Supplementary Figure 1 | Schematics of the meta-atom and decoupled control of wave parameters. **a**, Assignment of average pressure $p_{[i]}$ and displacement $q_{[jk]}$ for the membranes and sub-cells. **b,c**, Mapping of $(\rho, B^{-1})$ with the control of $(t_0, t_1)$ using **b**, the full solutions in (14) and **c**, their approximations (15) (right) at 1,300 Hz.
Supplementary Figure 2 | Band structures of the meta-atom. a, 2D band diagram (CMT and FEM) and b, 3D plot of Dirac cone (CMT) near the operation frequency at 1,300 Hz.
Supplementary Figure 3 | Characteristic motions of the meta-atom for a, ρ-dipolar and b, $B^{-1}$-monopolar modes. Displacement (gray lines) and momentum (black arrows) of the membranes. Also shown are the pressure field patterns (+: red, -: blue) at 1,300 Hz.
Supplementary Figure 4 | Shifting the center of decoupling operation away from the Dirac point. a, Schematics of the meta-atom in the background host medium. FEM obtained ($\rho$, $B^{-1}$) tuning map at 1,300 Hz are shown in b, ($\rho_c$, $B_c^{-1}$) = (0,0) without host ($a_h = a$) medium. c, ($\rho_c$, $B_c^{-1}$) = (0.2, 0.36) with ($\rho_h$, $B_h^{-1}$) = (1, 1). d, ($\rho_c$, $B_c^{-1}$) = (0.4, 0.72) with ($\rho_h$, $B_h^{-1}$) = (2, 2). $a_h = 6$cm, 7.5cm and 7.5cm for b, c, and d respectively.
Supplementary Figure 5 | Extension of $\rho$ tuning range. CMT calculated ($\rho$, $B^{-1}$) tuning maps at 1,300 Hz for different lattice constant, a, $a = 6$ cm, b, $a = 3$ cm and c, $a = 1$ cm. Grid spacings for $B^{-1}$ (red lines) are fixed to 0.2, and for $\rho$ (blue lines) are 0.2, 0.5 and 3, respectively.
Supplementary Figure 6 | Implementation of bianisotropy and CMT obtained $\xi$ values. a, Schematics of the asymmetric meta-atom for non-zero bianisotropy $\xi$ with $\Delta t_1 \neq 0$. b, Comparison of exact (16) and approximated (17) solution of $\xi$, as a function of $\Delta t_1$. $\rho = B^{-1} = 0, 1,300$ Hz.
Supplementary Figure 7 | Schematics of the asymmetric transmission between a, different impedances or b, waveguides widths, using matched zero index and bianisotropic metamaterials.
Supplementary Figure 8 | Schematics of the S parameters in the bianisotropic medium for determining effective wave parameters.
Supplementary Figure 9 | Wave parameter value maps in the $(\phi_R, \phi_T)$ coordinate, subject to the constraint of 50:50 power divisions for transmission and reflection. Required a, density $\rho$ and b, modulus $B^{-1}$, and c, bianisotropy $\xi$ are plotted. Required phase shifts $(\phi_R, \phi_T)$ for individual meta-atoms in the $40 \times 1$ meta-atom array, achieving extra-ordinary transmission and reflections are marked with red square ($\Delta \phi_T(x) \neq 0$) and red star ($\Delta \phi_R(x) \neq 0$), respectively ($a = 6$ cm, $\lambda = 26.4$ cm, and $f = 1,300$ Hz).
Supplementary Figure 10 | Required phase shifts \((\phi_R, \phi_T)\) for individual meta-atoms in a single sheet of a 40 × 1 array to achieve ordinary (blue marks) or extra-ordinary (red marks) transmission and reflection.
Supplementary Note 1. Derivation of effective macroscopic acoustic parameters from the homogenization theory

To derive equation (1) in manuscript, we start from the microscopic acoustic wave equations based on Newton’s law and Hooke’s law. Assuming \( \exp(i\omega t) \) time dependency of angular frequency \( \omega \),

\[
\nabla p(r) = -i\omega \rho_0 \nabla (r) = -i\omega \rho_0 v(r) - i\omega \rho_s (\rho_s - 1) v(r) \\
\nabla \cdot v(r) = -i\omega (r) = -i\omega B_0^{-1} p(r) - i\omega B_s^{-1} (B_s^{-1} - 1) p(r)
\]

where \( p(r), v(r), e(r) = B_0^{-1} B_s^{-1} p(r), \pi(r) = \rho_0 \rho_s v(r) \) denote the pressure, velocity, strain and momentum density field at \( r \), respectively. \( \rho_0 \) and \( B_0 \) are the density and bulk modulus of the air; \( \rho_s \) and \( B_s \) are the density and bulk modulus (normalized to the air) of the constituting materials in the unit cell.

