CONSTITUTIONAL SECESSION CLAUSES

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Abstract

Taking the view that constitutions are devices whereby people coordinate to specific equilibria in circumstances that allow multiple equilibria, we show that a constitutional secession clause can serve as such a device and, therefore, that such a clause is more than an empty promise or an ineffectual threat. Employing a simple three-person recursive game, we establish that under certain conditions, this game possesses two equilibria – one in which a disadvantaged federal unit secedes and is not punished by the other units in the federation, and a second equilibrium in which this unit does not secede but is punished if it chooses to do so.
1 Introduction

Of all the provisions that might be part of a federal state's constitution, perhaps none is more controversial than those that implicitly or explicitly deal with secession. The conventional wisdom is that allowing secession weakens a state. As Sunstein (1991:634) argues, a constitutional right to secede "would increase the risks of ethnic and factional struggle; reduce the prospects for compromise and deliberation in government; raise dramatically the stakes of day-to-day political decisions; introduce irrelevant and illegitimate considerations into these decisions; create dangers of blackmail, strategic behavior, and exploitation; and, most generally, endanger the prospects for long-terms self-governance."

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We should not be surprised, then, to see arguments defending nullification or veto clauses (Calhoun 1853, Buchanan and Tullock 1963) or, as with Yeltsin’s proposed constitution for Russia, over-representation of specific federal units in the legislature (Izvestia, April 30, 1993), as better ways to accommodate the demands of recalcitrant political sub-units that might not otherwise agree to form or join a federal state. Just as the adoption of an implementable provision allowing secession is credited with hastening the demise of the Soviet Union (by way of encouraging the Baltic Republics to resist Moscow), a liberal constitutional secession clause would seem to promote a union destined for disintegration.

Defending such suppositions, though, takes us to the core of a theory of constitutions. Just as we might ask how a constitution or any of its provisions is enforced, we can ask: Why would any prohibition or admission of secession influence behavior? If a federal unit chooses whether or not to secede on the basis of self-interest, how can such a clause influence that interest? If that choice is influenced by the responses of others in the federalism who also act out of self-interest, then why would a secession clause influence their self-interest or perceptions of it? Another way to formulate these questions is to consider Sunstein’s (1991) argument that a constitutional provision prohibiting secession is best interpreted as an agreement whereby federal units pre-commit to strategies that preclude secession and that require punishing those who defect. The unanswered question, though, is: How are such agreements maintained; how are pre-commitments enforced; why would a “parchment barrier” influence anything?

The game theoretic answer to these questions appeals to the idea of an equilibrium.
Agreements are maintained if and only if no one has a unilateral incentive to defect from the individual strategies that describe them. Complications can be added by considering other notions of equilibria, such as allowing coordinated defections or allowing players to renegotiate agreements as a game unfolds. But the essential idea is that constitutional provisions are self-enforcing if and only if abiding by them, including punishing those who defect, is in the self-interest of each participant when everyone else does the same.

However, even if agreeing not to secede or agreeing to punish those who do is sustainable as an equilibrium, we cannot say that setting this agreement to paper influences anything. If an agreement embodied in a constitution corresponds to a situation’s unique equilibrium, then we can employ any number of game-theoretic arguments that rationalize the concept of an equilibrium as a prediction about individual action to hypothesize that that outcome would be realized regardless of the words a constitution contains. In this event, we would be unable to reject the hypothesis that a federalism survives or fails merely as a product of a self-interest that is independent of constitutional provisions or guarantees. Thus, to establish the potential influence of constitutional provisions we must also show that the absence of any agreement can yield a different outcome. And to do this we must show two things. First, we must show that in the event of a constitutional prohibition of secession that provides for the punishment of defecting subunits, the subunits of a federation would, in fact, punish one of their number were it to try to secede and that the threatened punishment is sufficient to keep subunits from seceding. Second,
we must show that, barring such a prohibition – in effect, granting a constitutional right to secede – a different outcome is possible; namely, one in which states secede without incurring any punishment.

