A Micro-Econometric Analysis of Risk-Aversion and the Decision to Self-Insure

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Abstract

This study estimates a von Neumann-Morgenstern utility function using market data and micro-econometric methods. We investigate the decision whether to purchase insurance against the risk of telephone line trouble in the home. Using the choices of approximately 10,000 residential customers, we determine the shape of the utility function and the degree of risk-aversion. We find that risk-aversion varies systematically in the population and varies with the level of income and that the observed choice behavior is consistent with expected utility maximization. We are unable to detect the presence of ambiguity effects or over-weighting of low-probability events.
A Micro-Econometric Analysis of Risk-Aversion and the Decision to Self-Insure*

Charles J. Cicchetti† Jeffrey A. Dubin‡

1 Introduction

Whether expected utility theory is consistent with individual behavior is a question that has received considerable attention by economists, marketing scientists, and psychologists. The growing body of evidence, derived principally from laboratory experiments (see e.g. Mosteller and Nogee (1951), Coombs and Komorita (1958), Coombs and Huang (1976), and Grether and Plott (1979)), suggests that expected utility theory is frequently violated. Limitations of expected utility theory have led to the development of many alternative theories such as prospect theory (Kahneman and Tversky (1979)), and many others (see e.g. Camerer (1991) for a comprehensive survey). While tests of consistency of the various theories with observed behavior have begun (see e.g. Camerer (1989), Camerer and Kunreuther (1989), and Currim and Sarin (1989)), the conclusion of much of this analysis is that “no theory can explain all of the data, but prospect theory and the hypothesis that indifference curves fan out can explain most of them” Camerer (1989).

There are at least two reasons why the demise of expected utility theory may be premature. First, it has been observed that individual experiments and market experiments often produce dissimilar results. While laboratory experiments have shown that individuals may poorly estimate probabilities and violate the basic axioms of probability theory, experimental results in market settings have been more encouraging (Camerer (1987)). Second, experimental results are frequently categorized simply by whether or not they

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obey the theoretical restrictions of a given model.¹ The advantage of this method of hypothesis testing is that it does not require or impose specific forms of the utility or value function for the individual. But by avoiding the direct estimation of the utility function, researchers have ignored the potential for individual response error and for randomness or heterogeneity in preferences.

The purpose of the present analysis is to address both of the above concerns by estimating a von Neumann-Morgenstern utility function using market data and microeconometric methods. The empirical analysis we conduct is based on the decision whether to purchase insurance or to self-insure against the risk of telephone line trouble in the home. Using the choices of approximately 10,000 residential customers we determine the shape of the utility function and the degree of risk-preference. Our model for the choice of firm-insurance versus self-insurance is based on expected utility theory and random utility maximization. We allow for both state-dependent and status-quo effects in the estimation and test for local departures from the expected utility theory by allowing “ambiguity” in the underlying uncertainty.²

Laskey and Fischer (1987) have outlined four approaches to dealing with the problem of estimating utility functions in the presence of response error: they are (1) ignore the problem; (2) average multiple judgments; (3) employ consistency checks to eliminate problematic observations; and (4) fit preference models to the decision makers’ responses. The latter approach is in fact the least common method for assessing utility functions or for testing decision theories. Indeed the direct calibration of von Neumann-Morgenstern utility functions by statistical techniques has received very little attention in the literature.³

In the marketing literature, Hauser (1978) and Hauser and Urban (1977, 1979) have discussed the measurement of multi-attribute utility functions from preference data while Eliashberg and Hauser (1985) and Laskey and Fischer (1987) have discussed the modelling of risk-preferences in the presence of measurement error. Eliashberg and Hauser suggested a maximum likelihood procedure for estimating probabilistic choice wherein the probability that an alternative is selected arises from a random risk-parameter (which may be known to the individual but unknown to the analyst). Laskey and Fischer adopt a different perspective and introduce an additive disturbance in individuals’ responses which affects the accuracy of the response rather than an individual’s true preference.

³Camerer (1991) cites only two recent examples of studies which directly estimate utility functions. But early references to the statistical calibration of utility functions include Meyer and Pratt (1968) who provide a method for “fairing” deterministically a smooth function through observed responses, Fishburn and Kochenberger (1979) who use a minimum mean-squared method, and Currim and Sarin (1984) who rely on conjoint analyses. Tversky (1967) and Fischer (1976) also use forms of curve fitting to assess the utility function.
Empirical assessment of risk preference models with measurement error appear in Currim and Sarin (1989) and Daniels and Keller (1990). These studies fit either single risk parameters for each experimental subject or smooth approximations to the risk parameter as a function of experimental conditions.

Measurement of von Neumann-Morgenstern utility functions has also appeared in the finance and economics literatures. Friend and Blume (1975), in their classic study of risk preference, showed that in market equilibrium the ratio of the expected premium on risky financial assets to the variance of those assets is related to a function of individuals' risk-aversion and the value of all risky results. Their study found evidence of a constant relative risk-aversion level for a representative consumer of approximately two.

There have also been attempts in the economics literature to estimate life-cycle consumption and labor supply models. The studies by Altonji (1986), MacCurdy (1981, 1983), Browning, Deaton, and Irish (1985), Altug and Miller (1990), and Keane, Moffitt, and Runkle (1988) link individual level data to aggregate financial data. While these studies are able to demonstrate that utility is concave and increasing in leisure, their principal focus concerns intertemporal labor supply and the separability of consumption and leisure rather than the determination of the degree of risk-aversion or the shape of the underlying utility function.

Recently Viscusi and Evans (1990) and Evans and Viscusi (1991) have considered the estimation of state dependent utility functions from survey data. The data they employ are the percentage wage increase required to compensate a worker for a hypothetical change in the risk of a specific job occupation. Viscusi and Evans use the survey responses of 249 individuals to estimate the shape of a representative individual's utility function. In one approach, they use Taylor's approximations to the utility function and in another approach, they estimate a single parameter logarithmic utility function where the unknown parameter characterizes the health state.

Our empirical analysis is most similar to that of Viscusi and Evans but relies on mar-

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4Cox, Smith, and Walker (1988), like Currim and Sarin (1989), estimate individual-specific risk parameters using a log-concave constant relative risk-aversion model. Their data come from an experimental analysis of first-price auctions. They find strong support for the hypothesis that individuals are risk-averse in bidding and that the degree of risk-aversion varies from individual to individual.