By following a similar procedure used in Alù (homogenization theory for electromagnetic waves), we now derive the acoustic equations for the macroscopic fields (\( \overline{p}, \overline{v} \)) from (1). Noting that the effective parameters are independent of the origin of the coordinate in case of periodic system or finite metamaterial terminated at the same location of the unit cell, we use Floquet theory with \( \exp(-i\beta \cdot r) \) dependence. Averaging the field of \( p \) and \( v \) over the two-dimensional square lattice (lattice constant \( a \), unit cell domain \( S \), effective wavevector \( \overline{\beta} \)), equation (1) then becomes

\[

- i\overline{\beta} \overline{p} = -i\omega \rho_0 \nabla - \frac{i\omega \rho_0}{a^2} \int_S (\rho_s - 1) v(r) e^{i\beta \cdot r} dS \\

- i\overline{\beta} \cdot \overline{v} = -i\omega B_0^{-1} \overline{p} - \frac{i\omega B_0^{-1}}{a^2} \int_S (B_s^{-1} - 1) p(r) e^{i\beta \cdot r} dS
\]

where \( \overline{v} = \frac{1}{a^2} \int_S v(r) e^{i\beta \cdot r} dS \), and \( \overline{p} = \frac{1}{a^2} \int_S p(r) e^{i\beta \cdot r} dS \).

Now, using Taylor’s expansion in the long wavelength limit (\( a << 2\pi|\beta| \)),

\[

\int_S (\rho_s - 1) v(r) e^{i\beta \cdot r} dS = \int_S (\rho_s - 1) v(r) dS - i\beta \int_S \frac{r \times (\rho_s - 1) v(r)}{2} dS + \cdots \\

\int_S (B_s^{-1} - 1) p(r) e^{i\beta \cdot r} dS = \int_S (B_s^{-1} - 1) p(r) dS - i\beta \int_S \frac{r \times (B_s^{-1} - 1) p(r)}{2} dS + \cdots
\]
(2) is rewritten as

\[ -i \mathbf{B} \left[ \bar{p} - \frac{i \omega}{2a^2} \int \mathbf{r} \cdot (\rho_{sw} - 1) \mathbf{v}(\mathbf{r}) dS \right] = -i \omega \rho_0 \left[ \bar{v} + \frac{1}{a^2} \int (\rho_0 - 1) \mathbf{v}(\mathbf{r}) dS \right] \]

\[ -i \mathbf{B} \cdot \left[ \bar{\nabla} - \frac{i \omega}{2a^2} \int \mathbf{r} (B_s^{-1} - 1) p(\mathbf{r}) dS \right] = -i \omega B_0^{-1} \left[ \bar{p} + \frac{1}{a^2} \int (B_0^{-1} - 1) p(\mathbf{r}) dS \right], \quad \text{(4)} \]

where \( \rho_{sn} \) is \( \rho_s \) in the \( n \) (\( r \) or \( x \)) direction. Comparing the microscopic (1) and macroscopic (4) equations, we obtain the averaged fields \( p_{av}, \pi_{av}, v_{av}, \) and \( e_{av} \) as

\[ p_{av} = \bar{p} - \frac{i \omega}{2a^2} \int \mathbf{r} \cdot (\rho_{sw} - 1) \mathbf{v}(\mathbf{r}) dS, \quad \pi_{av} = \rho_0 \left[ \bar{v} + \frac{1}{a^2} \int (\rho_0 - 1) \mathbf{v}(\mathbf{r}) dS \right] \]

\[ v_{av} = \bar{v} - \frac{i \omega}{2a^2} \int \mathbf{r} (B_s^{-1} - 1) p(\mathbf{r}) dS, \quad \pi_{av} = \rho_0 \left[ \bar{v} + \frac{1}{a^2} \int (\rho_0 - 1) \mathbf{v}(\mathbf{r}) dS \right] \]

\[ e_{av} = B_0^{-1} \left[ \bar{p} + \frac{1}{a^2} \int (B_0^{-1} - 1) p(\mathbf{r}) dS \right], \quad \text{(5)} \]

which make the macroscopic acoustic equations (4) to take the usual forms of \(-i \mathbf{B} p_{av} = -i \omega \pi_{av}\), and \(-i \mathbf{B} \cdot v_{av} = -i \omega e_{av}\). Under good approximations of \( \bar{v} \sim \frac{1}{a^2} \int v_x dS \) and \( \bar{p} \sim \frac{1}{a^2} \int p dS \), we finally obtain the effective parameters of \( \rho_x \) and \( B^1 \):

\[ \rho_x = \frac{\pi_{av,x}}{\rho_{av,v,x}} = \frac{\bar{v}_x + \frac{1}{a^2} \int (\rho_{sw} - 1)v_x dS}{\bar{v}_x - \frac{i \omega}{2a^2} \int (B_s^{-1} - 1) p x dS} \sim \frac{\int \rho_0 v_x dS}{\int v_x dS - \frac{i \omega}{2} \int (B_0^{-1} - 1) p x dS}, \quad \text{and} \]

\[ B^1 = \frac{\pi_{av,x}}{\rho_{av,v,x}} = \frac{\bar{p} + \frac{1}{a^2} \int (B_s^{-1} - 1) p dS}{\bar{p} + \frac{i \omega}{2a^2} \int r (\rho_{sw} - 1)v_x dS} \sim \frac{\int B_0^{-1} p dS}{\int p dS + \frac{i \omega}{2} \int r (\rho_{sw} - 1)v_x dS}. \quad \text{(6)} \]

