Entanglement dynamics in critical random quantum Ising chain with perturbations

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We simulate the entanglement dynamics in a critical random quantum Ising chain with generic perturbations using the time-evolving block decimation algorithm. Starting from a product state, we observe super-logarithmic growth of entanglement entropy with time. The numerical result is consistent with the analytical prediction of Vosk and Altman using a real-space renormalization group technique.

Many-body localization (MBL) is an active area of research studying the effects of interactions added to an Anderson insulator [1,4]. One characteristic feature (among others) of MBL lies in the entanglement dynamics [5]: Starting from a product state, the entanglement entropy grows logarithmically with time. This is in contrast to the case of an Anderson insulator, in which the entanglement entropy remains bounded [6]. The logarithmic growth of entanglement in MBL systems has been well established: It was observed numerically [7], followed by theoretical explanations [8,9].

Besides many-body localization-delocalization transitions [11], it is also important to understand transitions between MBL phases. The critical random quantum Ising chain with generic perturbations is such an example, in which Vosk and Altman [8] predicted super-logarithmic growth of entanglement using a real-space renormalization group (RSRG) technique. RSRG is an analytical approach to the long-range or long-time physics of random spin chains [12,13]. It is believed and only believed to be asymptotically exact at “infinite-randomness” quantum critical points.

The main contribution of this paper is to observe numerically the super-logarithmic growth of entanglement (which was conjectured to be a universal feature at transitions between MBL phases) using the time-evolving block decimation (TEBD) algorithm [14].

Preliminaries. The entanglement entropy of a bipartite pure state $\rho_{AB}$ is the von Neumann entropy

$$S(\rho_A) = -\text{tr}(\rho_A \log_2 \rho_A)$$

of the reduced density matrix $\rho_A = \text{tr}_{B} \rho_{AB}$. It is the standard measure of entanglement for pure states.

The Hamiltonian of the random quantum Ising chain is

$$H_{\text{Ising}} = \sum_j J_j \sigma_j^z \sigma_{j+1}^z + h_j \sigma_j^x,$$

where $J_j$'s are independent and identically distributed (i.i.d.) and $h_j$'s are i.i.d. random variables. This model is non-interacting in the sense of having free-fermion representations. Its phase diagram is parametrized by

$$\delta = (\ln|\bar{h}| - \ln|\bar{J}|)/(\text{var} \ln |\bar{h}| + \text{var} \ln |\bar{J}|),$$

which describes the competition between $J$ and $h$ terms. The ferromagnetic ($\delta < 0$) and paramagnetic ($\delta > 0$) phases are separated by an infinite-randomness critical point [15]. The transition occurs in not only the ground state but also the excited eigenstates of the model [16].

The entanglement dynamics of $H_{\text{Ising}}$ was studied numerically in Ref. [17]: Starting from a product state, the entanglement entropy grows double-logarithmically with time and remains bounded at and away from the critical point, respectively. This is consistent with the analytical result obtained using RSRG [8].

Here we study the weakly interacting model [8,18]

$$H = H_{\text{Ising}} + \sum_j J'_j \sigma_j^x \sigma_{j+1}^x,$$

where $J'_j$’s ($|J'_j| \ll |J_j|, |h_j|$) are i.i.d. random variables. The integrability-breaking $J'$ terms respect the $Z_2$ symmetry of $H_{\text{Ising}}$. They do not affect the phase diagram, but change the asymptotic behavior of certain dynamical quantities.

Let $S(t)$ denote the entanglement entropy of $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ across a random cut in a random sample of $|\psi(t)\rangle$ for $t > 0$, where the initial state $|\psi(0)\rangle = |\uparrow\uparrow\uparrow\cdots\downarrow\downarrow\rangle$ is a random product state in the computational basis. Using RSRG, Vosk and Altman [8] predicted

$$\langle S(t) \rangle \sim c \ln^{\sqrt{3} - 1} t$$

for $\delta = 0$ (5)

$$\langle S(t) \rangle \sim c \ln t$$

for $\delta \neq 0$ (6)

in the limit $t \to \infty$, where $\langle \cdots \rangle$ denotes averaging over randomness. Ref. [8] did not work out the prefactor $c$ explicitly, but it is easy to see $c = 1/8$ for $\delta = 0$ from the calculations there.

Results. We simulate the dynamics of $|\psi(t)\rangle$ using TEBD, which is a quite efficient method due to the slow growth of entanglement [19]. For moderate disorder, we can do very large system size $n > 100$ with a moderate amount of computational resources. It suffices to use only one random sample of $|\psi(t)\rangle$ and average over all cuts provided that the chain is long enough.

The simulation is carried out in the following way. The probability density functions of the random variables $J_i, h_i, J'_i$ are $f(J_i), f(h_i), 25f(25J'_i)$, respectively, where $f(x) = 1/(4\sqrt{|x|})$ for $|x| \leq 1$ and $f(x) = 0$ otherwise. This
means moderate disorder and weak interaction in the sense of $|J| = |h| = 25|J|$ for typical values. The system size is 10,050, and open boundary conditions are used. To avoid boundary effects, we average only over 10,000 cuts in the bulk. We use a second-order Trotter decomposition with time step 0.02, and the truncation error per step is kept below $10^{-6}$.

The simulation result is shown in Fig. 1. On a semi-log plot, the $\langle S(t) \rangle$ versus $t$ curve slightly but clearly bends upward. This is qualitatively consistent with (5), which states that the entanglement entropy grows super-logarithmically with time.

It should be noted that (5) only keeps the leading term in the limit $t \to \infty$. At finite $t$, a necessary condition for the subleading terms to be completely negligible is $\ln^{(3-\sqrt{5})/2} t \gg 1$ [8]. Therefore, one cannot accurately confirm (5) even if $t \approx 10^6$.

Neglecting subleading terms, we demonstrate a power law relationship between $\langle S(t) \rangle$ and $\ln t$ from the data. As shown on a loglog-log plot in the inset of Fig. 1, the red line

$$\langle S(t) \rangle = 0.0709 \ln^{1.36} t$$  \hspace{1cm} (7)

appears to be a good fit, which is semiquantitatively consistent with (5). Noticeable differences between (5) and (7) are expected because the subleading terms are not completely negligible. Despite this, it makes a lot of sense that the prefactors in (5) and (7) are within a factor of 2.

It would be desirable to confirm (6) numerically for small $|\delta| > 0$. However, a necessary condition for the effects of a finite $\delta$ to be apparent is $t \gtrsim \exp(1/|\delta|)$ [8]. Indeed, we observe super-logarithmic growth of entanglement up to $t \approx 200$ for small $|\delta| > 0$ (data not shown).

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