5Estimation of risk parameters within asset-pricing models has been reported by Hall (1988), Breeden (1979), Breeden, Gibbons, Litzenberger (1989), Grossman, Melino, and Shiller (1987), Hansen and Singleton (1982, 1983), Mehra and Prescott (1988), and Eichenbaum, Hansen, and Singleton (1988). Related studies which also use a representative consumer and aggregate financial data, but relax the expected utility hypothesis, include Epstein and Zin (1987) and Weil (1989). While all of these studies attempt to determine the degree of relative risk-aversion, there can be some lack of identification between the degree of risk-aversion and the degree of intertemporal substitution (Hall (1988)).

6Edwards (1988) estimates a discrete-choice model for households' willingness-to-pay to prevent contamination of a potable supply of ground water. His approach is utility theoretic but relies on contingent, rather than actual values.
ket rather than contingent valuation data. First, we use first and second-order Taylor’s series expansions of the difference in utility states to examine the local properties of the underlying utility function; i.e., whether insurance is an inferior good and whether consumers, in fact, reveal risk-aversion. We then use a full information maximum likelihood procedure to estimate a fully parameterized structural model. Our results indicate that: (1) risk-aversion varies systematically in the population and varies with the level of income; (2) observed choice behavior is consistent with expected utility maximization; and (3) ambiguity effects and over-weighting of low-probability events are not present in this context.

In the next section we discuss the basic inside-wire maintenance (IWM) contract which allows a consumer to insure against potential telephone line trouble. In Section III we examine several theories behind the purchase of IWM contracts including service aspects, warranty aspects, and priority service aspects. In Section IV we develop a theory of inside-wire maintenance choice based on expected utility theory and discrete choice econometrics. Section V describes the data we use in the estimation while Section VI develops both the reduced-form and the structural-form estimates. We conclude in Section VII.

2 Inside Wire Maintenance Contracts

Inside-wire maintenance service contracts were created as a result of the deregulation of the telephone industry. Before 1982, when the telephone industry was forced to unbundle many of its traditional service arrangements such as the installation and repair of telephones and the inside wiring for telephones, all customers with basic telephone exchange service were charged a monthly fee which recovered installation and maintenance costs. The single monthly charge paid by customers did not separate charges paid for maintenance or for installation. After the 1982 Federal Communication Commission (FCC) divestiture order, regional phone companies were required to terminate many services such as phone repair and make optional some other services such as maintenance of inside home wiring. Phone customers could contract with the phone company to acquire IWM, they could hire a third party to do the work when it became necessary, or they could do it themselves.

In some parts of the country customers were “negatively enrolled” in IWM programs. Negative enrollment occurred when customers were notified (typically by a phone bill insert) that they would automatically be charged for basic IWM service unless they specifically notified the telephone company they did not want the maintenance contract. The negative enrollment aspect of inside-wire maintenance has brought challenges legally on the basis that many customers did not and would not have selected IWM contracts had
they been non-passively enrolled. These legal challenges have raised serious economic issues: which individuals would have chosen IWM for each method of enrollment, how much were customers willing to pay to avoid the risk of inside wire trouble, and were individuals rational in their assessments of the probability of service interruption?

It is important to consider that many customers were not negatively enrolled in the post-1982 period. Customers who initiated new phone service (perhaps by moving into or within the service region, or by adding new service) had the opportunity to choose from a variety of service features including IWM. We use a sample of individuals whose choices regarding IWM were non-passive to reveal the preferences of the population at large.

As we discuss in detail below, our data comes from the Mountain States Telephone and Telegraph Company (U.S. West) service area. During the 1980’s Mountain Bell customers faced a monthly probability of line trouble of less than 0.5 percent. The mean time to failure for line trouble was thus about 17 years. Typical charges for IWM were approximately $0.45 per month. When line trouble would occur repair charges averaged about $55.00. Therefore the expected cost of line trouble was about $0.27 per month. Mountain Bell customers therefore paid, on average, an amount greater than was actuarially fair. Yet, the market penetration of IWM among actively enrolled customers was well above 50 percent. In the next section we consider several theories (including risk-aversion) which help explain the purchase behavior of IWM.

3 Theories of Purchase Behavior for IWM Contracts

The inside-wire maintenance contract between customers and the phone company has elements of several different commodities. We discuss three potentially important elements: (1) IWM as a service contract; (2) IWM as a warranty and as insurance; and (3) IWM as priority service.

3.1 Service Contracts

The literature on service contracts (see e.g., Day and Fox (1985) and Fox and Day (1988)) indicates that service contracts are typically expensive compared with the protection they provide; the expected cost of repair varies between 25 and 50 percent of the cost of the contract. Renewals for service contracts are typically low because of the low frequency of

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contact between the consumer and organization offering the contract. Day and Fox find that individuals with relatively little experience with a product class are more likely to buy a service contract and that “the only perceived convenience associated with service contracts was that a customer would not have to seek out a service/repair person.” But some features of IWM might make it more successful in gaining and keeping customers as compared with more typical service contracts. First, the IWM renewals are frequent (once per monthly bill) and the renewal cost seemingly is inexpensive. Second, the renewal process is convenient and requires no effort from the individual. Third, some customers are unlikely to have had much personal experience with inside-wire trouble and may therefore over-estimate the probability of trouble. Finally, an IWM contract eliminates the need for a costly search for a repair service if line trouble develops. These features of IWM make the firm-insured state more desirable than the self-insured state.\(^8\)

### 3.2 Warranties and Insurance

IWM contracts have some features in common with warranties, but are perhaps better regarded as service contracts. As Emons (1989) observes, a warranty imposes more obligations on the seller than a service contract does. Also the method of payment for a warranty is different than that for a service contract—the warranty price typically is a part of the purchase price (Gill and Roberts (1989), Schwartz and Wilde (1983)). Warranties are usually limited in duration while IWM contracts are in theory renewable even if a customer allows coverage to lapse. The warranty literature (see e.g. Priest (1981)) includes four basic theories:

1. **Signaling Theory.** Sellers cannot control the intensity and care of use of the products they sell. Buyers, on the other hand, are often unable to discern (or have imperfect information) on the quality of products prior to purchase. Warranties can ameliorate this form of double moral hazard by acting as a signal to consumers of the quality of the product (Wiener (1985), Lutz (1989), and Shimp and Bearden (1982));

2. **Investment Theory.** The warranty imposes obligations on manufacturers to make investments in design and quality control so as to reduce failure rates and the costs associated with failure and repair. Warranties also impose obligations on buyers who make investments in the care of the product;

\(^8\)In an empirical analysis of the demand for repair service during warranty, Gerner and Bryant (1980) found that as the value of the spouses time increased, households demanded greater warranty comprehensiveness. This suggests, if anything, that service and warranty contracts are normal goods. The results of the study by Gerner and Bryant are however far from definitive. The value of time for the head of household had an opposite effect from that of the spouse and the value of non-wage income had no statistically significant effect.
3. *Exploitation Theory.* The warranty service is a marketing device to extract consumer surplus by exploiting the risk-averse nature of consumers; and

4. *Insurance Theory.* Warranties represent a response to the problem of efficient risk-sharing when firms are risk-neutral and consumers are risk-averse.

In the case of inside wire, the moral-hazard problem should be minimal since failure of inside wire should not be correlated with the intensity of use of telephone equipment. Buyers may have imperfect information on the quality of the inside wiring, but the quality is discrete in nature—either the phone system communicates or it does not. On the other hand, exploitation theory may be relevant. In 1986 U.S. West marketed two new IWM programs, “linebacker and linebacker plus” which provided backup phone instruments in the case of equipment failure. These programs were quite inexpensive for the phone company to offer and probably served as marketing vehicles to stimulate overall demand for insurance contracts.

The most applicable idea from the warranty literature is that of insurance. IWM contracts are insurance policies which fully cover the cost of replacement or repair of the phone equipment if it fails. Customers can not adopt to partially insure against the hazard of line failure so that the amount of insurance purchased is not relevant. Instead consumers must make a discrete decision on whether to firm-insure or to self-insure. Nonetheless, risk-averse individuals will have reservation values greater than the expected value of the gamble which they are willing to pay to avoid the uncertainty. Models of optimal insurance have been developed in Smith (1966), Lee and Pinches (1988), and Drèze and Modigliani (1972). The theoretical model of Szpiro (1985) is most applicable to IWM because in his model only full coverage is possible.

Empirical evidence on insurance purchases is largely laboratory experimental. Camerer and Kunreuther (1989) in an analysis of insurance markets found little evidence supporting prospect theory in favor of expected utility theory. Hogarth and Kunreuther (1989) discuss how ambiguity regarding the event probability can increase the premium above the actuarially fair value that individuals are willing to pay for insurance. Hershey and Shoemaker (1980), Kahneman and Tversky (1979), and Smith and Desvousges

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9Ehrlich and Becker (1972) contrast market insurance (firm-insurance) with self-insurance and self-protection. Self-insurance involves actions taken by the individual to reduce the magnitude of loss while self-protection involves actions taken to reduce the probability of a loss. Our own use of the term self-insurance refers to the individual’s state when he or she chooses not to buy market insurance.

10Recent theoretical literature on the demand for insurance is concerned with whether insurance is a Giften good; this literature asserts that insurance purchases are inferior (Borch (1986), Hoy and Robson (1981), and Briys, Dionne, and Eeckhoudt (1989)). But these studies do not cite any evidence for this widely accepted view.

11Our situation is also similar to that of Drèze (1981) who shows that the degree of absolute risk-aversion can be inferred from the amount of insurance purchased, the distribution of losses, the probability of loss, and the level of wealth.
(1987) have observed in a series of experiments that individuals overweight low probability events suggesting that fair insurance becomes more attractive for low probability events. Hershey and Shoemaker also find a strong context effect in which choices involving insurance were judged with greater risk-aversion than mathematically identical choices presented as standard gambles. Shogren (1990), also using experimental data, found evidence to support Hershey and Shoemaker. He also found that individuals tend to overestimate the impact of low probabilities, but that the degree of overestimation decreased with repeated market exposure.\(^\text{12}\)

In a non-experimental market analysis of self-insurance and self-protection, Brookshire, Thayer, Tschirhart, and Schulze (1985) explain housing price differentials for geographically distinct areas in California with differing likelihoods of experiencing earthquakes.\(^\text{13}\) Their model assumes that risk-averse individuals can self-insure by choosing to live in safer areas. In equilibrium housing price differentials should reflect the probability of an earthquake, the likely magnitude of loss, the wealth levels for representative consumers, and the degree of risk-aversion. Using a first-order Taylor series expansion of the utility function and auxiliary estimates of the key variables, the authors are able to demonstrate consistency of their hedonic estimates and those implied by expected utility theory.

### 3.3 Priority Service

Inside-wire maintenance also shares some aspects of priority service. Individuals who require service reliability or guaranteed access to the phone system may be willing to pay more for IWM contracts. But priority service offers gradations of service reliability to customers while inside wire contracts offer no gradations at all, merely a method of dealing with service outages once they occur. The priority service literature has also attempted to quantify outage costs—usually for electrical service (Hartman, Doane, and Woo (1981)). With telephone service there too may be costs associated with an outage other than the costs of repairs. If there is a perception on the part of customers that having an IWM contract reduces outage costs (perhaps through faster response time or less search time spent by the customer) then there may be a preference for IWM contracts.\(^\text{14}\)

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\(^{12}\) Another finding due to Kahneman and Tversky is that “people tend to underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty.” The “certainty effect” also noted in Viscusi, Magat and Huber (1987) may have direct relevance to the choice of IWM contracts because consumers may be willing to pay a premium for the certain elimination of risks associated with IWM failure over and above the amount they would be willing to pay to reduce the risk of this failure to some non-zero level.

\(^{13}\) A similar analysis appears in MacDonald, Murdoch, and White (1987) who use residential property values to study consumer behavior with respect to the uncertainty of land flooding.