Supplementary Note 2. Decoupling of the effective macroscopic acoustic parameters near the Dirac point

Near the Dirac point, the decoupling of \( \rho_x \) and \( B^1 \) can be achieved using a meta-atom having an inner sub-cell with radial symmetry and outer sub-cells supporting linear vibrations, constructed with a membrane, air, and solid walls (Fig. 1b in manuscript). Considering that the area (thickness) and compressibility of the membrane is much less than that of air such that \( s_m << s_0, B_{sm}^{-1} << B_{s0}^{-1} = 1 \), and noting that the mass of the membrane is much larger than that of air such that \( \rho_{sm}s_m >> \rho_{s0}s_0 \), we achieve,
\begin{equation}
\rho_x \sim \frac{\int \rho_m v_x dS}{\int_{S_m} v_x dS + \frac{i \omega}{2} \int_{S_m} px dS} = \frac{\int \rho_m v_x dS + \int \rho_m v_x dS}{\int_{S_m} v_x dS + \frac{i \omega}{2} \int_{S_m} px dS - \frac{i \omega}{2} B_{s0}^{-1} \int_{S_m} px dS} = \frac{\int B_{s0}^{-1} pdS}{\int_{S_m} px dS + \frac{i \omega}{2} \int_{S_m} \rho_{mx} v_x dS} = \frac{\int B_{s0}^{-1} pdS}{\int_{S_m} px dS + \frac{i \omega}{2} \int_{S_m} \rho_{mx} v_x dS + \int_{S_m} \rho_{mx} v_x dS}.
\end{equation}

where ISm(0) and OSm(0) refer to the integration region of the inner sub-cell and the outer sub-cell at the membrane (air), respectively. Now, near \(B^{-1} \sim 0\) with zero effective compressibility, \(\int_{OSm}(\rho_{mx} v_x) dS \sim 0\) because the radial movement of the outer membrane is impossible, and because the first term in the denominator vanishes from \(\int_{S0} pdS = B_{s0}^{-1} \int_{S0} px dS \sim 0\) (with \(B_{s0}^{-1} = 1\), and \(B^{-1} \sim 0\) while \(B^{-1}\) proportional to \(\int B_{s0}^{-1} pdS\)), we obtain

\begin{equation}
B^{-1} = \frac{\int B_{s0}^{-1} pdS}{\frac{i \omega}{2} \int_{S_m} \rho_{mx} v_x dS}.
\end{equation}

Similarly, near \(\rho_x \sim 0\), the outer- and inner- membrane should move out of phase but need to maintain the same momentum value. To control \(\rho\) with the outer cells we impose \(\int_{ISm} \rho_{mx} v_x dS \sim 0\), which can be achieved with a large \(\rho_m\) in the inner membrane because \(\rho_x \sim \int_{ISm} \rho_{mx} v_x dS + \int_{OSm} \rho_{mx} v_x dS \sim 0\). Additionally, for the membrane, \(s_m << s_0\) thus \(\int_{S_m} px dS << \int_{OSm} v_x dS\); thus,

\begin{equation}
\rho_x = \frac{\int_{S_m} \rho_{mx} v_x dS + \int_{S_m} \rho_{mx} v_x dS}{\int_{OSm} v_x dS}.
\end{equation}

**Supplementary Note 3. Coupled mode theory derivation of \((\rho, B^{-1})\) for a meta-atom having a sub-cell design**

For a specific geometry of Fig. 1b in the manuscript, a more rigorous solution for \((\rho, B^{-1})\) can be obtained by solving the coupled mode equation. Assuming an average displacement \(q_{[jk]}\) (i.e., membrane displacement directing from \(j\) to \(k\)) and pressure \(p_{[i]}\) (i.e., pressure in the sub-cell room of \(i\)) as in Supplementary Fig. 1a, we separately apply Newton’s law to the membranes and Hooke’s
law to the sub-cells to obtain the relation between $p_{[i]}$ and $q_{[j k]}$. Assuming Floquet’s boundary conditions with an acoustic refractive index of $(n_x, n_y)$, the coupling equations are then expressed as

$$B_0^{-1} p_{[i]} s_{[i]} = \sum_{k} b_{[j k]} q_{[j k]} - \sum_{j} b_{[j i]} q_{[j i]}, \quad -\rho_0 t_{[j k]} \alpha^2 q_{[j k]} = (p_{[k]} - p_{[i]}),$$

$$\frac{p_{[7]} + p_{[3]}}{p_{[3]} + p_{[1]}} = q_{[37]} = \exp(-i \beta_0 n_x a), \quad \frac{p_{[2]} + p_{[6]}}{p_{[8]} + p_{[4]}} = q_{[26]} = \exp(-i \beta_0 n_y a)$$

where $b_{[j k]}, t_{[j k]}$ and $s_{[i]}$ are the width, thickness and area of the membrane $[j k]$ and sub-cell $[i]$; the $B_0$, $\beta_0$ are the air modulus and propagation constant, $\rho_{Al}$ is the Aluminum density and $a$ is the lattice constant.

