analysis of radiation coupling from the tapered fiber is based on a meridional-ray-propagation model which includes power flow calculation with the Poynting vector. The simple meridional-ray model without modification is able to predict propagation delay and pulse dispersion of a step-index, multimode fiber.5,6

By reciprocity, it appears possible to utilize such a conically tapered fiber as a means of coupling light from sources with high numerical apertures into a fiber of a relatively small numerical aperture by entering through the tapered zone. Of course, no improvement of intrinsic source brightness is possible. In our application of this fiber as a dispersing element, it was found to be much simpler than any other scheme. Fibers which are normally bundled together for the purposes of directing light into remote regions for illumination might benefit from such a tapered end in order to achieve wide-angle illumination.

ACKNOWLEDGMENTS

This research was supported in part by National Institutes of Health Grant No. GM22550 (Burn Research Center). We would like to thank Professor Akira Ishimaru and Dr. Robert Smith for technical discussions. Computer programming assistance from Mr. W. K. Tsiang is also highly appreciated.

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Theory of Bragg fiber*

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(Received 16 January 1978; revised manuscript received 24 May 1978)

The possibility of using Bragg reflection in a cylindrical fiber to obtain lossless confined propagation in a core with a lower refractive index than that of the cladding medium is proposed and analyzed.

I. INTRODUCTION

The guiding of light in a slab with a refractive index lower than that of the substrate was proposed and analyzed by Yeh and Yariv in 1976.1 Experimental demonstration of this phenomenon in a sample fabricated by molecular beam epitaxy2 was recently performed.3 A general mode dispersion was derived for the planar Bragg waveguide with an arbitrary periodic layered medium as the substrate.1,4 However, it is found that in order to get the best Bragg confinement, the periodic layered medium must consist of alternating quarter-wave layers.5 This result is obtained by minimizing the outward flowing power in the guide. The same procedure will be applied in this paper to the cylindrical fiber.

Propagation of electromagnetic waves in cylindrically symmetric dielectric waveguides has become increasingly important in fiber optics communication. The guiding principle is similar to the planar slab waveguide. A dielectric fiber is capable of supporting confined modes provided the refractive index of the guiding region (core) is greater than that of the surroundings (cladding). This ensures the evanescent decay of optical waves as r goes to infinity. Instead of dealing with sine and cosine functions, one deals with Bessel functions of both kinds in the cylindrical regime. A great deal has been done on optical propagation in conventional fibers.6-8

In this paper we will show that, in principle, confined modes exist in a fiber with a low-index core, provided the core is surrounded by a suitably designed alternating cladding of high and low refractive indices (see Fig. 1). A fiber with this kind of cladding is called a Bragg fiber9 and is similar in principle to the Bragg planar waveguide, where the light is guided by a low-index slab. To treat this problem properly we introduce an optimization similar to the one discussed in Ref. 5. Instead of solving for the confined modes of a given fiber structure, we search for the fiber structure such that the modes have some defined properties. The guiding of electromagnetic waves in a fiber with a low-index core, especially the hollow waveguide,10 is not only important in optical communication, but also very useful in the guiding of uv or of soft x rays in a hollow Bragg cylindrical waveguide where almost any material is too lossy in that frequency regime. Another important application in fiber optics communication is the design of a

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single-mode fiber. We have shown that a single-mode Bragg waveguide can be designed with a guiding layer thickness much larger than a wavelength.

A similar conclusion is derived for cylindrical Bragg waveguides. A single-mode fiber is capable of transmitting a light pulse without broadening due to modal dispersion. Pulse broadening is a serious problem in digital fiber optics communication; where it places a limit on pulse repetition rate.

II. MATRICE METHOD IN CONCENTRIC STRATIFIED FIBER

In this section we will introduce a matrix method to compute the mode characteristics as well as the power flux of radially stratified fibers. The basic idea is to replace the boundary conditions by a matrix equation. Thus, each cladding interface is represented by a matrix. The introduction of this 4x4 matrix greatly simplifies the analysis.