Thus, to establish that constitutional prohibitions of secession or commitments to allow it can influence choices and outcomes, we must establish that there are at least two alternative equilibria and that constitutional language can direct us to one of these equilibria or to the other. To the extent that the constitutional bargain coordinates society to different equilibria of procedure and process (Lewis 1969, Hardin 1989, Ordeshook 1992) we can then argue that a constitution not only has the capacity to influence action, but that it can do so in a wholly endogenous, self-enforcing way.

We argue here, in fact, that secession clauses can influence behavior. We do this by offering a simple three-person recursive game that allows federal units to secede and that confronts others with the choice of punishing or not punishing a seceding unit. We then establish that this game has two equilibria. In the first – which corresponds to a constitutional right to secede – a federal unit secedes only if it fails to receive a share of the benefits from federation that equals or exceeds what it can secure acting alone and the other units in the federation do not punish it for doing so in this circumstance. In the second equilibrium – which corresponds to a constitutional prohibition of secession – a subunit does not secede even though it may receive less than this share because it is punished if it defects. And since these equilibria coexist for a range of parameter values in our model, we infer that a constitution can coordinate the players to either equilibrium.
2 A Simple Model of Secession

Choosing a model that captures the processes surrounding secession is difficult since the analysis of federalism encompasses nearly all aspects of political-institutional design. Our purposes are served, however, by considering a simple possibility that, instead of trying to capture all potentially relevant aspects of the problem, focuses on what we believe are the core dimensions of secession. Specifically, we want a model that admits of these considerations:

1. A federalism that is “profitable” in the aggregate, so that subunits can earn more acting in concert than each can earn acting separately.

2. Benefits from federation that are not allocated among the subunits in accordance necessarily with their individual contributions – in proportion to what each subunit might earn if it were to become a sovereign entity. Indeed, some subunits may earn less than what they can earn as a sovereign entity, and they may thereby regard themselves as disadvantaged by confederation.

3. The opportunity to punish subunits that try to secede by forcing them to remain in federation, but with a reduced share of the benefits from confederation.

4. The absence of states in the federation that are sufficiently powerful to sustain the federation unilaterally.
5. Punishments that are costly to those who administer them.

6. A continual and ongoing threat of secession. There is never the "permanent" elimination of the possibility of secession owing to the creation of some new technology of confederation or binding commitment.

This last consideration – the continual threat of secession – warrants additional comment. Briefly, the things with which constitutions deal are not single events that, once resolved, can be ignored thereafter. Constitutions treat problems and processes that persist over time and that cannot be resolved with a single choice – the maintenance of a separation of powers, of national defense, of a common domestic market, of civil liberties, and so on. Because a subunit can postpone secession and because an unsuccessful attempt at secession need not preclude a second attempt, any model of a constitution’s role must be dynamic – it must view the situation as part of some ongoing process. A model that merely gives one subunit a one-time choice of seceding and not seceding and subunits a one-time choice of punishing and not punishing cannot be adequate for our purposes.

Thus, with these six things in mind, suppose the federation consists of three subunits (our analysis can be generalized to larger federations in obvious ways), denoted by $I = \{1, 2, 3\}$. Next, suppose each subunit holds an initial resource endowment, $\pi_i$, which measures what subunit $i \in I$ can secure in the event of the dissolution of the federation. However, because of economies of scale or other advantages of being in a larger unit,
suppose the total payoff for all units if the federation is maintained is $\pi = K \sum_{i=1}^{3} \pi_i$, where, in accordance with consideration 1, $K > 1$.

Next, suppose subunit 1 can decide whether or not to secede at any stage of the federation’s existence, so that its choice set is $S_1 = \{0, 1\}$, where 0 corresponds to “not secede” and 1 corresponds to “secede”. If 1 chooses to secede, subunit 2 and 3 must then choose between punishing and not punishing 1, so their choice sets are $S_j = \{0, 1\}$, for $j = 2, 3$, where 0 corresponds to “not punish” and 1 corresponds to “punish”.\(^1\) Finally, in accordance with consideration 4, we assume that punishment maintains the federation only if both subunits punish an attempted secession. A unilateral decision to punish cannot thwart the seceding unit’s intent, so if either subunit 2 or 3 fails to punish, the federation is dissolved. On the other hand, if 2 and 3 both choose to punish, the federation is preserved, at least temporarily.