To find the eigenmodes of the system, we rewrite (10) in the form of a linear system $(A x = 0)$, where $A$ is the 17 x 17 matrix, and $x = [p, q]$ is the vector consisting of $p$'s and $q$'s. Focusing on the $x$ direction $(n_x = n, n_y = 0)$, (10) can be written as

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix} \begin{pmatrix}
p_{[0]}
\end{pmatrix} = \begin{pmatrix}
0
\end{pmatrix}$$

(11)

where $C_i = \frac{B_0 d_i}{s_i}, C_0 = \frac{B_0 d_o}{s_0}, M_i = -\frac{\omega^2 a}{2a_m} [\rho_{Al} f_i + \frac{\rho_0 s_0}{2a_m} + \frac{\rho_0 s_1}{4a_m}], M_o = -2\omega^2 [\rho_{Al} f_0 + \frac{\rho_0 s_0}{2a_m} + \frac{\rho_0 s_1}{4a_m} + \frac{\rho_0 s_2}{6a_m}]$ and $q_{[e]} = \frac{b}{a} q_e$.

It should be noted that the matrix $A$ in (11) consists of the coefficients from Hooke’s law (row 1 to 5), Newton’s law (row 6 to 13), and Floquet’s boundaries (row 14 to 17).
To achieve a non-zero physical null space of \( x = [p, q] \), Det(A) should vanish, leading to the analytical expression for effective \( n \):

\[
\exp(i\beta na) = \frac{1}{2C_0} \{M_0(M_1 + C_0)(M_1 + C_0 + 4C_1) + 2C_0[M_1(M_1 + C_0) + C_0(4M_1 + C_0)] \pm \sqrt{[2M_0C_0 + M_0(M_1 + C_0)][M_1 + C_0 + 4C_1][2M_0C_0(M_1 + C_0) + 4C_0C_1(2M_1 + C_0) + M_0(M_1 + C_0)(M_1 + C_0 + 4C_1)]} \}.
\]

(12)

The effective impedance \( Z \) then also becomes

\[
ZZ = \frac{2i\omega(M_1 + C_0)Z}{P_{\{1\} + P_{\{2\}}}} = \pm \frac{2i\omega(M_1 + C_0)Z}{\sqrt{[2M_0C_0 + M_0(M_1 + C_0)][2M_0C_0(M_1 + C_0) + 4C_0C_1(2M_1 + C_0) + M_0(M_1 + C_0)(M_1 + C_0 + 4C_1)]}}.
\]

(13)

Using \( \rho = n/Z \) and \( B^{-1} = nZ \), we finally obtain rigorous solutions for \( \rho \) and \( B \) in analytical forms.

Of critical importance is in solving the inverse problem, to find the design parameters of the membranes \((M_O, M_I)\) from the target wave parameters of \( n \) and \( Z \). From (12) and (13), we obtain,

\[
M_O = -2C_0 \pm \frac{2\omega[1 + \cos(\omega n/a)]}{Z_n \sin(\omega n/a)} \pm \frac{4C_1 \omega}{C_0} \left[ \frac{1 \pm \frac{C_0 \omega^2}{4C_1 \omega} Z_n \sin(\omega n/a)}{\sin(\omega n/a)} \right] \frac{1}{C_0 Z_n} \sqrt{\frac{1 \pm \frac{C_0 \omega^2}{4C_1 \omega} Z_n \sin(\omega n/a)}{\sin(\omega n/a)}}
\]

(14)

Simplifying (14) in the long wavelength limit \([\exp(i\beta na) \sim 1 + i\beta na - 1/2(\beta na)^2]\) and by setting \( \pm_1 = + \) and \( \pm_2 = - \) for the condition of \((\text{Im}[n] \geq 0, \text{Re}[Z] \geq 0)\), for a first order approximation we achieve decoupled equations for \( M_O(\rho) \) and \( M_I(B^{-1}) \) that is equivalent to equation (3) in the manuscript:

\[
-(\rho_0 \rho + \rho_0 \rho_0) = \frac{1}{2} M_O - C_0 + \frac{2\omega^2}{8C_1} Z_n \sin(\omega n/a) - \frac{4C_1 \omega}{C_0} \left[ \frac{1 \pm \frac{C_0 \omega^2}{4C_1 \omega} Z_n \sin(\omega n/a)}{\sin(\omega n/a)} \right] \frac{1}{C_0 Z_n} \sqrt{\frac{1 \pm \frac{C_0 \omega^2}{4C_1 \omega} Z_n \sin(\omega n/a)}{\sin(\omega n/a)}}
\]