We consider a fiber with the index profile given by

$$n(r) = \begin{cases} n_g, & 0 < r < r_1 \\ n_a, & r_1 < r < r_2 \\ \vdots & \vdots \\ n_v, & r_v < r < r_{v+1} \\ \end{cases} \quad (1)$$

In particular, we will consider a fiber with a low-index core and alternating low- and high-index cladding. The geometry of this structure is sketched in Fig. 1. The index profile is then given by

$$n(r) = \begin{cases} n_0, & 0 < r < r_1 \\ n_1, & r_1 < r < r_2 \\ n_2, & r_2 < r < r_3 \\ \vdots & \vdots \\ n_v, & r_v < r < r_{v+1} \\ \end{cases} \quad (2)$$

We take the z axis as the direction of propagation, so that every field component has the form

$$\psi(r, \theta, z, t) = \psi(r, \theta) e^{i(\omega z - \alpha t)}, \quad (3)$$

where $\psi$ can be $E_z$, $E_r$, $E_\theta$, $H_z$, $H_r$, $H_\theta$. $\omega$ is the angular frequency and $\alpha$ is the propagation constant.

From waveguide theory we know that the transverse field components can be expressed in terms of $E_z$ and $H_z$.12

$$E_r = \frac{i \beta}{\omega \mu_e - \beta^2} \left( \frac{\partial}{\partial r} E_z + \frac{\omega \mu}{\beta} \frac{\partial}{\partial r} H_z \right); \quad (4)$$

$$E_\theta = \frac{i \beta}{\omega \mu_e - \beta^2} \left( \frac{\partial}{\partial \theta} E_z + \frac{\omega \mu}{\beta} \frac{\partial}{\partial \theta} H_z \right); \quad (5)$$

$$H_r = \frac{i \beta}{\omega \mu_e - \beta^2} \left( \frac{\partial}{\partial r} H_z + \frac{\omega \epsilon}{\beta} \frac{\partial}{\partial r} E_z \right); \quad (6)$$

$$H_\theta = \frac{i \beta}{\omega \mu_e - \beta^2} \left( \frac{\partial}{\partial \theta} H_z + \frac{\omega \epsilon}{\beta} \frac{\partial}{\partial \theta} E_z \right). \quad (7)$$

$E_z(r, \theta)$ and $H_z(r, \theta)$ satisfy the wave equation

$$\left( \nabla^2 - \frac{(\omega \mu_e - \beta^2)}{\mu_e} \right) \begin{bmatrix} E_z \\ H_z \end{bmatrix} = 0. \quad (8)$$

where $\nabla^2 = \nabla^2 - \beta^2/\alpha^2$ is the transverse Laplacian operator. The general solutions can be written

$$E_z = [A_1 J_1(\kappa r) + B_1 Y_1(\kappa r)] \cos(\theta + \phi_1), \quad (9)$$

$$H_z = [C_1 J_1(\kappa r) + D_1 Y_1(\kappa r)] \cos(\theta + \phi_1), \quad (10)$$

where $A, B, C, \phi$, and $\psi$ are constants, $l$ is an integer, and

$$k = (\omega \mu_e - \beta^2)^{1/2}. \quad (11)$$

We now consider the boundary conditions at a general cladding interface at $r = \rho$. The solution of the wave equation is taken as

$$E_z = [A_2 J_2(\kappa r) + B_2 Y_2(\kappa r)] \cos(\theta + \phi_2), \quad (12)$$

and

$$H_z = [C_2 J_2(\kappa r) + D_2 Y_2(\kappa r)] \cos(\theta + \phi_2), \quad (13)$$

where

$$\kappa_i = [(\omega/c^2 \epsilon_i \mu_i - \beta^2)^{1/2}, \quad i = 1, 2. \quad (14)$$