Figure 1 shows the game tree that describes these choices,\(^2\). But to this figure we have added dashed lines that indicate the way in which our model accommodates consideration 6. Specifically, if subunit 1 chooses not to secede, the game repeats itself so that 1

\(^1\)We appreciate, of course, that a wholly general model allows any subunit to secede, but because our model is sufficient to illustrate the role of a constitutional secession clause, we prefer not to allow the attendant mathematical complexity to obscure this initial exploration of the potential influence of such a clause.

\(^2\)We use $p_i$ in Figure 1 to denote the probability that subunit i chooses to secede or punish. But this notation is an analytic convenience since we only consider pure strategies. That is, in looking for equilibria, we only allow $p_i = 0$ or 1.
confronts the same choice of seceding versus not seceding in the next period. That is, choosing not to secede does not preclude the possibility that 1 will choose to secede at some later date. Similarly, if 1 chooses to secede and if subunits 2 and 3 choose to punish, the game again repeats. After incurring its punishment – a one-time reduction in payoff – subunit 1 is confronted again with the choice between seceding and not seceding. Only if 1 chooses to secede and 2 or 3 fail to punish is there no need to consider the issue of secession – the federation in this instance is dissolved.

Insofar as the payoffs entered in Figure 1 are concerned,

1. if subunit 1 does not secede, subunit $i$ gets $a_i \pi$ for that period and the game proceeds to the next period. Thus, $\alpha_i$ denotes subunit $i$’s share of the total value of the federation;

2. if subunit 1 chooses secession and if 2 and 3 punish, the payoffs for that period are $\beta_1 \pi$, $\beta_2 \pi - C_2$ and $\beta_3 \pi - C_3$ and the game repeats. Thus, in accordance with consideration 5, the punishing players incur a cost of punishment ($C_j > 0, j = 2, 3$) whereas the punishment itself is a one-period reallocation of the pie such that $\sum_{i=1}^{3} \beta_i = 1$, and $\beta_1 < \alpha_1$;

3. a unilateral decision to punish also yields some small cost, $\epsilon$, which we can suppose derives, for example, from the political costs of mobilizing public opinion to an ineffectual end;
4. finally, if subunits 2 or 3 choose not to punish, the federation dissolves, and each subunit gets its per-period endowment, \( \pi_i \), forever.

Our game, then, is recursive and its solution requires that we specify *continuation values* consistent with the choices of the different subunits. For example, if 1 chooses not to secede, then it gets \( \alpha_1 \pi \) in that period plus the value of playing the game further, say \( \pi \), discounted by one period. If it chooses to secede, but its actions are blocked by 2 and 3, then it gets \( \beta_1 \pi \) for that one period plus the discounted value of \( \pi \). And if it chooses to secede but 2 or 3 fail to punish, then it gets \( \pi_1 \) forever. Thus, we must solve for \( \pi \), as well as for \( \pi_2 \) and \( \pi_3 \), such that the choices implied by these values and by the other specified payoffs yield decisions that are consistent with these values – that is, these values must be self-fulfilling prophesies.

Because our game allows for infinite repetitions, it, like the repeated prisoners' dilemma, allows for an infinite variety of strategies. For example, subunit 1 could try to secede at every turn until it is punished, say, \( X \) times, at which point it abandons the idea of secession. Similarly, subunits 2 and 3 could select strategies that punish the first \( Y \) attempts at secession, and then allow it thereafter. However, since we are interested in establishing the possibility of multiple equilibria, we consider only the simplest possibility, namely stationary strategies. Briefly, a stationary strategy is one that requires a player to make history-independent choices. Thus, with a stationary strategy, a player makes the same
choice at any two equivalent decision nodes in the game’s extensive form. This restriction means, then, that the equilibria our analysis uncovers are, in all likelihood, only a small subset of the possible equilibria in our game. But this restriction also means that if a constitutional secession clause can influence outcomes in our analysis, then there is an even greater variety of possible influences than our analysis suggests directly.