(15)

From the CMT (11), the dispersion relation \( \omega(k) \) also can be obtained. Replacing \( M_I \) and \( M_O \) with \( m_I \omega^2 \) and \( m_O \omega^2 \) respectively, the CMT now becomes the function of \( \omega \) and \( k \). Directly solving Det[A]=0 in terms of wavenumber \( \omega(k) \), we obtain,
\[
\omega = 0, \pm i \frac{\sqrt{C_0}}{\sqrt{m_i}} \pm \sqrt{(m_i + m_o)C_0},
\]
\[
\pm \sqrt{-\frac{2C_0}{m_o} - \frac{2C_0}{m_i} - \frac{4C_1}{m_i} + \frac{2^{1/3}}{m_i m_i} P + \frac{2^{1/3}}{m_i} Q} / 3,
\]
\[
\pm \sqrt{-\frac{2C_0}{m_o} - \frac{2C_0}{m_i} - \frac{4C_1}{m_i} + \frac{2^{1/3}}{m_i m_i} \left( -\frac{1 - \sqrt{3}i}{2} \right) P + \frac{2^{1/3}}{m_i} \left( -\frac{1 + \sqrt{3}i}{2} \right) Q} / 3,
\]
\[
\pm \sqrt{-\frac{2C_0}{m_o} - \frac{2C_0}{m_i} - \frac{4C_1}{m_i} + \frac{2^{1/3}}{m_i m_i} \left( -\frac{1 + \sqrt{3}i}{2} \right) P + \frac{2^{1/3}}{m_i} \left( -\frac{1 - \sqrt{3}i}{2} \right) Q} / 3.
\]

where
\[
II = 2 \left[ m_{i0}^2 + 3 m_{i0} m_{i1} - 6 m_{i1} m_{i1} - 8 m_{i1} \right] C_{i0}^2 C_i + 12 \left[ m_{i0}^2 - m_{i0} m_{i1} + 4 m_{i1} m_{i1} \right] C_{i0}^2 C_i + 54 m_{i0}^2 m_i \left[ \cosh(k, a) + \cosh(k, a) \right] C_{i0}^2 C_i + 48 \left[ -m_{i0}^2 + 2 m_{i0} m_{i1} \right] C_{i0}^2 C_i^2 - 128 m_{i0} C_i^3
\]
\[
P = \left[ II + \sqrt{II^2 - 4 \left[ m_{i0}^2 + 2 m_{i0} m_{i1} + 4 m_{i1} \right] C_{i0}^2 C_i^2 + 4 \left[ m_{i0}^2 - 2 m_{i0} m_{i1} \right] C_{i0} C_i + 16 m_{i0} C_i^2 \right] \right]^{1/2}
\]
\[
Q = \left. \left( m_{i0} + 2 m_{i0} m_{i1} / m_{i0} \right) C_{i0}^2 + 4 \left( m_{i0} - 2 m_{i1} \right) C_{i0} C_i + 16 m_{i0} C_i^2 \right. \right]
\]

(16)

Excluding negative valued and trivial solutions, we get solutions corresponding to 5 lowest bands; two flat bands and three dispersive bands (Supplementary Fig. 2a). Focusing on the Dirac point \((\rho, B^{-1}) = (0, 0)\), we plot band diagram near 1,300Hz as shown in Supplementary Fig. 2b; exhibiting the Dirac cone and a third flat band (corresponding to the acoustic transverse mode\(^6\)) providing a triple degeneracy at \(k = 0\).

**Supplementary Note 4. Characteristic motions of dipolar (\(\rho\)) and monopolar (\(B^{-1}\)) eigenmodes at the Dirac point**

Combining the eigensolution sets of \(v\) and \(p\) that were calculated in Supplementary Note 3 meanwhile considering the excitation directions, it is possible to extract the dipolar-\(\rho\) (linear vibrations) and monopolar-\(B^{-1}\) (radial vibrations) modes, of the proposed structure. In Supplementary Fig. 3, we plot the CMT-calculated eigenmode patterns and displacement of membranes (gray lines) near the Dirac point, which clearly shows negligible vibrations of the inner membranes for the \(\rho\) - dipolar mode and negligible vibrations of the outer membrane for the \(B^{-1}\) - monopolar mode. Black arrows indicate the momentum of the individual membranes. These results confirm the proposed ansatzs and discussions regarding Eq. (2) in the manuscript.
Supplementary Note 5. Decoupled operation away from the Dirac point and Extension of the tuning range

