ARE QUARKS DIRAC PARTICLES?

H. David Politzer
California Institute of Technology, Pasadena, California 91125

ABSTRACT

Calculations of scaling properties of hadronic semi-inclusive processes are discussed using an analysis of the infrared structure of perturbative QCD.

INTRODUCTION

There can be no doubt of the permanent significance of the Dirac equation. A modest embellishment, of growing popularity over the last fifty years, is the idea that the interactions of relativistic spin-1/2 particles can be described by a local gauge principle. A crucial problem, then, is to identify what are the fundamental particles and what are the fundamental gauge groups. Historically, the prediction of the electron magnetic moment was a triumph, while the anomalous moment of the proton was really the first indication of its composite structure.

We can ask today whether the quarks, which we know from hadron spectroscopy and the probings of leptonic currents, are indeed fundamental Dirac particles. This is another way of asking, "Is QCD, the colored-quark-gluon gauge theory of strong interactions, a valid description of hadrons?"

There are only two canonical processes for which we have extracted detailed predictions from QCD. They are inclusive leptoproduction and e^+e^- annihilation. It is imperative that we find methods of making further predictions— to test the theory more stringently, and to provide a reliable framework and reference point for the discovery and interpretation of new phenomena (e.g., in the weak interactions).

Very tentative attempts to generate new predictions have been undertaken. Most studies have simply considered the first non-trivial corrections to Born graphs. I wish to describe here an analysis, applicable to all orders, of the requisite properties of QCD perturbation theory for semi-inclusive processes. This is a collaborative effort, and the work is in progress. I firmly believe we have the elements of the necessary proofs, but the ε's and δ's are yet to be filled in.

THEOREM

The result we are aiming at is the following: the cross section, \( \sigma \), for any process of the form \( a + b + X \) where \( a \) and \( b \) are sets of hadrons, currents, and/or jets with all invariants (except

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for masses) large is of the form

\[
d\sigma(P_i) = \int \pi dz \, f_i(z_i, \Lambda_i) d\tilde{\sigma}(z_i P_i, \Delta_i) + O(1/s)
\]

where \(P_i\) are the particle momenta; \(f_i\) are as yet uncalculable functions measured experimentally; \(d\tilde{\sigma}\) is a reduced cross section calculable in perturbation theory; \(\Lambda_i\) is a dimensionful parameter introduced to make the separation of \(f_i\) and \(d\tilde{\sigma}\) precise; and \(s\) is the common scale of the large invariants. \(\Delta_i\) essentially limits the transverse momenta absorbed into \(f_i\) and provides an infrared cut-off on \(d\tilde{\sigma}\). An important net result is that the \(f_i\) are process independent.

The significance of establishing eq. (1), at least to all orders of perturbation theory, goes beyond simply justifying the important lowest order calculations\(^1\) which already possess that form. A uniform analysis assures self-consistency. So it is possible to avoid double counting, under counting, and plain wrong counting with confidence. (I mention this because many of the preliminary phenomenological analyses have such flaws.) One such lesson from the present analysis is that the 1/s terms are not process independent. They include final state interactions, for example. Hence a careful phenomenological fit of 1/s effects in one process is not of much significance at present because we would not know its implications for another process.

An underlying assumption, not presently amenable to proof, is that the hadron wave functions are sufficiently soft to guarantee that processes that vanish like 1/s for quark and gluon scattering will continue to do so for their bound states, hadrons. Hence we concentrate on quark-gluon-current inclusive scatterings in perturbation theory. Equation (1) is represented in Fig. 1, the appropriate discontinuity of a forward amplitude.

The obstacle to progress in the past was that in gauge theories, the infrared sensitive pieces do not organize them-

Fig. 1: The inclusive cross section for \(a + b + X\).
selves as suggested in Fig. 1. Specifically there are two-particle irreducible graphs, which apparently would be included in $d\sigma$, which are infrared divergent in the limit that all masses go to zero.

In fact, the graphical location and organization of infrared divergences is gauge dependent. In axial gauges ($n_{\mu}A^\mu = 0$ for some $n_\mu$, preferably not $n^2 = 0$ because of $\log n^2$ terms) there is a zero associated with colinear emissions, which appears to render the two particle irreducible graphs infrared finite. The detailed arguments will be presented elsewhere.