With $v_i$ representing the value of the game for subunit $i$, and using the usual definitions of an equilibrium, the strategy 3-tuple $(p_1^*, p_2^*, p_3^*)$ is a stationary Nash equilibrium if and only if $v_i(p_1^*|p_i^*, \ldots, p_i^*) \geq v_i(1 - p_i^*|p_i^*, \ldots, p_i^*)$, and $\pi_i = v_i(p_i^*|p_i^*, \ldots, p_i^*)$, where $p_i^* \in \{0, 1\}$ and $i = 1, 2, 3$.

Next, let $\delta$ be the discount factor for all players (we avoid excessive subscripts by assuming that all subunits have the same discount factor). Then Proposition 1, which employs the following shorthand,

$$\Pi_j^b = \beta_j \pi - \frac{\pi_j}{1 - \delta} + \frac{\delta}{1 - \delta} \alpha_j \pi,$$

where $j = 2, 3$, establishes that our game has three stationary Nash equilibria.

**Proposition 1** For the infinitely repeated secession game, the following are stationary Nash equilibria and their conditions:

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3See Myerson (1991) for precise definitions and Baron and Ferejohn (1989) and Niou and Ordeshook (1991) for an application of the ideas of continuation values and stationary strategies in a political context.
Stationary Nash Equilibria, Their Conditions and Aggregate Values

<table>
<thead>
<tr>
<th>Equilibria</th>
<th>Subunit 1</th>
<th>Subunit 2</th>
<th>Subunit 3</th>
<th>Aggregate Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0, 0)</td>
<td>(\alpha_1 \geq \frac{\pi_1}{\pi})</td>
<td>-</td>
<td>-</td>
<td>(\sum_{i=1}^{3} \bar{v}<em>i = \frac{K \sum</em>{i=1}^{3} \pi_i}{1-\delta})</td>
</tr>
<tr>
<td>(1, 0, 0)</td>
<td>(\alpha_1 &lt; \frac{\pi_1}{\pi})</td>
<td>-</td>
<td>-</td>
<td>(\sum_{i=1}^{3} \bar{v}<em>i = \frac{\sum</em>{i=1}^{3} \pi_i}{1-\delta})</td>
</tr>
<tr>
<td>(0, 1, 1)</td>
<td>(\alpha_1 \geq \beta_1)</td>
<td>(C_2 &lt; \Pi_2^{b})</td>
<td>(C_3 &lt; \Pi_3^{b})</td>
<td>(\sum_{i=1}^{3} \bar{v}<em>i = \frac{K \sum</em>{i=1}^{3} \pi_i}{1-\delta})</td>
</tr>
</tbody>
</table>

Notice now that the two equilibria \((0, 0, 0)\) and \((1, 0, 0)\) form a pair in which, regardless of 1’s actions, subunits 2 and 3 do not punish, and subunit 1’s decision depends solely on whether the benefits its derives from confederation are at least as great as what it can secure acting alone. Thus, one of these two equilibria prevails whenever subunits agree to allow secession in the event that a subunit finds it in its self-interest to secede. Since the conditions under which either \((0, 0, 0)\) or \((1, 0, 0)\) is an equilibrium span the full range of parameter values (aside from the restriction on \(\alpha_i\), there are no other restrictions on parameters other than that costs are indeed costs), the existence of any additional equilibria point to the need for coordination in order to ensure the realization of any equilibrium. And, in fact, Proposition 1 establishes that there is such an equilibrium, \((0, 1, 1)\), in which subunit 1’s share, \(\alpha_1\), can be less than its proportionate share, \(\frac{\pi_1}{\pi}\), but in which 1 is deterred from seceding owing to the threat of punishment.