To shift the center of decoupling operation away from the Dirac point \((\rho, \frac{1}{B}) = (0, 0)\), we place the meta-atom in the host medium of \((\rho_h, \frac{1}{B_h})\) (Supplementary Fig. 4a). In the frame of the effective medium theory, the shifted center of decoupling operation is calculated from \((\rho_c, \frac{1}{B_c}) = (\rho_h(1-a/a_h), B_h(1-a^2/a_h^2))\), well agreeing with the FEM results (Supplementary Fig. 4b-d). For example, with air \((\rho_h, \frac{1}{B_h}) = (1, 1)\) and \(a_h = 7.5\, \text{cm}\), it was possible to shift the decoupled operation point to \((\rho_c, \frac{1}{B_c}) = (0.2, 0.36)\); mixing with \((\rho_h, \frac{1}{B_h}) = (2, 2)\) we also get \((0.4, 0.72)\).

For the extension of effective parameters’ tuning ranges, we first focus on the extension of \(\rho\), with Eq. (3) in the manuscript and (15). With \(\rho \sim \rho_{\text{Al}t_O}/a\) and \(\frac{1}{B} \sim \rho_{\text{Al}t_1} a\) (as \(s_O \sim a^2\)), the tuning range of \(\rho\) is extended (while keeping \(\frac{1}{B}\) at same value) by increasing \(t_O\) and \(t_1\), and using smaller unit cell \(a\). For example, we get \(\rho = -36\sim36\) and \(\frac{1}{B} = -1\sim1\) with \(a = 1.0\, \text{cm}\) & thicker \(t_O\) and \(t_1\) (Supplementary Fig. 5), as intuitively expected for the smaller and heavier meta-atom. The same equation (3) in the manuscript and (15) can be used to change the tuning range of \(\frac{1}{B}\). Nonetheless, it is noted that the tuning range of \(\frac{1}{B} = -1\sim1\) is already not small, because \(\frac{1}{B}\) is normalized to Air \((B_{\text{Air}}^{-1} : B_{\text{Solid or Liquid}}^{-1} = 10^4 \sim 10^6 : 1)\).

Supplementary Note 6. Coupled mode theory derivation of bianisotropy \(\xi\) for a meta-atom having a asymmetric sub-cell design

Decoupled control over the bianisotropy \(\xi^{9-11}\) of the acoustic meta-atom also can be achieved with the asymmetric assignment of the inner membrane thickness (non-zero \(\Delta t_1 = t_{1-\text{Right}} - t_{1-\text{Left}}\)), as shown in Supplementary Fig. 6. The analytical formula for \(\xi\) is obtained by solving the coupled mode equations using a similar process to that described in Supplementary Note 3. Solving for \(Z_{+}\) and \(Z_{-}\) for \(+x\) and \(-x\) propagating waves, respectively, \(\xi = \ln\left(\frac{Z_{+} - Z_{-}}{Z_{+} + Z_{-}}\right)\) becomes

\[
\xi = -2\frac{C_0^2}{a\omega} (M_t + C_o + 2C_1) \Delta M \frac{1}{F} \log\left(\frac{1}{(2C_0^2(M_t + C_o)C_1)(G - F)}\right)
\]  

(17)
where
\[
\begin{align*}
F &= \sqrt{(M_1 + C_o)(H - K + 2C_i(2H - K + 2C_i(2H - K))} \\
G &= (M_1 + C_o)(H - K) + 2C_i(2H - K + 2M_iC_o + C_o^2) \\
H &= (M_1 + C_o)(2M_iC_o + M_o(M_1 + C_o)) \\
K &= (M_o + 2C_o)\Delta M^2
\end{align*}
\]
and \(\Delta M = \frac{a_0\omega^2}{a_{in}}\Delta t_i\).

In a good approximation to the first order \(\Delta t_i\) near the Dirac point \(\rho = B^{-1} = 0\), the bianisotropy \(\xi\) of the meta-atom is solely determined by the structural asymmetry \(\Delta t_i\), as shown below, which is independent (completely decoupled) from \(t_i\) or \(t_o\):

\[
\xi = \frac{4c_0C_i}{a_0^\omega}\Delta M^2 + 16C_i^2 \log(1 + \frac{\Delta M^2}{8C_i^2} + \frac{\Delta M^2}{C_i^2})
\]

Supplementary Note 7. Condition of complete tunneling between asymmetric impedance waveguides with bianisotropic meta-waveguide