\(^{14}\) Hartman, Doane, and Woo also identify a strong preference for the status quo. In many cases customers must be compensated for switching reliability regimes even when the alternative entails more
Our theory of IWM contract choice most closely follows the discrete insurance model. To the extent that other motivations are present (status quo effects, ambiguity effects, certainty effects, priority effects, etc.) we generalize the expected utility model by allowing state dependence in the utility function, unobserved random preference, and ambiguous probabilities. These issues are taken up below when we operationalize a structural model for IWM purchase.

4 A Theory of Inside Wire Maintenance Choice

Customers who choose inside-wire maintenance replace the uncertain possibility of having to pay for inside-wire repair (both the direct charges for repair and the indirect costs of finding a repair service) with a guaranty of not having to bear the cost of repair at the expense of a fixed monthly fee. Risk-aversion and differing levels of income in the customer class lead some individuals to prefer firm-insurance to the alternative of self-insurance. In this section we develop a theory of IWM choice and consider how properties of the utility function can be inferred from a population of non-identical consumers who reveal their preferences by their choices.

We begin with a von Neumann-Morgenstern utility function \( U(W) \) which is increasing in wealth, \( U'(W) > 0 \). Let \( C \) denote the cost of repair for the uninsured individual, \( R \) denote the monthly fee for IWM, and \( p \) denote the exogenous probability of line trouble. The utility with insurance is \( U(W - R) \). The expected utility under self-insurance is \( pU(W - C) + (1 - p)U(W) \). For individuals with identical utility functions who face identical costs and probabilities, the decisions regarding insurance must be identical. Individuals will insure provided the utility with insurance exceeds the expected utility under self-insurance. In the population, individuals are heterogeneous and not all components of utility are observable. We account for these differences empirically through three sources: (1) differing levels of risk-aversion, (2) differing levels of income, and (3) alternative specific preferences. Specifically we specify a utility function \( U(W; s, i, \epsilon_i) \) where \( s \) represents characteristics of the decision maker, \( i \) represents the state in which utility occurs (insured or self-insured), and \( \epsilon_i \) represents a random component of indirect utility.

To generate a specific probabilistic choice system for observed choices, we follow McFadden (1983) and assume random utility maximization with additively separable random errors.15 We therefore write the indirect utility function as:

\begin{align*}
\text{reliable service than they already receive. Possible explanations for status quo effects are discussed in Samuelson and Zeckhauser (1988). Since all telephone customers were once in a regime which provided inside wire maintenance protection, there may have been lingering status quo effects which positively influenced the decision to contract for IWM service when the choice was offered.}
\end{align*}
\[
U(W; s, i, \epsilon_i) = \bar{U}(W; s, i) + \epsilon_i.
\] (1)

Under the hypothesis of random utility maximization, the individual chooses to insure provided:

\[
U(W - R; s, i, \epsilon_1) \geq pU(W - C; s, 2, \epsilon_2) + (1 - p)U(W; s, 2, \epsilon_2)
\] (2)

or

\[
\bar{U}(W - R; s, 1) + \epsilon_1 \geq [p\bar{U}(W - C; s, 2) + (1 - p)\bar{U}(W; s, 2)] + \epsilon_2.
\] (3)

If we further assume that the \(\epsilon_i\) are independent extreme-value distributed, then the probability associated with the event in equation (3) is given by

\[
q = \text{probability that individual buys insurance} = \text{Prob} [\epsilon_2 - \epsilon_1 \leq \Delta V] = 1/(1 + \epsilon^{-\Delta V})
\] (4)

where

\[
\Delta V = \bar{U}(W - R; s, 1) - [p\bar{U}(W - C; s, 2) + (1 - p)\bar{U}(W; s, 2)].
\] (5)

The probability of purchasing IWM depends on the indirect utility difference \(\Delta V\) which is a function of the individuals wealth level \(W\), the costs \(R\) and \(C\), the probability of having line trouble \(p\), and the degree of risk-aversion as embodied in the function \(\bar{U}\).

To illustrate the foregoing ideas consider the following example. Suppose strict utility is given by the quadratic function \(U(W) = bW - W^2\) where \(b\) is a parameter affecting the curvature of the utility function. The function \(U(W)\) exhibits positive marginal utility of income provided \(W < b/2\). The degree of absolute risk-aversion is positive in this range with \(R_a = 1/(b/2 - W)\) and increasing in \(W\). The latter property is often too restrictive in empirical applications. For this example, we will simply fix the income level and ask whether curvature in the utility function alone (a positive risk-aversion level) can be consistent with the observed distribution of preferences in the population. Since the choice probability \(q\) is constant in this case we can write the log-odds probability as:

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Elements in common with both Eliashberg and Hauser and Lasky and Fischer. Since our model assumes random utility maximization (McFadden (1983)), it shares the error structure of Lasky and Fischer. But we interpret the randomness as do Eliashberg and Hauser—as unobserved effects known to the individual but unknown to the econometrician—rather than as response errors.
\[ \Delta V = \log\left(\frac{q}{1-q}\right). \]  

We solve for \( b \) using:

\[
\Delta V = U(W - R) - [pU(W - C) + (1 - p)U(W)] \\
= [-W^2 + (b + 2R)W - bR - R^2] - [-W^2 + (b + 2C)W - pC(b + C)]
\]

which implies

\[
b = 2W - \left[\frac{R^2 - pC^2 + \Delta V}{R - pC}\right].
\]

Using the value of \( b \), we can calculate the degree of risk-aversion. We can also calculate the cost of risk from not being on the inside-wire maintenance program. This is given by the amount \( R^* \) which solves the equation

\[
U(W - R^*) = pU(W - C) + (1 - p)U(W);
\]

i.e., \( R^* \) is the certainty equivalent of the gamble.\(^{16}\)

This equality produces a quadratic equation in \( R^* \); the solution depends on the level of income \( W \), the log-odds of the market penetration for IWM, \( \Delta V \), and the constants \( p \), \( C \), and \( R \). Intuitively we determine the risk-premium by explaining the market preference for insurance in the face of the actuarial unfair return.

