Large Bragg Reflection from One-Dimensional Chains of Trapped Atoms
Near a Nanoscale Waveguide
- Supplemental Material -

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S1. SPECTRA SIMULATION

The light propagation through a one-dimensional chain, as illustrated in Fig. 1 can be described by the transfer matrix formalism [1]. For an atom at position z one can introduce the matrix $M_a$ that relates components of the backward- and forward-traveling electric field on the right side $E_R^{\text{back}}(z)$ and $E_R^{\text{forw}}(z)$ to the ones on the left side $E_L^{\text{back}}(z)$ and $E_L^{\text{forw}}(z)$:

$$
\begin{pmatrix}
E_L^{\text{back}}(z) \\
E_L^{\text{forw}}(z)
\end{pmatrix} = M_a \cdot \begin{pmatrix}
E_R^{\text{back}}(z) \\
E_R^{\text{forw}}(z)
\end{pmatrix}.
$$

(1)

The boundary conditions for the wave equation allows to obtain the following expression for the matrix $M_a$:

$$
M_a = \frac{1}{t} \begin{pmatrix} t^2 - r^2 & r \\ -r & 1 \end{pmatrix},
$$

(2)

where $t$ and $r$ are single-atom transmission and reflection coefficients respectively. The transfer matrix $M_p$ for propagation between neighboring atoms is simply given by

$$
M_p = \begin{pmatrix} e^{ikd} & 0 \\ 0 & e^{-ikd} \end{pmatrix},
$$

(3)

where $d$ is the distance between the two atoms. The transfer matrix for the full ensemble is obtained as a product of matrices $M_a$ and $M_p$ as follows:

$$
M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = (M_a \cdot M_p)^N.
$$

(4)

The transmission and reflection coefficients of the atomic chain are finally given by:

$$
T = \left| \frac{M_{11}}{M_{22}} \right|^2, \\
R = \left| \frac{M_{12}}{M_{22}} \right|^2.
$$

(5)

The single-atom transmission and reflection coefficients $t$ and $r$ depend on the radiative decay rates. Each atom has two channels of decay $\Gamma = \Gamma_{1D} + \Gamma'$ that can be associated with the guided mode $\Gamma_{1D}$ and the radiation modes $\Gamma' \sim \Gamma_0$ where $\Gamma_0$ is the radiative decay in free-space. The decay to the guided mode can occur in forward $\Gamma_{1D}^{\text{forw}}$ and backward $\Gamma_{1D}^{\text{back}}$ directions. Given the polarization of the guided light and positions of the atoms, $\Gamma_{1D}^{\text{forw}}$ and $\Gamma_{1D}^{\text{back}}$ can be equal or asymmetric [2].

In the case of quasilinear polarization along $y$, i.e. orthogonal to the atomic array, the decay rates in the forward and backward directions are equal $\Gamma_{1D}^{\text{forw}} = \Gamma_{1D}^{\text{back}} = \Gamma_{1D}/2$. In this symmetric case, the reflection and transmission coefficients can be expressed as [3]:

$$
r = -\frac{\Gamma_{1D}}{\Gamma - 2i\delta},
$$

(6)

$$
t = 1 + r
$$

where $\delta$ is the detuning from the atomic resonance. In accordance with [2] the explicit expression of the transfer matrix $M_a$ can be written as follows:

$$
M_a = \frac{1}{\Gamma_0 - 2i\delta} \begin{pmatrix} (\Gamma_0 - 2i\delta)^2 - \Gamma_{1D}^2 & -\Gamma_{1D} \\ -\Gamma_{1D} & \Gamma_0 - 2i\delta \end{pmatrix}.
$$

(7)

In the case of asymmetric scattering in forward and backward directions, a situation that occurs if the guided mode is quasilinearly polarized along the $x$-direction (pointing towards the atomic chain), the full decay rate can be written as $\Gamma = \Gamma' + \Gamma_{1D}^{\text{forw}} + \Gamma_{1D}^{\text{back}}$. The explicit

![Figure 1](image-url)
expressions for these coefficients can be found in accordance with [4] as follows:

\[
    r = -\frac{2\sqrt{\Gamma_{1D}^{\text{forw}} \cdot \Gamma_{1D}^{\text{back}}} \cdot (\Delta - 2i\delta)}{\Gamma - 2i\delta}, \\
    t = 1 - \frac{(\Gamma_{1D}^{\text{forw}} + \Gamma_{1D}^{\text{back}})}{\Gamma - 2i\delta}
\]

These expressions finally provide the transfer matrix \( M_a \) for the asymmetric case as:

\[
    M_a = \frac{1}{\Gamma_0 - 2i\delta} \left( \frac{(\Gamma_0 - 2i\delta) - 4\Gamma_{1D}^{\text{forw}} \cdot \Gamma_{1D}^{\text{back}}}{\Gamma - 2i\delta} \right)
\]

S2. EFFECT OF INHOMOGENEOUS BROADENING

The dipole trapping of the atoms can result in a resonance shift and an inhomogeneous broadening. This effect is limited in the reported experiment: the shift has been measured equal to 3 MHz and the broadening to \( \sigma_\delta \sim 3 \text{ MHz} \). To study the effect of these parameters, we vary randomly the detuning for each atom of the chain, \( \delta_i = \delta + \delta_b \), in accordance with the Gaussian distribution:

\[
    f(\delta_b) = \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\frac{\delta^2}{2\sigma_\delta^2}}
\]

where \( \sigma_\delta \) is the standard deviation. Each atom has an individual transfer matrix \( M_{a_i} \), and the total transfer matrix for the chain can be read as:

\[
    M = (M_{a_1} \cdot M_p) \cdot (M_{a_2} \cdot M_p) \cdot \ldots \cdot (M_{a_N} \cdot M_p)
\]

In the main manuscript, simulations include the shift but not the broadening. The measured broadening has indeed small effect, below what can be measured given the precision of our measurements. Figure 2 provides the simulations of the reflection spectra for different broadenings \( \sigma_\delta \).