In this section, we consider the problem of perfect tunneling between two different impedance waveguides (region I and region III, \(Z_I \neq Z_{III}\)), with the introduction of bianisotropic media \((\rho, B^{-1}, \xi)\) in their interface (region II, Supplementary Fig. 7, left). Assuming the incident wave from the left, we convert the well-known electromagnetic wave equations of bianisotropy\(^9\) into the acoustic wave equations; by replacing \(E\) and \(H\) with \(v\) and \(p\), and applying the boundary conditions we get,

\[
\begin{align*}
v_I &= v_{I+}e^{-ik_{I+}n} + v_{I-}e^{ik_{I-}n} \\
p_I &= \frac{v_{I+}}{Z_+}e^{-ik_{I+}n} - \frac{v_{I-}}{Z_-}e^{ik_{I-}n} \\
v_{I+}(0) &= 1 + R, v_{I+}(d) &= \sqrt{\frac{Z_{III}}{Z_I}}T, p_{I+}(0) = 1 - R, p_{I+}(d) = \sqrt{\frac{1}{Z_I Z_{III}}}T
\end{align*}
\]

where \(n\) is the refractive index; \(Z_+\) and \(Z_-\) are the impedances for the + and - directions \((Z_+ \neq Z_-\) when \(\xi \neq 0\), respectively; \(R\) and \(T\) denote the reflection and transmission coefficients across region II. To achieve 100% transmittance with zero phase shift, we impose \(R = 0\) and \(T = 1\) to obtain
Eliminating $v_{II+}$ and $v_{II-}$ in (20), we get

\[
\begin{pmatrix}
Z_e \frac{v_{II+}}{Z_+} + v_{II+} e^{-ik_{0}nd} &= \frac{\sqrt{Z_{III}}}{Z_+} \\
Z_e \frac{v_{II-}}{Z_-} + v_{II-} e^{ik_{0}nd} &= \frac{1}{Z_- Z_{III}}
\end{pmatrix}
\]  

(20)

Additional equations can be obtained from the other case, where the wave propagates from the region of input impedance $Z_{III}$ to the region of the output impedance $Z_{I}$. By exchanging $Z_{I}$ and $Z_{III}$ and $Z_+$ and $Z_-$ in (21), we get two additional equations, and together with (21) we achieve the following expressions for $\exp(ik_{0}nd)$:

\[
\begin{pmatrix}
\frac{Z_{I}(Z_{I}+Z_{-})}{Z_{I}(Z_{I}+Z_{+})} e^{-ik_{0}nd} + \frac{Z_{III}(Z_{I}-Z_{-})}{Z_{I}(Z_{I}+Z_{+})} e^{ik_{0}nd} &= \frac{\sqrt{Z_{III}}}{Z_{I}} \\
\frac{Z_{I}+Z_{-}}{Z_{I}(Z_{I}+Z_{+})} e^{-ik_{0}nd} - \frac{Z_{III}-Z_{+}}{Z_{I}(Z_{I}+Z_{+})} e^{ik_{0}nd} &= \frac{1}{\sqrt{Z_{I} Z_{III}}}
\end{pmatrix}
\]  

(21)

It is evident that (22) holds only if $Z_{+} = 0$ and $Z_{-} = \infty$ (or if $Z_{+} = \infty$ and $Z_{-} = 0$). This leads to two sets of solutions $(n, Z_{+}, Z_{-}) = (1/2\log(Z_{III}/Z_{I}))/ik_{0}d, 0, \infty)$ and $(1/2\log(Z_{I}/Z_{III}))/ik_{0}d, \infty, 0)$, which are in fact physically identical in terms of $(\rho, B^{-1}, \xi) = (2n/(Z_{+}+Z_{-}), 2Z_{+}Z_{-}n/(Z_{+}+Z_{-}), (Z_{+}+Z_{-})/(Z_{+}+Z_{-})) = (0, 0, 1/2\log(Z_{I}/Z_{III}))/k_{0}d)$. It is important to note that when the impedances of the input and output waveguides are identical ($Z_{I} = Z_{III}$), all solutions of $\rho, B^{-1}$ and $\xi$ should be zero concurrently; enforcing region II to be matched zero index material (for the initial condition of $R = 0$ and $T = 1$). In contrast, when $Z_{I} \neq Z_{III}$, the required bianisotropy $|\xi|$ increases for larger impedance mismatch $Z_{I}/Z_{III}$.

The condition for complete transmission between different width waveguides ($w_1 \neq w_2$, right of Supplementary Fig. 7) is also obtained by replacing $Z$ with $w$, which leads to the solution $(\rho, B^{-1}, \xi) = (0, 0, 1/2\log(w_1/w_2)/k_{0}d)$. It is noted that the proposed bianisotropic impedance conversion is intrinsically different from the resonator-based impedance matching\textsuperscript{12}, as our condition is determined only by the ratio of impedances $(Z_{III}/Z_{I})$, meanwhile the resonator approach involves both the matching of impedances $(Z_{I}Z_{III})^{1/2}$ and resonance $n_{eff} \cdot d = \lambda/4$.  