The boundary conditions at $r = \rho$ are that $E_z$, $H_z$, $E_\theta$, and $H_\theta$ are continuous at the interface. Thus a $4 \times 4$ matrix can be found which relates $A_1, B_1, C_1, D_1$ to $A_2, B_2, C_2, D_2$, i.e.,

$$\begin{pmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{pmatrix} = M \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix}. \quad (15)$$

Derivation of $M$

In terms of fields (12) and (13) the continuity of $E_z$ gives

$$[A_1 J_1(\kappa \rho) + B_1 Y_1(\kappa \rho)] \cos(\theta + \phi_1) = [A_2 J_1(\kappa \rho) + B_2 Y_1(\kappa \rho)] \cos(\theta + \phi_2). \quad (16)$$

FIG. 1. Bragg fiber.
This equation has to be satisfied for all \( \theta \) which implies
\[
\phi_1 = \phi_2
\]  
(17)
and similarly from the continuity of \( H_z \)
\[
\psi_1 = \psi_2.
\]  
(18)
Thus continuity of \( E_z \) and \( H_z \) gives
\[
A_1J_1(k_1\rho) + B_1Y_1(k_1\rho) = A_2J_1(k_2\rho) + B_2Y_1(k_2\rho),
\]  
(19)
\[
C_1J_1(k_1\rho) + D_1Y_1(k_1\rho) = C_2J_1(k_2\rho) + D_2Y_1(k_2\rho).
\]  
(20)
In terms of the fields (12), (13), and (5), the continuity of \( E_\theta \) gives
\[
\frac{1}{k_1^2} \left( \frac{-l}{\rho} \right) [A_1J_1(k_1\rho) + B_1Y_1(k_1\rho)] \sin(\theta + \phi)
\]
\[
- \frac{\omega_{11}}{\beta_1} k_1 [C_1J_1(k_1\rho) + D_1Y_1(k_1\rho)] \cos(\theta + \phi)
\]
\[
= \frac{1}{k_2^2} \left( \frac{-l}{\rho} \right) [A_2J_2(k_2\rho) + B_2Y_1(k_2\rho)] \sin(\theta + \phi)
\]
\[
- \frac{\omega_{21}}{\beta_2} k_2 [C_2J_1(k_2\rho) + D_2Y_1(k_2\rho)] \cos(\theta + \phi),
\]  
(21)
where the primed quantities are the derivatives with respect to their own argument. Again, this equation has to be satisfied for all \( \theta \). From (19) and (20)
\[
(1/k_1^2)[A_1J_1(k_1\rho) + B_1Y_1(k_1\rho)]
\]
\[
\neq (1/k_2^2)[A_2J_1(k_2\rho) + B_2Y_1(k_2\rho)],
\]  
(22)
\[
(\omega_1/k_1)[C_1J_1(k_1\rho) + D_1Y_1(k_1\rho)]
\]
\[
\neq (\omega_2/k_2)[C_2J_1(k_2\rho) + D_2Y_1(k_2\rho)],
\]  
(23)
provided \( k_1 \neq k_2 \). Thus we conclude from (21)–(23) that
\[
\sin(\theta + \phi) = \pm \cos(\theta + \phi)
\]  
(24)
or equivalently,
\[
\phi = \psi \pm \pi/2.
\]  
(25)
Continuity of \( H_\theta \) and Eq. (7) gives
\[