These conclusions can be summarized formally by two corollaries that follow by simple algebra from the conditions set forth in Table 1 in the proof of Proposition 1 (see the Appendix). Briefly, this table establishes that each subunit has two thresholds in making a decision. Letting \(M_1 = \max \{\beta_1, \frac{\pi_1}{\pi}\}\) and \(m_1 = \min \{\beta_1, \frac{\pi_1}{\pi}\}\), whereas for \(j = 2\) and 3
letting \( M_j = \max\{\Pi_j^a, \Pi_j^b\} \), \( m_j = \min\{\Pi_j^a, \Pi_j^b\} \), then,

**Corollary 1** When \( \alpha_1 \geq M_1 \), \( p_1 = 0 \) is a dominant strategy for 1. When \( \alpha_1 < m_1 \), \( p_1 = 1 \) is a dominant strategy for 1. Similarly, for subunit 2 and 3, when \( C_j \geq M_j \), \( p_j = 0 \) is dominant strategy; when \( C_j < m_j \), \( p_j = 1 \) is dominant strategy, \( j = 2, 3 \).

Thus, if the conditions set forth in Corollary 1 are satisfied, a Constitutional specification of secession rights cannot influence choices and outcomes. However, Corollary 2 identifies the conditions under which no dominant strategy exists so that \((0, 1, 1)\) and \((1, 0, 0)\) are stationary equilibria simultaneously.

**Corollary 2** When \( \alpha_1 \in (\beta_1, \frac{\alpha_1}{\pi}) \), \( C_2 \in (0, \Pi_2^a) \), and \( C_3 \in (0, \Pi_3^b) \), both \((0, 1, 1)\) and \((1, 0, 0)\) are equilibria to the secession game.

### 3 Conclusions

Corollary 2 establishes that there is at least one non-trivial circumstance under which a Constitutional secession provision can influence eventual outcomes – when subunits, in Sunstein’s (1991) terms, can believably pre-commit to allow or prohibit secession and when a subunit’s decision whether or not to secede depends on prior coordinating agreements. Specifically, if \( \alpha_1 < \frac{\pi_1}{\pi} \) – if subunit 1 gets less than its security value from
acting alone – but not if $\alpha_1 < \beta_1$ – if 1’s share does not become so low that it would actually gain from the punishment. Thus, there is a range of values of $\alpha_1$ in which a subunit is disadvantaged and is thereby likely to demand a provision that allows for secession whereas, because they are advantaged in such a circumstance, the remaining subunits prefer a clause that prohibits secession.

Naturally, there are several extensions to our model that must be considered before we can use it to utter definitive conclusions about the influence of constitutional secession clauses. Although our analysis establishes the need for coordination, it cannot explain why a federation would form in the first place in the specific circumstance in which a constitution’s coordination function is required – that circumstance being where one subunit is permanently disadvantaged. Thus, after expanding the analysis to $n > 3$ countries, we should allow any and all countries to secede from the federation at every stage of the game. Second, we should also consider non-stationary strategies that allow subunits to implement more sophisticated patterns of choices, since doing so may expand the range of parameter values for which constitutional secession clauses can influence outcomes. Third, we should allow some stochastic indeterminacy in the determination of payoffs in each period of play. We can then combine this extension with the second to consider constitutional provisions that allow for conditional secession. Taking Buchanan’s (1991) suggestion that constitutional secession clauses need not fit some unitary mode, we can explore the influence of clauses that allow secession, for example, if a subunit’s rewards from confederation fall below some level for a pre-specified period of time. Finally,
we should add an analysis of bargaining so that the allocation of federation resources, 
\((\alpha_1, \alpha_2, ..., \alpha_n)\), becomes an endogenous vector that can be reconfigured in each time pe-
riod. This last extension would allow us to ascertain whether the form of a constitution's 
secession clause can influence eventual payoffs and in this way we can begin to understand 
the role of constitutions generally as determinants of political outcomes.