Supplementary Note 8. Transmittance of bianisotropic impedance conversion as a function of $\xi$

In this section, we calculate the transmittance for the problem shown in Supplementary Fig. 7 when the wave parameters of the medium in region II are $(\rho, B^{-1}, \xi) = (0, 0, \xi)$. For this case, the wave equations in (19) are reduced to

$$\begin{align}
    v_{II} &= v_{II, e^{ik_0 d}}, \\
    p_{II} &= p_{II, e^{-ik_0 d}}, \\
    v_{II}(0) &= 1 + R, v_{II}(d) = \sqrt{\frac{Z_{III}}{Z_1}} T, \\
    p_{II}(0) &= \frac{1 - R}{Z_1}, p_{II}(d) = \sqrt{\frac{1}{Z_1 Z_{III}}} T. \\
\end{align}\quad (23)$$

Solving (23) and also replacing $Z$ with $w$, the transmittance $T$ as a function of $\xi$ is derived as

$$T = \frac{2}{\sqrt{\frac{Z_{III}}{Z_1} e^{-k_0 d} + \frac{Z_1}{Z_{III}} e^{k_0 d}}} = \frac{2}{\sqrt{\frac{w_2}{w_1} e^{-k_0 d} + \frac{w_1}{w_2} e^{k_0 d}}}.$$ \quad (24)

From (24), the maximum transmittance ($T = 1$) is shown to occur when $\exp(2k_0 \xi d) = Z_{III}/Z_1$, which is equivalent to the primary result obtained in Supplementary Note 7.

Supplementary Note 9. Determination of $\rho, B^{-1}$ and $\xi$ for the target $\phi_R, \phi_T$

With the target $S_{11} (= \sqrt{1/2\exp(i\phi_R)})$, $S_{21} (= \sqrt{1/2\exp(i\phi_T)})$, and $S_{22} (= \sqrt{1/2\exp(i\phi)})$ for the bianisotropic meta-atom (Supplementary Fig. 8), it is straightforward to calculate $\rho, B^{-1}$ and $\xi$. Adapted from the corresponding solutions in electromagnetic waves we have,

$$\begin{align}
    n &= \frac{\cos^{-1}(1 - S_{11} S_{22} + S_{21}^2)}{2S_{21}}, \\
    \xi &= \frac{S_{11} - S_{22}}{2\sin(nk_0 d)}, \\
    B^{-1} &= \frac{-\sin(nk_0 d)}{\sin(nk_0 d)} \frac{1 + S_{11} S_{22} + S_{11} S_{22} - S_{21}^2}{2S_{21}}, \\
    \rho &= \frac{n^2 + \xi^2}{B^{-1}}. \\
\end{align} \quad (25)$$

Assuming the lossless case (i.e., $n$ is real or pure imaginary), we obtain real-valued $\xi$ and $B^{-1}$ if $\text{Im}[(S_{11}-S_{22})/S_{21}] = 0$ and $\text{Re}[(1+S_{11}+S_{22}+S_{11} S_{22} - S_{21}^2)/2S_{21}] = 0$. Then we get,
\[
\operatorname{Im}\left(\frac{S_{11} - S_{22}}{S_{21}}\right) = \operatorname{Im}\left(\frac{e^{i\phi} - e^{i\theta}}{e^{i\phi}}\right) = \sin(\phi_R - \phi_T) - \sin(\phi - \phi_T) = 2\sin(\frac{\phi_R - \phi}{2})\cos(\frac{\phi + \phi_R - 2\phi_T}{2}) = 0
\]

\[
\operatorname{Re}\left(\frac{1 + S_{11} + S_{22} + S_{11}S_{22} - S_{21}^2}{2S_{21}}\right) = \frac{1}{2\sqrt{2}}\left[\cos(\phi_T) + \cos(\phi + \phi_R - \phi_T)\right] + \frac{1}{2}\left[\cos(\phi_R - \phi_T) + \cos(\phi - \phi_T)\right] = 0.
\]

Both equations in (26) are satisfied when \(\phi = \pi - \phi_R + 2\phi_T\). Using the target values \(\phi_R, \phi_T\), and \(\phi = \pi - \phi_R + 2\phi_T\) to calculate \(S_{11}, S_{21},\) and \(S_{22}\), then the required values of \((\rho, B^{-1}, \xi)\) are determined using (25). Figure 4a in the manuscript and Supplementary Fig. 9 show the phase shift contour \((\phi_R, \phi_T)\) in the parameter octant space of \((\rho, B^{-1}, \xi)\) and the maps of each required wave parameters for the decoupled manipulation of the phase shift for \((\phi_R, \phi_T)\) subject to the constraint of 50:50 power division. For example, Supplementary Fig. 10 shows the required phase shifts \((\phi_R, \phi_T)\) of an individual meta-atom (in a single sheet of 40 × 1 array) achieving ordinary \((\Delta\phi(x) = 0, \text{blue marks})\) or extra-ordinary \((\Delta\phi(x) \neq 0, \text{red marks})\) reflection and transmission. From the target \((\phi_R, \phi_T)\) values in Supplementary Fig. 10, the calculation of \((\rho, B^{-1}, \xi)\) are obtained from Eqs. (25); then, the top-down determination of the corresponding \((t_O, t_I, \Delta t_I)\) is straightforward.

**Supplementary References**