Risk-aversion need not be the only explanation for why individuals in aggregate prefer insurance to self-insurance. If, for example, the degree of absolute risk-aversion declines in income, then the amount an individual would be willing to pay to avoid a fixed size gamble declines as income increases i.e. if \( R'(W) \leq 0 \) then \( dR^*/dW \leq 0 \). A proof of this result is provided by Mossin (1968) or one can easily verify that convexity of the function \( f(p) = U(W - R^*(p)) - [pU(W - C) + (1 - p)U(W)] \) in the interval \( p \in [0, 1] \) is sufficient for \( dR^*/dW \leq 0 \).

Knowing whether \( R^* \) declines as income increases is not sufficient to determine whether \( \Delta V \) will decrease; although \( \Delta V = U(W - R) - U(W - R^*) \), the income level \( W \) is increasing at the same time as \( R^* \) is declining. Is there a set of conditions under which

\(^{16}\)Willingness to pay in this context also has an interpretation as an option price. See e.g. Schmalensee (1972) and Freeman (1984).
discrete insurance purchases are inferior? To answer this question we examine $\partial \Delta V / \partial W$. From equation (5),

$$
\frac{\partial \Delta V}{\partial W} = U'(W - R) - [pU'(W - C) + (1 - p)U'(W)]
$$

$$
= U'(W - R) - U'(W - R^*)[1 - dR^*/dW]
$$

$$
= [U'(W - R) - U'(W - R^*)] + U'(W - R^*)dR^*/dW. \quad (7)
$$

When the individual's maximum willingness to pay $R^*$ is greater than the market cost of discrete insurance $R, U'(W - R^*) > U'(W - R)$ as long as utility is strictly concave. Since the first term in equation (7) is negative, a sufficient condition for $\partial \Delta V / \partial W \leq 0$ is that $dR^*/dW \leq 0$. But this is only a sufficient condition and it is possible for $\partial \Delta V / \partial W \leq 0$ when $dR^*/dW$ close to zero. When the individual's maximum willingness to pay $R^*$ is less than the market cost $R$, the first term in equation (7) is positive and it is necessary that $dR^*/dW$ be negative (and sufficiently so) in order that $\partial \Delta V / \partial W < 0$.

The above analysis demonstrates that an empirical observation of discrete insurance purchases as inferior does not limit a priori the class of admissible utility functions. This observation notwithstanding, a family of utility functions which allows non-constant absolute risk-aversion may be a good candidate for structural estimation when it is reasonably assured that the individual's willingness to pay exceeds the market price.

5 Description of Data

Our analysis is based on a random sample of 25,099 observations of residential customers in the Mountain Bell Colorado service area. The sample was taken from customer records in July 1990. Mountain Bell maintains records of its customers for the purposes of billing and telemarketing. Billing records contain information on the presence of or lack of various service options and the size of the monthly bill. We use the billing records to define two variables. First, any household which subscribes to an IVM program and pays a monthly service charge for this option is assigned a value of one. All other households are assigned a value of zero. Second, we use the billing records to define a typical monthly bill (BILL) which is equal to the average amount paid by the household for service and tolls (this includes the amounts paid for special service features such as inside-wire maintenance, call waiting, and the like).

Mountain Bell maintains twenty-four variables for each customer matched either from company records or from census data. In the case of monthly income, a categorical variable matched at the block group/enumeration district level (a sub-categorization of
census tract) is the basis for our continuous variable (MINC). We also use information from company records to determine when a customer initiated telephone service. Customers with phone service prior to March 1982 were passively enrolled in basic inside-wire maintenance service, while subscribers who started phone service after March 1982 were actively enrolled. To only include households who actively decided about the IWM option, we restrict the sample to those households who began service after March 1982. This eliminates a total of 9,663 households from the analysis. We also define a dummy variable (POST86) to indicate whether a household initiated phone service after or prior to 1986. This variable is used to capture differences in the types of inside-wire maintenance service options available during the period 1982-1990. Prior to 1986 Mountain Bell offered only a basic inside-wire service contract. In 1986, Mountain Bell introduced a new type of IWM. To allow for the shift in demand which may have resulted from the introduction of these new forms of IWM, we use the treatment variable POST86.

We have further eliminated from the sample any individuals who are employees of the phone company and receive service at no charge (148 households), and have also eliminated households for which data is missing (4,644 households). After these deletions, our analysis sample consists of 10,644 households.

In addition to billing and telemarketing records, we have used company records to determine the frequency of line trouble for differing service zones in the metropolitan Boulder/Denver area. Mountain Bell maintains “trouble tickets” which are records of individual trouble calls serviced by the company. The trouble tickets contain the nature of the trouble and the phone number for the lines which were serviced. Some 350,000 trouble tickets for Colorado residential and business customers were used to estimate the probability of inside-wire trouble in various regions of Colorado.

We divided the metropolitan Boulder/Denver area into eight geographic regions by selecting geographically adjacent phone number prefixes according to the Boulder addition of the White and Yellow pages for 1990. In principal, trouble probabilities could be calculated for each individual prefix. In practice, however, too few trouble events or phone numbers were found for any given prefix to produce reliable estimates of the underlying trouble probabilities. Trouble probabilities (TPROB) were determined by finding the average number of monthly trouble occurrences by zone (in eight zones and a residual zone) for the years 1982-1986 and then dividing by the average number of customers in the corresponding region who were covered under the company’s IWM program (the latter data was also provided by Mountain Bell). Observations of trouble events were limited to the years 1982-1986 due to data availability, but should be representative for later years as well. Trouble probabilities were assigned to individual customer records based on the prefix of the individual’s phone number. A histogram for the trouble probability is shown in Figure 1. Variable definitions and sample statistics are summarized in Table 1.
As can be seen from Table 1, 57 percent of households subscribe to inside-wire maintenance. Nearly 70 percent of households acquired service after 1986 which reflects the high degree of turnover in the residential population. Households averaged about $20,000 in annual income and had typical monthly phone bills of about $25.00. Monthly trouble probabilities (summarized in Table 2) ranged from 0.00318 to 0.00742 (Zones 5 and 4 respectively). The differences in zonal trouble probabilities reflect differences in the vintage of the underlying housing stock and differences in the vintage of phone equipment.\textsuperscript{17}

6 Estimation

We now examine the consistency of the observed choices of inside-wire maintenance with expected utility theory. We also illustrate how reduced-form estimation of the utility function can be used to recover key structural parameters.