It is generally true, of course, that describing desirable extensions of a model is easier 
than actually implementing them. Of course, our analysis allows some reasonable guesses 
about things since the influence of parameters here conforms to intuition and since we 
can see no reason why that intuition would be contradicted by a more general analysis. 
For example, regardless of a model's ultimate form, no state should prefer a constitution 
that prohibits of secession if it believes that it would be permanently disadvantaged in 
the federation. From 1's point of view the equilibrium \((0, 1, 1)\) merely opens the door to 
exploitation whereas \((-,-,0)\) ensures against this possibility. Unfortunately, our present 
model does not allow us to answer other questions about a secession clause's ultimate 
impact on outcomes – most notably those that concern the distribution of benefits. In a 
model with renegotiated terms of confederation (renegotiated values of \(\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)\) 
and stochastic shocks to \(\alpha\), does allowing secession force states to negotiate more equitable 
values of \(\alpha\) or does the mere act of prohibiting secession force states, in equilibrium, to 
pre-commit to strategies whereby only wholly equitable distributions prevail. That is, we 
cannot yet preclude the seemingly perverse possibility that one way to achieve equitable 
outcomes is to preclude states from seceding when in-equitable outcomes arise.
In addition, predicting the types of strategy n-tuples that might exist in equilibrium in a more complicated model is difficult owing to the fact that in such a model, the assumption of stationarity is less palatable. In a simple model such as the one we offer here, this assumption almost certainly fails to exclude any interesting possibilities. But in a model in which subunits are allowed to renegotiate distributions of resources or in which nature can intervene with random shocks, stationarity precludes demands for inter-temporal compensation or strategies that postpone secession until it is revealed that nature is biased against one subunit or another.

Nevertheless, such extensions, although desirable from the point of view of understanding the ultimate implications of a constitutional secession clause, are unlikely to undermine the central conclusion of this essay. Specifically, that conclusion is: the ongoing processes of federalism occasion more than one equilibrium outcome, and at least two such outcomes correspond to a pre-commitment, embodied in the provisions of a constitution, allowing or prohibiting secession.
Appendix. Proof of Proposition 1

Proof: The values of the game for the three subunits are as follows,

\[
\begin{align*}
\overline{v}_1 &= \max\{v_1(S), v_1(\hat{S})\} \\
&= \max\{p_2p_3(\beta_1\pi + \delta v_1) + (1 - p_2p_3)\frac{\pi_1}{1-\delta}, \alpha_1\pi + \delta v_1\}, \\
\overline{v}_j &= \max\{v_j(P), v_j(\hat{P})\} \\
&= \max\{p_1p_k(\beta_j\pi - C_j + \delta v_j) + p_1(1 - p_k)(\frac{\pi_j}{1-\delta} - \epsilon) + (1 - p_1)(\alpha_j\pi + \delta v_j), \\
&\quad p_1\frac{\pi_j}{1-\delta} + (1 - p_1)(\alpha_j\pi + \delta v_j)\},
\end{align*}
\]

where \(j, k = 2, 3\), and \(j \neq k\). Applying the definition of stationary Nash equilibrium,

\[
\begin{align*}
p_1 = 1 \iff \\ v_1(S) > v_1(\hat{S}) \\
&\quad \overline{v}_1 = v_1(S),
\end{align*}
\]

\[
\begin{align*}
p_1 = 0 \iff \\ v_1(S) \leq v_1(\hat{S}) \\
&\quad \overline{v}_1 = v_1(\hat{S}),
\end{align*}
\]

\[
\begin{align*}
p_j = 1 \iff \\ v_j(P) > v_j(\hat{P}) \\
&\quad \overline{v}_j = v_j(P),
\end{align*}
\]

\[
\begin{align*}
p_j = 0 \iff \\ v_j(P) \leq v_j(\hat{P}) \\
&\quad \overline{v}_j = v_j(\hat{P}),
\end{align*}
\]
where \( j = 2, 3 \). Simplifying these conditions,