\frac{1}{k_1^2} \left( \frac{-l}{\rho} \right) [C_1J_1(k_1\rho) + D_1Y_1(k_1\rho)] \sin(\theta + \phi)
\]
\[
+ \frac{\omega_{11}}{\beta_1} k_1 [A_1J_1(k_1\rho) + B_1Y_1(k_1\rho)] \cos(\theta + \phi)
\]
\[
= \frac{1}{k_2^2} \left( \frac{-l}{\rho} \right) [C_2J_1(k_2\rho) + D_2Y_1(k_2\rho)] \sin(\theta + \phi)
\]
\[
+ \frac{\omega_{21}}{\beta_2} k_2 [A_2J_1(k_2\rho) + B_2Y_1(k_2\rho)] \cos(\theta + \phi).
\]  
(26)
From (24) or (25) we classify the waves into two categories:
I
\[
E_z = [A_1J_1(k\rho) + BY_1(k\rho)] \cos \theta,
\]
\[
H_z = [C_1J_1(k\rho) + DY_1(k\rho)] \sin \theta;
\]  
(27)
II
\[
E_z = [A_1J_1(k\rho) + BY_1(k\rho)] \sin \theta,
\]
\[
H_z = [C_1J_1(k\rho) + DY_1(k\rho)] \cos \theta.
\]  
(28)
The boundary conditions for these two categories are summarized below: For category I,
\[
A_1J_1(k_1\rho) + B_1Y_1(k_1\rho) + 0 = 0 \quad (1 \to 2),
\]  
(29)
\[
\frac{\omega_{11}}{k_1^2} A_1J_1(k_1\rho) + \frac{\omega_{11}}{k_1^2} B_1Y_1(k_1\rho)
\]
\[
+ \frac{l}{k_1^2} C_1J_1(k_1\rho) + \frac{l}{k_1^2} D_1Y_1(k_1\rho) = 0 \quad (1 \to 2),
\]  
(30)
\[
0 + 0 + C_1J_1(k_1\rho) + D_1Y_1(k_1\rho) = 0 \quad (1 \to 2)
\]  
(31)
\[
\frac{l}{k_1^2} A_1J_1(k_1\rho) + \frac{l}{k_1^2} B_1Y_1(k_1\rho)
\]
\[
+ \frac{\omega_{11}}{k_1^2} C_1J_1(k_1\rho) + \frac{\omega_{11}}{k_1^2} D_1Y_1(k_1\rho) = 0 \quad (1 \to 2),
\]  
(32)
where (1 \to 2) means the same functional form with subscript 1 replaced by 2, and vice versa. For category II there are similar equations except that the coefficient \( l/k_1^2 \) is replaced by \( -l/k_1^2 \). Equations (29)–(32) can be written as a matrix equation
\[
M(1,\rho) \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = M(2,\rho) \begin{pmatrix} A_1 \\ B_2 \\ C_2 \\ D_2 \end{pmatrix}
\]  
(33)
with
\[
M(i,\rho) = \begin{pmatrix} J_1(k_i\rho) & Y_1(k_i\rho) \\ \frac{\omega_{11}}{k_i^2} J_1(k_i\rho) & \frac{\omega_{11}}{k_i^2} Y_1(k_i\rho) \\ \frac{l}{k_i^2} J_1(k_i\rho) & \frac{l}{k_i^2} Y_1(k_i\rho) \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{\omega_{11}}{k_i^2} \end{pmatrix}
\]  
(34)
where \( i = 1,2 \). We notice that when \( l = 0 \), the matrix is reducible. In other words, we can have pure TE or pure TM waves when \( l = 0 \).