6.1 Reduced-Form Estimation

Estimation of the probabilistic choice model given by equation (4) requires either a direct specification of the function $\Delta V$ or an approximation of this function. This section takes the latter approach. Since,

$$\Delta V = f(W, p) = [U(W - R) - U(W)] - p[U(W - C) - U(W)]$$

a second-order Taylor series expansion of $\Delta V$ in wealth and the probability of trouble is given by

$$f(W, p) \approx f(W_0, p_0) + f_W(W - W_0) + f_p(p - p_0)$$

$$+ \frac{1}{2} f_{WW}(W - W_0)^2 + f_{Wp}(W - W_0)(p - p_0) + \frac{1}{2} f_{pp}(p - p_0)^2.$$ 

The partial derivatives in this case are:

$$f_W = [U'(W - R) - U'(W)] - p[U'(W - C) - U'(W)].$$

\textsuperscript{17}The mean trouble probability reported in Table 1 and the aggregate trouble probability reported in Table 2 differ slightly due to differences in the distribution of customers between the billing and trouble-ticket data files.
The reduced-form estimation method uses the linear and quadratic terms in $W$ and $p$ as explanatory variables in a binary logit model for choice of IWM service. A negative value for $f_w$ indicates that insurance is inferior and has implications for the properties of the underlying utility function as discussed above. A positive estimate of $f_p$ shows that the utility is increasing in wealth. A negative value of $f_{wp}$ reveals concavity in the utility function. Similarly the signs and magnitudes of the coefficients on the second-order terms provide additional information about the shape of the utility function.

In Table 3 we present the reduced-form logit models. In each model we include an alternative specific construct and a dummy variable for those households which acquired telephone service after 1986. These two terms are entered to allow for state dependence in the utility structure, i.e., they represent shifts in tastes and preferences which may be attributable to the insured versus uninsured states. In all cases the POST86 variable has a positive and significant coefficient which affirms the hypothesis that consumers found the IWM insurance plans collectively more valuable after 1986.

Models 1 and 2 are estimated as baseline cases and do not include the income and probability covariates. In Model 3 we include income, the trouble probability, and the size of the monthly telephone bill. Our hypothesis is that consumers with higher average bills require more reliable service and are willing to pay a premium to insure reliability. The coefficient of BILL is significant and positive in the various models. This confirms our hypothesis that BILL provides a measure of the importance of service reliability to consumers.

The coefficient of income in Model 3 shows that discrete insurance purchases are inferior. As income rises the probability that an individual will self-insure increases. The sign of the trouble probability variable in Model 3 is also consistent with the theory. An increase in the trouble probability variable increases the demand for IWM and reduces the probability of self-insurance. Model 4 is a simple reprise of Model 3 using only the first-order terms from the Taylor series expansion. Note that the centering of the income and probability terms around their mean values shifts the estimated intercept coefficient. Finally, Model 5 adds the second-order terms to Model 4. Here again the coefficients on the linear income and probability terms are consistent with the theory. The coefficient on the squared probability term is not significant (also consistent with the predictions of the theory). The cross-partial term also has the correct sign for a population of risk-averse individuals, but the estimated coefficient is not significant at conventional levels. Finally,
the squared income term is positive and significant which indicates that the propensity to self-insure is increasing at a decreasing rate.\footnote{Viscusi and Evans (1990) found their second-order Taylor series terms to be insignificant.}

In principal the estimated coefficients from the reduced-form estimation can be used to reconstruct the utility function. To illustrate the approach we use the estimates from Model 4 and the negative exponential utility function. Suppose then that \( U(W) = -ae^{-bW} \) with \( a, b > 0 \). Then \( U''(W) = abe^{-bW} > 0 \), and \( U''(W) = -ab^2e^{-bW} < 0 \). Thus absolute risk-aversion is constant with: \( R_a(W) = -U''/U' = b \). For the negative exponential utility function,

\[
\begin{align*}
    \psi &= \frac{b(e^R - 1) - bp}{(e^{bC} - 1) - bp} \\
    e^{bC}(\psi + bp) &= be^bR = \psi + bp - b.
\end{align*}
\]

From Model 4 in Table 3, \( f_p = 89.36 \) and \( f_w = -28,810.0 \) (after a change in scale to reflect the units of income). Assuming values of \( C = 55.00 \) for the fixed repair cost and \( R = 0.45/month \) for the cost of insurance, and sample averages \( \text{MINC} = 1699.40 \) and \( \text{TPROB} = 0.00477 \) implies a numerical solution of \( b = 0.01936 \). The willingness-to-pay \( R^* \) satisfies \( U(W - R^*) = pU(W - C) + (1 - p)U(W) \) and has a solution \( R^* = \frac{1}{b} \log[1 + p(e^{bC} - 1)] \). Since \( R^* \) exceeds the market cost \( R, \partial \Delta V/\partial W < 0 \). In this case, increases in income lower the probability that insurance is purchased even though absolute risk-aversion is constant.

Given the lack of precision attached to some of the estimates in Model 5, a similar mapping of reduced-form to structural-form models does not seem warranted.
6.2 Structural-Form Estimation

We now consider the structural estimation of a utility function consistent with the revealed preferences of individuals who made choices regarding IWM service. The class of utility functions we adopt allows risk-aversion to vary among individuals with the same level of income and to vary with the level of income for otherwise identical individuals. We specify utility to be a member of hyperbolic absolute risk-aversion (HARA) class with: \(^{19}\)

\[
U(W) = a_1 \cdot (W + a_2)^L. \tag{8}
\]

This utility class contains several well known utility functions as special cases including linear, quadratic, negative exponential, power, and log. The degree of absolute risk-aversion for the utility function in equation (8) is

\[
R_o(W) = \frac{1 - L}{W + a_2}
\]

which is declining in both \(W\) and \(L\). Monotonicity and concavity require that \(0 < L < 1\).