**IMPLICATIONS**

In a qualitative way, eq. (1) is a justification of the parton picture for hard processes. The $f_i$ are universal distribution and decay functions that are convoluted with a parton cross section $d\sigma$. The quantitative differences from a most naive parton picture can be enormous, however. They arise from the following new features: Gluons, as well as quarks, must be considered as active partons. And more significantly, the computable $d\sigma$ now includes inelastic parton processes. These can radically alter even the most qualitative predictions, e.g., for specific phenomena that are vanishingly small in the Born approximation.

An issue that inevitably causes confusion is to what extent are radiative corrections already included in the naive parton model. A precise answer is offered by eq. (1) and Fig. 1. Indeed, $d\sigma$ is not the sum of all Feynman graphs. Parts of various graphs must be absorbed into the $\tilde{f}_i$, to render $d\sigma$ calculable (i.e., infrared insensitive). But to make the $\tilde{f}_i$ universal, process independent functions only those radiative corrections that render physically indistinguishable states (within a criterion set by $A_f$) get lumped into $\tilde{f}_i$. (From this point of view, eq. (1) is no doubt an immediate consequence of the Lee-Nauenberg theorem.) An example of such indistinguishability is a massless quark state compared to a quark and gluon state, both moving in the same direction. Such states can be identical in all quantum numbers. But once the quark-gluon system has an opening angle, it has a non-zero invariant mass. The lesson of eq. (1) is that the effects of radiation at finite angles is not yet included in the naive parton model but must be added into $d\sigma$.

Phenomenological arguments based purely on leading log calculations may be misleading because the kinematic regions in which the leading logs are genuinely large get reinterpreted and reabsorbed somewhere else. The relative sizes of the parts left over may be subtle. For example, a calculation of all Feynman graphs to a given order may yield an expression like

$$ag^4 \log \frac{Q^2}{p^2} + bg^4 \log \frac{Q^2}{p^2} + cg^2 \log \frac{Q^2}{p^2} + dg^2 + eg^4$$

where $Q^2 \gg p^2 \approx 0$. But the corresponding contribution to $d\sigma$ would look very similar but with $p^2$ replaced by $A$. However, $\log Q^2 / A$ will typically not be enormous. Hence the "leading log" terms are not...
obviously dominant.

Experimental consequences, in as much as they differ from the naive parton model, have been discussed at length elsewhere\textsuperscript{1, 4}. I wish only to mention the salient features. There are logarithmic scaling violations implicit in $d\sigma$ of the type encountered in electroproduction. To make them explicit, note that the product $f(z, \Delta) d\sigma(s, y, \Delta)$ is independent of $\Delta$, where $s$ is a scale and $y$ are dimensionless ratios of large invariants. If we take $\Delta = \epsilon_4 s$ with $\epsilon_4$ fixed, the "logarithmic" scaling violations have now moved to the $f(z, \epsilon_4 s)$ which depend on $s$ in the way familiar from electroproduction.

Inelastic parton processes are of particular importance for weak and electromagnetic effects in $p - p$ collisions because of the relative abundance of quarks and gluons compared to antiquarks. Thus processes like quark + gluon + quark + W boson (or quark + $\mu^+\mu^-$) will be competitive in $pp$ (as well as quark + quark + W + X). As a consequence $p^2_{T}$'s are expected to rise linearly with $s$.

Finally, the predictions for hadrons at large $p_T$, e.g., $p + p \rightarrow \pi + X$, are radically different from any previous expectations. The predictions lie dramatically above the linear fit to existing data, i.e., $p^2_{T}$; and they lie dramatically below the $p^2_{T}$ expected on the basis of one gluon exchange. The latter difference is due to a piling up of many logarithmic effects from next order corrections. These "go away" asymptotically but are expected to give an effective $p^2_{T}$ for the region $6 \leq p_T \leq 30$ GeV. This abundance of hadrons at large $p_T$ (relative to the phenomenological $p^2_{T}$) if observed will provide a horrendous background to weak and electromagnetic physics at high energies.

But such an observation will also provide striking evidence that the quarks are indeed Dirac particles. I would regard this as an honor for the quarks.

REFERENCES

   C. T. Sachrajda, CERN preprints TH.2416, 2459 (1977-78);
4. There are already too many to list accurately.