The matrix in Eq. (15) can be written, using (33), as
\[
M = M^{-1}(2,\rho) M(1,\rho).
\]  
(35)
Using (34) and after some matrix manipulation, the matrix elements \( m_{ij} \) in (36) are obtained as

\[
\begin{align*}
m_{11} &= j_1(y)j_1(y) - (k_{21}/k_{12})j_1(x)j_1(y), \\
m_{12} &= j_1(y)j_1(y) - (k_{21}/k_{12})j_1(x)j_1(y), \\
m_{13} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{14} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{21} &= (k_{21}/k_{12})j_1(x)j_1(y) - j_1(x)j_1(y), \\
m_{22} &= (k_{21}/k_{12})j_1(x)j_1(y) - j_1(x)j_1(y), \\
m_{23} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{24} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{31} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{32} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{33} &= j_1(x)j_1(y) - (k_{21}/k_{12})j_1(x)j_1(y), \\
m_{34} &= j_1(x)j_1(y) - (k_{21}/k_{12})j_1(x)j_1(y), \\
m_{41} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{42} &= (\beta_1/\omega_2)(1/\gamma - 1/\chi)j_1(x)j_1(y), \\
m_{43} &= (k_{21}/k_{12})j_1(x)j_1(y) - j_1(x)j_1(y), \\
m_{44} &= (k_{21}/k_{12})j_1(x)j_1(y) - j_1(x)j_1(y).
\end{align*}
\]

Again we find that the transfer matrix \( M \) is block diagonalized when \( l = 0 \). In this case the matrix equation (15) can be written as two separate equations

\[
\begin{align*}
\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} &= M_{TM} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}, \\
\begin{pmatrix} C_2 \\ D_2 \end{pmatrix} &= M_{TE} \begin{pmatrix} C_1 \\ D_1 \end{pmatrix}.
\end{align*}
\]

The matrix method described above can be employed to obtain the mode dispersion relations for any conventional fibers. We will, however, use this technique to design a Bragg fiber in the next section.

III. BRAGG FIBERS

An optimization procedure similar to that of Ref. 5 will be introduced to design an optimum Bragg fiber. This procedure is to minimize the outflowing flux by properly choosing the thicknesses of the claddings. Without loss of generality we will consider TE modes (\( I = 0 \) and \( E_z = 0 \)) only. The only nonvanishing components of the field for TE waves are

\[
\begin{align*}
H_z &= [C J_0(kr) + D Y_0(kr)] e^{i(\delta z - \omega t)}, \\
E_\theta &= -i\omega / k^2 \partial / \partial r H_z, \\
H_r &= i\beta / k^2 \partial / \partial r H_z,
\end{align*}
\]

where \( C \) and \( D \) are constants. \( C \) and \( D \) are real in each layer because the field is normalized such that \( H_z = e^{i(\delta z - \omega t)} \) at \( r = 0 \). The radial component of the time averaged Poynting flux is given by

\[
S_r = (\frac{1}{2}) \Re [E_\theta H^*],
\]

\[
= (\frac{1}{2}) \Re \left[ -i \omega / \mu \left( C J_0(kr) + D Y_0(kr) \right) \times \left[ C* J_0(kr) + D* Y_0(kr) \right] \right]
\]

\[
= 0 \quad \text{for all } r.
\]

This implies

\[
\text{outflowing flux} = \text{inflowing flux}.
\]

It can be shown that

\[
\text{outflowing flux} = \text{inflowing flux} \propto (C^2 + D^2) \omega / k^2.
\]

Equation (44) is exactly true mathematically for the electromagnetic modes of our fiber structure either with a finite or infinite number of layers. This, however, does not mean that the propagation is lossless in a Bragg fiber of finite number of annular claddings. Mathematically, \( S_r = 0 \) means that these modes have standing-wave pattern in the radial direction. These standing-wave radiation modes can only be excited by an infinitely extended source.18 It is thus obvious that in an actual experimental excitation of Bragg fiber modes of a finite Bragg fiber structure there is a net outward power flow because \( (S_r)_{\text{outward}} \) is not reflected and constitutes a loss. Therefore it is legitimate to calculate the loss based on the outflowing flux \( (S_r)_{\text{outward}} \).