The arguments to the binary logit function for the choice probability are \(\Delta V\) and the state dependent variables: Constant and POST86. To allow the level of risk-aversion to vary across individuals we take the parameter \(L\) to be a linear function in the monthly bill: \(^{20}\)

\[
L = b_1 + b_2 \cdot \text{BILL}. \tag{9}
\]

This choice is consonant with our findings in the reduced-form analysis where we found that customers with larger bills were more likely to purchase IWM contracts. We therefore expect the coefficient \(b_2\) to be negative which implies that as BILL increases the degree of risk-aversion increases. Since the willingness-to-pay will increase with the level of absolute risk-aversion, \(\Delta V = U(W - R) - U(W - R^*)\) will increase making it more likely that insurance will be purchased.

Since ambiguity or over-weighting effects may be present, we embed the observed probability within a transformation that allows individuals to consistently under or over-estimate the true trouble probability. We specify the log-odds ratio for the subjective

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\(^{19}\)See e.g. Merton (1971) for a discussion of the HARA class.

\(^{20}\)Viscusi and Evans explore heterogeneity in their estimation by taking key parameters to be linear functions of individual characteristics. None of the characteristic data was significant (at the five percent level) in their estimation.
probability of line trouble to be a linear function of the log-odds ratio of the true line trouble probability:

\[ \log \left( \frac{p}{1-p} \right) = c_1 + c_2 \cdot \log \left( \frac{TPROB}{1-TPROB} \right). \]  

(10)

This is equivalent to specifying a logit probability for \( p \) as a function of a constant term and the log-odds of TPROB. The specification in equation (8) allows the subjective probability to be constant \((c_2 = 0)\), equal to the actual probability \((c_1 = 0, c_2 = 1)\), or consistently biased with measurement of the bias reflected in the coefficient values for \( c_1 \) and \( c_2 \). The model we estimate by FIML is then

\[ Q_i = [\text{individual } i \text{ purchases IWM}] \]

\[ = \frac{1}{1 + e^{-(d_1+d_2 \cdot \text{POSTS} + \Delta V_i)}}. \]  

(11)

In all eight parameters are estimated: \( a_1 \) and \( a_2 \) which characterize the utility function in equation (8), \( b_1 \) and \( b_2 \) which characterize risk-aversion in equation (9), \( c_1 \) and \( c_2 \) which characterize subjective probability in equation (10), and \( d_1 \) and \( d_2 \) which characterize state dependent effects in equation (11).

The results of the estimation are presented in Table 4. The models we present vary depending on whether the subjective probability is constrained or not constrained in its relationship to the actual probability. The results are similar across the specifications. In Model 3, for example, the parameters of the utility function \( a_1 \) and \( a_2 \) are both significant. The sign of \( a_1 \) shows that the probability of purchasing insurance increases with \( \Delta V \) as expected. The significance of the term \( a_2 \) rejects the power utility function in favor of the more general HARA class.

The coefficients \( b_1 \) and \( b_2 \) for the risk parameter \( L(b_1, b_2) \) are each significant and \( b_2 \) has the hypothesized negative sign. Predicting the risk-aversion parameter \( L \) for each sample observation, we find that \( L \) is positive in about 78 percent of all households (consistent with expected utility theory), but negative in the remaining 12 percent (inconsistent with expected utility theory). The distribution of \( L \) is shown in Figure 2. \( L \) has an average value of 0.37 with a minimum of -8.86, a maximum of 2.87, and a standard deviation of 1.28. In about 41 percent of the cases \( L \) lies in the unit interval. In these cases individuals experience declining absolute risk-aversion. With only 12 percent of the cases having point-predictions which are non-positive, the overall pattern of results is remarkably consistent with expected utility.

\[ 21 \text{The parameter } a_0 \text{ is not determined independently of the scale of the logit model.} \]
The estimated values for the parameters $c_1$ and $c_2$ reveal that consumers use a subjective probability which is nearly identical to the actual probability: $c_1$ is not statistically different from zero while $c_2$ is significant but not significantly different from one. There is thus slight evidence that consumers overestimate the small trouble probability event, but not to any significant degree.

The estimates of the coefficients $d_1$ and $d_2$ are similar to those we obtained in the reduced-form estimation for the alternate specific constant and the POST86 dummy variable. The only difference is that the intercept is significant from zero in the structural estimates.

7 Conclusions

Day and Fox (1985) hypothesized that “relatively affluent persons, especially those who also are well educated, are less likely to perceive real value in appliance service contracts, because these persons generally tend to be less risk-averse, i.e., more likely to self-insure, than consumers on restricted budgets.” Our estimation has confirmed the presence of risk-aversion and has also revealed that IWM purchases are for the most part inferior.

The results we have obtained are also very encouraging for expected utility theory. The coefficients determined in the structural estimation were significant and in accord with our expectations. For most consumers increases in income lead to increases in the likelihood that self-insurance was selected. On the other hand, consumers who had higher phone usage were less likely to self-insure.

The structural models provided significant improvement in overall fit as compared with the reduced-form models, as evidenced by the significant improvement in the log-likelihoods at convergence (note that Model 5 of Table 3 and Model 3 of Table 4 have the same degrees of freedom). Moreover, the structural logit model correctly predicts the choices of about 65 percent of the cases which is significant since the frequency of observed selection was only 57 percent choosing IWM versus 43 percent not doing so.

The HARA utility class was also found to perform better than other common choices such as log and negative exponential in the sense that imposing these functional forms either lead to problems with non-convergence or models which converged to implausible values (such as everywhere non-monotonic utility). This may explain some of the difficulty encountered by Viscusi and Evans (1990). Using log-utility, Viscusi and Evans were able to solve for the exact risk premium in closed form. Their estimation was accomplished using non-linear least squares. For other forms of the utility function, Viscusi and Evans were not able to solve directly for the risk-premium and were therefore unable to estimate their model by non-linear least squares. Their attempts at full-information maximum likelihood methods were non-convergent.
Our estimation sample does not reflect the greater participation levels for IWM which prevailed in the 1980's. In order to gauge the willingness-to-pay by individuals to avoid telephone line trouble we have made an adjustment to the alternative specific constant in the structural logit model to reflect a 70 percent market penetration. In the 1980's, the median individual was willing to pay about $0.55 per month to avoid inside wire trouble.
# Table 1
## Sample Statistics and Variable Definitions

