We propose a method for measuring the (partial) width $\Gamma$ of the decay
\[ \omega^0 \rightarrow \pi^+ + \pi^- + \pi^0, \]
using the fact that the cross section for
\[ \pi^- + \pi^0 \rightarrow \omega^0 + \pi^- \]
is related to $\Gamma$ in a fairly model-independent way. The cross section for (2) turns out to be of the order of a millibarn for a width $\Gamma$ of a few hundred keV at about 200 MeV (c.m.) above the threshold. The well-known Chew-Low extrapolation method may be used to deduce the desired cross section for (2) from a study of the reaction
\[ \pi^- + p \rightarrow \pi^- + \omega^0 + p. \]

We discuss the processes (1) and (2) under two extreme dynamical assumptions. First we assume that the $\omega-3\pi$ vertex is zero-ranged in the sense that it is given by the centrifugal barrier alone as follows:
\[ \left( \frac{f_{\omega 3\pi}}{m^3} \right) \epsilon_{\mu \nu \lambda \sigma} \epsilon^{\omega}_{\mu \nu} k^{(+)}_{\nu} k^{(-)}_{\lambda} k^{(0)}_{\sigma}, \]
where $k^{(+)}$, $k^{(-)}$, and $k^{(0)}$ refer to the pion four-momenta, and $\epsilon^{\omega}_{\mu \nu \lambda \sigma}$ stands for the polarization vector of the $\omega$ meson. The factor $m^3$ is inserted to make the coupling constant $f_{\omega 3\pi}$ dimensionless. The decay width $\Gamma$ can be calculated to be
\[ \Gamma = \left( \frac{f_{\omega 3\pi}^2}{4\pi} \right) \left( \frac{m_{\omega} - 3m_{\pi}}{2^3 \times 3^3 \lambda_{\omega} \mu_{\omega}} U(m_{\omega}) \right), \]
where $U(m_{\omega})$ is a relativistic correction factor which approaches unity as $m_{\omega} \rightarrow 3m_{\pi}$. For $m_{\omega} = 787$ MeV, we find numerically that $U = 1.6$. The differential cross section and the total cross section for (2) are given by
\[ \frac{d\sigma}{d\Omega} = (f_{\omega 3\pi}^2/4\pi) p_{\pi}^2 \sin^2 \theta / 16\pi m^6, \]
\[ \sigma = (f_{\omega 3\pi}^2/4\pi) p_{\pi}^2 / 6m^6, \]
where $p_{\pi}$ is the momentum of one of the incident (outgoing) particles in the center-of-mass system. For a $\Gamma$ of 400 keV, we obtain a cross section of 4.0 mb at a total c.m. energy of 1.1 BeV (173 MeV above the $\omega\pi$ threshold). For a total c.m. energy $\geq 1.2$ BeV, the predicted cross section with $\Gamma = 400$ keV becomes comparable to the $\rho$-wave unitarity limit for inelastic processes, $3\pi\lambda^2$, so that the zero-range model must break down.

The most general expression for the $\omega-3\pi$ vertex with all particles on the mass shell is of the form
\[ \epsilon_{\mu \nu \lambda \sigma} \epsilon^{\omega}_{\mu \nu} k^{(+)}_{\nu} k^{(-)}_{\lambda} k^{(0)}_{\sigma} F(x, y), \]
where $x$ and $y$ are two independent invariant scalars that can be constructed from the external four-momenta. In the zero-range model considered in the preceding paragraph, $F(x, y)$ has been taken to be constant. In contrast, Gell-Mann, Sharp, and Wagner suggest a model of $\omega$ decay in which the dispersion representation for $F(x, y)$ is assumed to be dominated by $\rho$-meson intermediate states. We call this the $\rho$-dominance model since we can visualize the decay interaction as proceeding via
\[ \omega \rightarrow \rho + \pi, \]
followed by
\[ \rho \rightarrow 2\pi. \]

Factors that enter in a diagram at the vertices (8) and (9) are, respectively,
\[ \left( \frac{f_{\omega \rho \pi}}{m_{\rho}} \right) \epsilon_{\mu \nu \lambda \sigma} \epsilon^{\omega}_{\mu \nu} k^{(\rho)}_{\nu} \epsilon^{(\rho)}_{\lambda} k^{(\rho)}_{\sigma}, \]
and
\[ f_{\rho \pi \pi} \epsilon^{(\rho)}_{\mu} k^{(\pi 1)}_{\mu} - k^{(\pi 2)}_{\mu}. \]

We then have
\[ \Gamma = \left( \frac{f_{\omega \rho \pi}}{4\pi} \right) \left( \frac{f_{\rho \pi \pi}^2}{4\pi} \right) \left( m_{\omega} m_{\rho}^2 (m_{\omega} - 3m_{\pi})^4 W(m_{\omega}) \right), \]
where $W(m_{\omega})$ is a relativistic correction factor which has been numerically estimated to be about 3.6 at $m_{\omega} = 787$ MeV. The constant $f_{\rho \pi \pi}^2/4\pi$ is about 2 for a $\rho$ width of 100 MeV; the only remaining constant, $f_{\omega \rho \pi}^2/(4\pi m^2)$, can be estimated from...
the \( \pi^0 \) lifetime à la Gell-Mann and Zachariasen\(^4\) if the \( \omega \) and \( \rho \) are coupled universally to the conserved hypercharge current and the isospin current\(^5\) with the coupling constants \( f_\omega \) and \( f_\rho \), or, equivalently, if \( \omega \) and \( \rho \) dominate the dispersion integrals for the isoscalar and isovector charge form factors for every strongly interacting particle. The coupling constant for \( \pi^0 \) decay, defined as in Eq. (10), can then be written as

\[
f_{\pi^0 \gamma} = e^2 f_\omega \rho / \rho f_\omega .
\]