Optimization of outflowing flux

Referring to Fig. 2, we consider the fields on both sides of a general cladding interface. The \( z \) component of magnetic field is taken as 

\[
H_z = \begin{cases} H_2(k_{1r}), & r < \rho \\ (C J_0(k_{2p}) + D Y_0(k_{2p})), & r > \rho 
\end{cases}
\]

where \( H_z(r) \) satisfies the wave equation

\[
\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) - k_0^2 \right] H_z(k_{1r}) = 0
\]

and is determined by the boundary condition at \( r = 0 \). In terms of fields (40), (41), (42), and (47), the continuity of \( E \) and \( H \) gives

\[
C J_0(k_{2p}) + D Y_0(k_{2p}) = H_z(k_{1p}) = H(k_{1p}),
\]

\[
(\mu_2/k_0^2) [C k_2 J_0'(k_{2p}) + D k_2 Y_0'(k_{2p})] = (\mu_1/k_0^2) k_1 H_z(k_{1p}) \equiv (\mu_1/k_0^2) k_1 H'(k_{1p}),
\]

where primed quantities are the derivatives with respect to their own argument. For the sake of simplicity in algebraic manipulation, we will drop the functional arguments and the \( z \) subscript. Thus we have

\[
C J_0 + D Y_0 = H,
\]

\[
C J_0' + D Y_0' = (k_{2p1} / k_{1p1}) H'.
\]

Solving for \( C \) and \( D \) from (51) and (52) we obtain

\[
C = (\pi k_{2p}/2) [H_y(k_{1p}) - (k_{2p1} / k_{1p1}) P_{1r} H'],
\]

\[
D = (\pi k_{2p}/2) [(k_{2p1} / k_{1p1}) J_0 H' - J_0 H],
\]

where we have used the Wronskian of Bessel functions

\[
J_0(x) Y_0(x) - J_0'(x) Y_0'(x) = 2 / \pi x.
\]
The magnitude of $C$ and $D$ depends on the interface radius $\rho$. The purpose of optimization procedure is to find a $\rho$ such that the outward flux (46) is a minimum.

$$\frac{\partial}{\partial \rho} (C^2 + D^2) = 0.$$  

By carrying out the partial differentiation with respect to $\rho$, we obtain

$$\frac{\partial}{\partial \rho} (C^2 + D^2) = \frac{1}{2} \left( \frac{\pi k_2 \rho}{2} \right)^2 \left[ k_2 \left( \frac{\mu_1}{\mu_2} - 1 \right) H^2 \right]$$

$$- k_1 \frac{k_2 \mu_1}{k_1 \mu_2} \left( 1 - \frac{k_2 \mu_1}{k_1 \mu_2} \right) H^2 \left( J_0 J'_0 + Y_0 Y'_0 \right)$$

$$+ H \left[ k_1 \left( 1 - \frac{k_2 \mu_1}{k_1 \mu_2} \right) \left( J_0^2 + Y_0^2 \right) \right]$$

$$- k_2 \left( \frac{\mu_1}{\mu_2} - 1 \right) \frac{k_2 \mu_1}{k_1 \mu_2} \left( J_0^2 + Y_0^2 \right).$$  

(57)

A similar expression can be obtained for TM waves by replacing $H$ by $E$ and $\mu_i$ by $\epsilon_i$, $i = 1, 2$.

Equation (57) is an exact general expression. If the fiber materials are pure dielectric, (57) can be simplified by using $\mu_2 = \mu_1$,

$$\frac{\partial}{\partial \rho} [C^2 + D^2] = \frac{1}{2} \left( \frac{\pi k_2 \rho}{2} \right)^2 \left( 1 - \frac{k_2^2}{k_1^2} \right)$$

$$\times H\left[ k_1 H(J'_0^2 + Y'_0^2) - k_2 H'(J_0^2 + Y_0^2) \right].$$  

(58)

A further simplification can be made if we note that, for $x \gg 1$,

$$\frac{J_0(x)J'_0(x) + Y_0(x)Y'_0(x)}{J_0^2(x) + Y_0^2(x)} \sim O(1/x).$$  

(59)