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IWM</td>
<td>Inside wire maintenance</td>
<td>0.571</td>
<td>0.0</td>
<td>1.0</td>
<td>0.495</td>
</tr>
<tr>
<td></td>
<td>service option</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINC</td>
<td>Monthly Income ($)</td>
<td>1699.4</td>
<td>312.5</td>
<td>6250.0</td>
<td>613.8</td>
</tr>
<tr>
<td>BILL</td>
<td>Monthly bill ($)</td>
<td>25.30</td>
<td>5.0</td>
<td>100.0</td>
<td>10.42</td>
</tr>
<tr>
<td>TPROB</td>
<td>Trouble Probability</td>
<td>0.00477</td>
<td>0.00318</td>
<td>0.00742</td>
<td>0.00093</td>
</tr>
<tr>
<td>POST86</td>
<td>Service Acquired After 1986</td>
<td>0.687</td>
<td>0.0</td>
<td>1.0</td>
<td>0.463</td>
</tr>
</tbody>
</table>

Number of Observations: 10,644
Table 2  
Monthly Trouble Probabilities, 1982-1986

<table>
<thead>
<tr>
<th>Region</th>
<th>Trouble Probability (TPROB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>0.004419</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.005049</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.004349</td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.007424</td>
</tr>
<tr>
<td>Zone 5</td>
<td>0.003183</td>
</tr>
<tr>
<td>Zone 6</td>
<td>0.004189</td>
</tr>
<tr>
<td>Zone 7</td>
<td>0.003938</td>
</tr>
<tr>
<td>Zone 8</td>
<td>0.005614</td>
</tr>
<tr>
<td>All Others</td>
<td>0.004756</td>
</tr>
<tr>
<td>Aggregate</td>
<td>0.004848</td>
</tr>
</tbody>
</table>
Table 3
Reduced-form Logit Models
Dependent Variable: IWM

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.286</td>
<td>-0.402</td>
<td>-0.939</td>
<td>-1.003</td>
<td>-0.996</td>
</tr>
<tr>
<td></td>
<td>(14.64)*</td>
<td>(-11.37)</td>
<td>(-5.83)</td>
<td>(-15.56)</td>
<td>(-14.67)</td>
</tr>
<tr>
<td>POST86</td>
<td>1.015</td>
<td>0.984</td>
<td>0.984</td>
<td>0.983</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(23.58)</td>
<td>(22.55)</td>
<td>(22.55)</td>
<td>(22.50)</td>
<td></td>
</tr>
<tr>
<td>BILL</td>
<td>0.0249</td>
<td>0.0249</td>
<td>0.0248</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.61)</td>
<td>(11.61)</td>
<td>(11.51)</td>
<td></td>
<td></td>
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<tr>
<td>MINC (10^3 $)</td>
<td>-0.288</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-8.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPROB</td>
<td>89.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.72)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINC–MINC</td>
<td>-0.288</td>
<td>-0.318</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-8.03)</td>
<td>(-7.87)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPROB–TPROB</td>
<td>89.36</td>
<td>146.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.72)</td>
<td>(3.23)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(MINC–MINC)^2</td>
<td>0.0718</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.84)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(TPROB–TPROB)^2</td>
<td>-34671</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.57)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(MINC–MINC) x</td>
<td>-3.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(TPROB–TPROB)</td>
<td>-0.07</td>
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</tr>
<tr>
<td>Log-Likelihood</td>
<td>-7269.8</td>
<td>-6984.1</td>
<td>-6852.8</td>
<td>-6852.8</td>
<td>-6847.1</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>10,644</td>
<td>10,644</td>
<td>10,644</td>
<td>10,644</td>
<td>10,644</td>
</tr>
</tbody>
</table>

*t-statistics in parentheses.
Table 4
Structural Logit Models
Dependent Variable: IWM

1 Purchase Insurance (57.1%) 0 Self-Insurance (42.9%)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1935.1</td>
<td>1935.1</td>
<td>1949.4</td>
</tr>
<tr>
<td></td>
<td>(18.23)*</td>
<td>(18.24)</td>
<td>(18.22)</td>
</tr>
<tr>
<td>a2</td>
<td>0.6849</td>
<td>0.6456</td>
<td>0.6475</td>
</tr>
<tr>
<td></td>
<td>(3.632)</td>
<td>(4.283)</td>
<td>(4.253)</td>
</tr>
<tr>
<td>b1</td>
<td>3.491</td>
<td>3.498</td>
<td>3.497</td>
</tr>
<tr>
<td></td>
<td>(21.33)</td>
<td>(20.98)</td>
<td>(14.75)</td>
</tr>
<tr>
<td>b2</td>
<td>-0.121</td>
<td>-0.123</td>
<td>-0.124</td>
</tr>
<tr>
<td></td>
<td>(-12.62)</td>
<td>(-13.63)</td>
<td>(-12.89)</td>
</tr>
<tr>
<td>c1</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.0021</td>
</tr>
<tr>
<td>Constrained</td>
<td></td>
<td>Constrained</td>
<td>(-0.0012)</td>
</tr>
<tr>
<td>c2</td>
<td>1.0</td>
<td>1.0093</td>
<td>1.0096</td>
</tr>
<tr>
<td>Constrained</td>
<td></td>
<td>(72.41)</td>
<td>(2.71)</td>
</tr>
<tr>
<td>d1</td>
<td>-0.054</td>
<td>-0.050</td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td>(-1.17)</td>
<td>(-1.06)</td>
<td>(-1.12)</td>
</tr>
<tr>
<td>d2</td>
<td>0.992</td>
<td>0.992</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td>(22.66)</td>
<td>(22.65)</td>
<td>(22.57)</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>-6791.1</td>
<td>-6790.9</td>
<td>-6790.9</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>10,644</td>
<td>10,644</td>
<td>10,644</td>
</tr>
</tbody>
</table>

*t-statistics in parentheses.
Figure 1: Distribution of Trouble Probabilities
Figure 2: Distribution of Risk Parameter
References


Ham, et al. v. Mountain States Telephone and Telegraph Co., aka U.S. West Communications, a Colorado corporation, dba U.S. West Communications; Plaintiffs’ Trial Data Certificate; Case No. 87-CV-231246, District Court, City and County of Denver, State of Colorado.