From these considerations and unitary symmetry\(^6\) (which requires \( f_\rho / 4\pi = f_\omega / 4\pi \)), Gell-Mann, Sharp, and Wagner\(^7\) estimate \( \Gamma \approx 400 \) kev.

Along similar lines we can discuss the \( \omega \) production process (2) by keeping only one-\( \rho \)-meson states that appear in the \( s, t \), and \( u \) channels. In this \( \rho \)-dominance model, the production cross section directly measures the product \( f_{\rho \pi^0} f_{\rho \pi^0} \), hence \( \Gamma \). The differential cross section is given by

\[
\frac{d\sigma}{d\Omega} = \left( f_{\rho \pi^0} \right)^2 \frac{f_{\rho \pi^0}^2}{4\pi} \left( \frac{p_i \rho f_\rho \sin^2 \theta}{m^2} \right)^2 \times \left( \frac{1}{s-m^2} \frac{1}{l-m^2} \frac{1}{u-m^2} \right)^2,
\]

where\(^7\)

\[
s = 4E_{\pi^0}^2, \\
t = 2m_{\pi^0}^2 - 2E_{\pi^0} E_{\pi^0} + 2p_i \rho f_{\rho} \cos \theta, \\
u = 2m_{\pi^0}^2 - 2E_{\pi^0} E_{\pi^0} - 2p_i \rho f_{\rho} \cos \theta, \\
E_{\pi^0} = (p_i^2 + m_{\pi^0}^2)^{1/2}, \ E_{\pi^0} = (p_f^2 + m_{\pi^0}^2)^{1/2}.
\]

The expression (14) can be integrated analytically to give the total cross section as a function of energies. The numerical results are shown in Fig. 1 where the solid curve represents the \( \omega \) production cross section expected for \( \Gamma = 400 \) kev. Also shown is \( 3\pi^2 \) (dashed curve).

We may remark that for a given value of \( \Gamma \), the \( \rho \)-dominance model predicts a cross section four times smaller than the zero-range model at \( s^{1/2} = 1.1 \) Bev. If we are to choose between the two models, the \( \rho \)-dominance model is probably the more reasonable. In any case, it is gratifying that at least the order of magnitude of \( \Gamma \) can be determined from the production cross section in a model-independent way.

\[\text{FIG. 1. The solid curve represents the cross section for } \pi^+ + \pi^0 \rightarrow \omega + \pi^- \text{ for } \Gamma = 400 \text{ kev as predicted by the } \rho \text{-dominance model. The dashed curve represents the } \rho \text{-wave unitarity limit for inelastic processes, } 3\pi^2.\]

It is hardly necessary to emphasize that the well-known Chew-Low method\(^2\) can be used to extract the desired cross section for (2) from a study of the reaction (3). If the \( \omega \) width is really of the order of 400 kev or greater, experiments along this line seem feasible in a hydrogen bubble chamber with a \( \pi^- \) beam of \( \rho_{\text{lab}} \approx 2-3 \) Bev/c.\(^8\)

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\(^3\)Equation (5) does not agree with Eqs. (3) and (4) of G. Feinberg [Phys. Rev. Letters 8, 151 (1962)]. Our expression does agree with the \( m_{\pi^-} \rightarrow \infty \) limit of Eq. (1) of reference 3.

\(^4\)M. Gell-Mann, D. Sharp, and W. D. Wagner, Phys. Rev. Letters 8, 261 (1962). Thanks are due to the authors of this reference for many helpful discussions.

\(^5\)M. Gell-Mann and F. Zachariasen, Phys. Rev. 124,
and \( \omega \) become "universal" in the unitary symmetry limit. Note that our \( f_\rho \) is equal to \( 2 \gamma_\rho \) of Gell-Mann. \(^1\)

\(^1\)Recall also the well-known relation

\[ s + t + u = 3m_\pi^2 + m_\eta^2. \]

\(^2\)The reaction (3) is currently being studied by the Wisconsin and the Berkeley hydrogen bubble-chamber groups. This work has been stimulated in part by their experimental programs.

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**ERRATUM**

**DECAY MODES AND WIDTH OF THE \( \eta \) MESON.**


On page 116, ninth line of text, \( \Gamma(p^+p^-p^0) \) should read \( \Gamma(s) \) (all modes).

In line 16 of the legend of Fig. 1, \( \sigma(K^-p\to\Lambda\pi^0\pi^-) \) should read \( \sigma(K^-n\to\Lambda\pi^0\pi^-) \).

In the ninth line of the legend of Fig. 2, \( K^-p\to\Lambda\pi^+\pi^-\pi^0 \) should be replaced by \( K^-d\to\rho\pi^+\pi^-\pi^- \).