In practical application $x = k_2 \rho$ is a large number (i.e., $k_2 \rho \gg 1$) even in the first cladding interface. Therefore, the last term in (58) can be neglected. Thus we obtain

$$\frac{\partial}{\partial \rho} [C^2 + D^2] = \frac{1}{2} \left( \frac{\pi k_2 \rho}{2} \right)^2 \left( 1 - \frac{k_2^2}{k_1^2} \right)$$

$$\times k_1 [J_0^2(k_2 \rho) + Y_0^2(k_2 \rho)] H_0(k_1 \rho) H'_0(k_1 \rho).$$  

(60)

Equation (60) is similar to the corresponding equation for planar geometry. The same conclusions are obtained that minimization occurs at $E_0 = H_0' = 0$ if $k_0 > k_1$, and $E_0 = H_0 = 0$ if $k_2 < k_1$. In a transition through the interface the field amplitude decreases by a factor of $k_1/k_2$ if $k_2 > k_1$ and $E_0 = 0$. The field amplitude, however, does not increase for $k_2 < k_1$ provided the interface is located at the local maximum of $E_0$, i.e., $E_0' = 0$. Thus by employing alternating cladings with different refractive indices, the field amplitude will decrease by a factor of $k_1/k_2$ per cladding pair, provided the interfaces are located according to the minimization procedure. As a result the field amplitude decreases exponentially as $r$ increases. Theoretically, a confined mode is obtained if the alternating cladding is infinite. However, it is important to note that the field decay is nearly complete in several pairs of cladings so that practical structures with, say, ten pairs of cladings, are a good approximation to the infinite alternating cladings.

The calculated field distribution for a typical Bragg fiber mode is shown in Fig. 3. We notice that each pair of cladings is exactly of “half-wave” thickness. The half-wave thickness of a given layer means that the transverse Bessel function field solution undergoes a full variation between two adjacent zeros in that thickness. Since the zeros of Bessel function are not equally spaced, the cladding thicknesses are not identical. However, they tend to become identical as $r$ becomes large. This is due to the asymptotic form of the Bessel functions which approaches that of sine and cosine functions. The index profile shown in Fig. 3 looks almost periodic in the cladding region.

The leakage due to finite number of cladings will be discussed in Sec. IV.
IV. MODE CHARACTERISTICS AND LEAK CONSIDERATION

A Bragg fiber is usually designed to minimize the leakage for some particular mode. Again, we have the problem of leakage due to finite number of claddings. As a result, some higher-order undesired modes may be supported by the same Bragg fiber with larger attenuation coefficients. However, it can be shown numerically that these undesired modes are very lossy. Therefore, the Bragg fiber can be employed as a mode filter to select some particular mode from an ensemble of modes.

To study the mode characteristics and leakage problem, we start from a Bragg fiber structure optimized for some particular ray angle $\theta$ [or equivalently $\delta = (\omega/c)n \cos \theta$]. Then we study the amplitude reduction factor for any other ray angle $\theta$. The amplitude reduction factor is defined as

$$\eta = \frac{(C^2 + D^2)_{r=\infty}}{(C^2 + D^2)_{r=0}}. \quad (61)$$

This quantity is proportional to the ratio of the radial outward power flow in the last cladding region to the power flow in the core and is thus a direct measure of the attenuation coefficient $\alpha$ of the mode. (We can show that $\alpha \approx \eta$.) Given a Bragg fiber structure, $\eta$ is a function of ray angle $\theta$ and number of pairs of claddings,

$$\eta = \eta(\theta, N). \quad (62)$$

Figure 4 shows the curves of $\eta$ vs $\theta$ for various $N$'s. The structure is optimized for the fundamental mode. The minima in the curves are virtual modes of the structure. We see that the fundamental mode has the best degree of confinement. All the higher-order modes are less confined and hence more lossy. This is the basic property of a mode filter.

In conclusion, by extending the theory of Bragg reflection to cylindrical geometry we find that it is possible, in principle, to fabricate optical fibers with a multiannulus cladding whose indices may be higher than that of a core. Such fibers are strongly mode selective and can operate single mode even with large core diameter. A procedure for designing the optimum Bragg fibers is given and some of their properties discussed.