S3. EFFECT OF DISORDER INDUCED BY THE FILLING FACTOR

The filling factor of the trapping sites can induce randomness in the distribution of the atoms across the lattice. We consider here two parallel lines of atoms, as obtained with a nanofiber, with at most one atom per site. Given a filling factor per site \( f \), at each trapping axial position, there are either no atom at all, one atom on the upper chain, one atom on the lower chain or one atom in both upper and lower chains. This situation can be simulated by a single random array with, for each site, either no atom (probability \( (1-f)^2 \)), one atom (probability \( 2f(1-f) \)) or two atoms (probability \( f^2 \)). As shown on Fig. 3, a filling factor of 50%, which is the ideal case in the collisional blockade regime, leads here to the same spectrum than a full single array for the same number of atoms. When the filling factor decreases, the spectrum is narrower and the maximum reflection decreases. In the main manuscript, a filling factor \( f = 0.3 \) was used to fit the experimental data.

In the inset of figure 4 of the main manuscript, we finally provided the maximal reflection as a function of the number of remaining trapped atoms. For this simulation, we started from two parallel arrays with a filling...
factor $f = 0.3$ and subsequently added random loss of the atoms. If one denotes $1 - \eta$ the probability for an atom to be lost, the new probability for having one atom is $2nf(1 - \eta f)$ and for two atoms $\eta^2 f^2$.

S4. TRAPPING POTENTIAL AND EFFECT OF DISORDER INDUCED BY IMPERFECT AXIAL LOCALIZATION

Following the calculation presented in [5, 6], we calculated the trapping potential and trapping frequency in the axial direction. Figure 4 shows the axial dependence of the trapping potential $U_{\text{trap}}(x, y = 0, z)$ for atoms trapped at a distance $x_0 = 234$ nm from the fiber surface. The $U_{\text{trap}}$ values for the $\{6S_{1/2}, F = 4\}$ ground state manifold and $\{6S_{1/2}, F = 5\}$ excited state manifold are plotted as blue and red lines respectively. For this calculation, we use $\Delta \lambda = 0.2 \text{ nm}$ and a power of $2 \times 1.9 \text{ mW}$ for the red detuned beams, and a power of $2 \times 4 \text{ mW}$ for the blue detuned beams. The potential depth at minimum is $U_{\text{min}} = -0.15 \text{ mK}$ and the trap frequency in the axial direction is $\nu_z / 2\pi = 258 \text{ kHz}$.

In the case of $\Delta \lambda = 0.12 \text{ nm}$ and a trapping power of $2 \times 1 \text{ mW}$, the potential depth at minimum is $U_{\text{min}} = -0.1 \text{ mK}$ and the axial trap frequency is reduced to $\nu_z / 2\pi = 215 \text{ kHz}$. Both axial frequencies for this near resonant trap are comparable with previous nanofiber-based traps [7, 8].

Using these calculated frequencies, we compute the root mean square of the spatial spread in the axial direction in the harmonic approximation:

$$\sigma_z = \sqrt{(k_B T) / (m_C \nu_z^2)},$$

where $k_B$ is Boltzmann constant, $T$ is the temperature, $m_C$ is the Cesium atomic mass and $\nu_z$ is the axial frequency of the trap. In our case, the temperature of the atomic cloud is estimated to $T = 20 \text{ mK}$ from a time-offlight measurement. Thus, $\sigma_z$ is equal to $22 \text{ nm}$ and $26 \text{ nm}$ for $\Delta \lambda = 0.2 \text{ nm}$ and $\Delta \lambda = 0.12 \text{ nm}$ respectively. For atoms cooled in the ground state, the axial spread would be $12 \text{ nm}$ and $13 \text{ nm}$.

The reduction of Bragg reflected intensity due the spread of the atomic position in the potential wells is usually estimated by a so-called Debye-Waller factor [10]. This factor is given by $f_{\text{DW}} = e^{-4k^2\sigma_z^2}$, where $k$ is the guided mode wavenumber. In our case, we find values of $0.89$ and $0.85$ for $\Delta \lambda = 0.2 \text{ nm}$ and $\Delta \lambda = 0.12 \text{ nm}$ respectively (0.96 for atoms cooled to the motional ground state). In Fig. 5 we provide the simulated spectra taking into account a Gaussian spread $\sigma_z$ for the atom position. As it can be seen, this close-to-unity Debye-Waller factor has a very limited effect in our configuration where Bragg reflection is obtained out of resonance while it would be an important factor close to resonance [10]. We note that the reduction at resonance is larger than the Debye-Waller factor, which is valid in the single-scattering regime.

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