\[
\begin{align*}
    p_1 &= 1 \iff p_2 p_3 \beta_1 \pi + (1 - p_2 p_3) \frac{\pi_1}{1 - \delta} > \frac{1 - p_2 p_3 \delta}{1 - \delta} \alpha_1 \pi, \\
    p_1 &= 0 \iff p_2 p_3 \beta_1 \pi + (1 - p_2 p_3) \frac{\pi_1}{1 - \delta} \leq \frac{1 - p_2 p_3 \delta}{1 - \delta} \alpha_1 \pi, \\
    p_j &= 1 \iff p_k \beta_j \pi + p_k \delta (1 - p_1) \alpha_j \pi \frac{1}{1 - \delta + \delta p_1} - \frac{p_k \pi_j}{1 - \delta + \delta p_1} - (1 - p_k) \epsilon > p_k C_j, \\
    p_j &= 0 \iff p_k \beta_j \pi + p_k \delta (1 - p_1) \alpha_j \pi \frac{1}{1 - \delta + \delta p_1} - \frac{p_k \pi_j}{1 - \delta + \delta p_1} - (1 - p_k) \epsilon \leq p_k C_j,
\end{align*}
\]

where \( j, k = 2, 3 \) and \( j \neq k \). Note that since we do not consider mixed strategies, we assume that \( i \in I \) chooses \( p_i = 0 \) when it is indifferent between \( p_i = 0 \) and \( p_i = 1 \).

Substituting \( p_i = 0 \) or \( 1 \) in (1), (2), (3) or (4), there are eight possible cases, described in Table 1 along with the conditions required for them to be stationary Nash equilibria.

We employ the following shorthand in Table 1:

\[ \Pi_j^a = \beta_j \pi - \pi_j, \text{ and } \Pi_j^b = \beta_j \pi - \frac{\pi_j}{1 - \delta} + \frac{\delta}{1 - \delta} \alpha_j \pi, \text{ where } j = 2, 3. \]

Table 1. Eight Possible Cases and Conditions Required to Be Equilibria
<table>
<thead>
<tr>
<th>Possible Cases</th>
<th>Subunit 1</th>
<th>Subunit 2</th>
<th>Subunit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0, 0)</td>
<td>$\alpha_1 \geq \frac{\pi_1}{\pi}$</td>
<td>$\epsilon \geq 0$</td>
<td>$\epsilon \geq 0$</td>
</tr>
<tr>
<td>(0, 1, 0)</td>
<td>$\alpha_1 \geq \frac{\pi_1}{\pi}$</td>
<td>$\epsilon &lt; 0$</td>
<td>$C_3 \geq \Pi_3^b$</td>
</tr>
<tr>
<td>(0, 0, 1)</td>
<td>$\alpha_1 \geq \frac{\pi_1}{\pi}$</td>
<td>$C_2 \geq \Pi_2^b$</td>
<td>$\epsilon &lt; 0$</td>
</tr>
<tr>
<td>(0, 1, 1)</td>
<td>$\alpha_1 \geq \beta_1$</td>
<td>$C_2 &lt; \Pi_2^b$</td>
<td>$C_3 &lt; \Pi_3^b$</td>
</tr>
<tr>
<td>(1, 0, 0)</td>
<td>$\alpha_1 &lt; \frac{\pi_1}{\pi}$</td>
<td>$\epsilon \geq 0$</td>
<td>$\epsilon \geq 0$</td>
</tr>
<tr>
<td>(1, 1, 0)</td>
<td>$\alpha_1 &lt; \frac{\pi_1}{\pi}$</td>
<td>$\epsilon &lt; 0$</td>
<td>$C_3 \geq \Pi_3^b$</td>
</tr>
<tr>
<td>(1, 0, 1)</td>
<td>$\alpha_1 &lt; \frac{\pi_1}{\pi}$</td>
<td>$C_2 \geq \Pi_2^a$</td>
<td>$\epsilon &lt; 0$</td>
</tr>
<tr>
<td>(1, 1, 1)</td>
<td>$\alpha_1 &lt; \beta_1$</td>
<td>$C_2 &lt; \Pi_2^a$</td>
<td>$C_3 &lt; \Pi_3^a$</td>
</tr>
</tbody>
</table>

From this table we can see that (0, 1, 0), (0, 0, 1), (1, 1, 0), (1, 0, 1) and (1, 1, 1) cannot be stationary equilibria since otherwise we must contradict the assumption that $\epsilon > 0$, and $\alpha_1 \geq \beta_1$. Therefore, the only equilibria are (0, 0, 0), (0, 1, 1) and (1, 1, 1).

Q.E.D.
References


