1) and (2). We first consider the baryons, which are constructed from three quarks. From Eqs. (1) the W spin is the same as the ordinary σ spin for these particles; the baryon octet has $W=\frac{1}{2}$, and the decuplet has $W=\frac{3}{2}$. The same holds for the corresponding antibaryons. We next consider the four meson states, constructed from quark-antiquark pairs. The two vector-meson states $V_{\pm 1}$ with spin projections ± 1 on the z axis have $W_z = \pm 1$, and therefore $W=1$. The vector-meson state V_0 with zero spin projection on the z axis and the pseudoscalar meson state P_0 both have $W_z = 0$. The W-spin classification of these two states is opposite to that of ordinary spin; the V_0 has $W=0$, and the P_0 has $W=1$. This difference results from the negative sign appearing in Eqs. (2b) and (2c), and may be checked by applying the appropriate W-spin lowering and raising operators to the states $V_{\pm 1}$. The W-spin triplet is therefore (V_{+1}, P_0, V_{-1}); the W-spin singlet is V_0. The photon transforms under W spin like the two vector-meson states $V_{\pm 1}$, and under $SU(6)_W$ like the members of the SU(3) octet in the SU(6) 35 which are a W-spin vector and U-spin scalar.

Calculations of physical processes using W-spin conservation and invariance under $SU(6)_W$ are formally identical to the usual $SU(6)_\sigma$ calculations, except for the interchange of the zero-helicity meson states. This interchange plays a crucial role in enabling predictions different from $SU(6)_\sigma$ to be obtained for relativistic processes. Consider, for example, the decay $\rho \to 2\pi$, which is forbidden by $SU(6)_\sigma$ because a particle of spin 1 cannot decay into two spin-zero particles. In $SU(6)_W$ we take the ρ to be initially at rest and choose our z axis in the direction of the outgoing pions. The latter both have $W=1$, $W_z=0$, and can couple to a total $W=0$ or 2. The V_0 polarization state of the ρ has $W=0$, $W_z=0$, and its decay into two pions is therefore allowed by W-spin conservation.

Examination of other three-meson couplings shows that all the following are forbidden by W-spin conservation: $(P_0P_0P_0)$ and $(V_0P_0V_0)$. The latter is just the W-spin analog of the forbidden VPP coupling in σ spin. These W-spin results are in agreement with experiment, in contrast to the $SU(6)_\sigma$ predictions which are clearly invalid for finite momenta. Note that the two couplings forbidden by $SU(6)_W$ are ordinarily forbidden by parity and angular-momentum conservation, which are generally considered to be outside the internal symmetries.

Another coupling forbidden by $SU(6)_\sigma$ in disagreement with experiment at finite momenta is $N^* \to N + \pi$. This is allowed by W spin as the pion has $W=1$. The corresponding coupling forbidden by W spin is (B^*BV_0). This selection rule is not observable as a decay, but might be checked in the analysis of peripheral reactions.

Since $SU(6)_\sigma$ and $SU(6)_W$ are both subgroups of SU(12), predictions from both are valid in an SU(12)-invariant theory at all momenta, provided that the proper classifications of particle states are used. The classification of particles of arbitrary momentum can be obtained from states at rest by an appropriate "boost" operator.[4] Such boosting procedures may be convenient in $SU(6)_W$ to treat two- and three-dimensional processes. The usual unboosted $SU(6)_\sigma$ is valid only in the nonrelativistic limit. Thus predictions from $SU(6)_W$ must agree with those from $SU(6)_\sigma$ at zero momentum. Any process for which these predictions disagree must be forbidden at zero momentum by SU(12). It should be noted that the W-spin classification of particles moving in the z direction is independent of the momenta of the particles, although the general SU(12) classification contains momentum-dependent factors.

Selection rules obtained in $SU(6)_\sigma$ for strange-particle production and the decay and production of the φ meson[5] remain valid in $SU(6)_W$. These are obtained by defining quark W spins $W_{p'}$, $W_{n'}$, and $W_{\lambda'}$ analogous to the ordinary quark spins. The Johnson-Treiman[6] relations for forward scattering obtained in SU(6) have been checked in $SU(6)_W$ by explicit calculations and found to be valid.[7]

B spin.—Since the 15 Dirac matrices are generators of the group SU(4), another set of three Dirac matrices can be found which constitute an SU(2) Lie algebra commuting with W spin. These are

$$B_1(12) = i\sigma_z\gamma_5/2, \tag{3a}$$

Table I. W- and B-spin properties of Dirac matrices.

		B-spin vector components 1	2	3	B-spin scalar
W-spin vector components	x	$\gamma_0\gamma_5\sigma_y$	$-i\gamma_5\sigma_y$	σ_x	$\gamma_0\sigma_x$
	y	$-\gamma_0\gamma_5\sigma_x$	$i\gamma_5\sigma_x$	σ_y	$\gamma_0\sigma_y$
	z	$i\gamma_5$	$\gamma_0\gamma_5$	$\gamma_0\sigma_z$	σ_z
W-spin scalar		$i\gamma_5\sigma_z$	$\gamma_0\gamma_5\sigma_z$	γ_0	1

$$B_2(12)=\sigma_z\gamma_0\gamma_5/2, \tag{3b}$$

$$B_3(12)=\gamma_0/2. \tag{3c}$$

The relation between the B-spin subgroup $SU(2)_B$ and SU(12) is illustrated by the decompositions[8]

$$SU(12)\supset SU(6)_W\otimes SU(2)_B,$$

$$SU(12)\supset SU(3)\otimes SU(4)$$
$$\supset SU(3)\otimes SU(2)_W\otimes SU(2)_B. \tag{4}$$

Unlike W spin, B spin is not invariant under Lorentz transformations. However, B_1 is just the generator of Lorentz transformations in the z direction. Thus Lorentz transformations in the z direction only rotate the B spin of a particle without changing its magnitude. Quarks and antiquarks at rest have $B=B_3=+\frac{1}{2}$. Mesons and baryons at rest are eigenstates of B and B_3, with $B_3=+B$. $B=1$ for mesons in the SU(6) 35, and $B=\frac{3}{2}$ for baryons and antibaryons in the SU(6) 56 and 56*. Corresponding states of finite momentum in the z direction have the same eigenvalue of B as the state at rest, but are mixtures of different eigenstates of B_3. Individual quark B spins $B_{p'}$, $B_{n'}$, and $B_{\lambda'}$ can also be defined and are conserved in any SU(12)-invariant theory. An $SU(6)_B$ subgroup of SU(12) can also be defined.

The 16 Dirac matrices transform under W spin and B spin as indicated in Table I. This table exhibits the transformation properties of the baryon currents used in weak interactions, and can therefore be used to classify the associated spurions according to W and B spin.[9]

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[3]This can be verified by direct calculation or by noting that the W spin for a quark moving in the z direction is just the relativistic spin operator defined in M. E. Rose, Relativistic Electron Theory (John Wiley & Sons, Inc., New York, 1961), p. 72.

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[6]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

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[8]The decomposition $SU(12)\supset SU(6)_\sigma\otimes SU(2)_D$ has been considered by Bég and Pais (see reference 1).

[9]D. Horn et al., to be published.