SOPHISTICATED SINCERITY: VOTING OVER ENDOGENOUS AGENDAS*

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Abstract

The empirical findings on whether or not legislators vote strategically are mixed. This is at least partly due to the fact that to establish any hypothesis on strategic voting, legislators' preferences need to be known; and these are typically private data. In this note it is shown that, under complete information, if decision-making is by the amendment procedure and if the agenda is set endogenously, then sophisticated (strategic) voting over the resulting agenda is observationally equivalent to sincere voting. The voting strategies, however, are sophisticated. This fact has direct implications for empirical work on sophisticated voting.
Introduction

Over the past decade, considerable theoretical attention has been devoted to voting in committees. The focus has been on two related questions. First, given an arbitrary agenda, what is the structure and consequence of strategic (sophisticated) voting, and how does this differ from sincere (naive) voting? Second, given strategic voting and an arbitrary status quo, what final outcomes can a monopoly agenda-setter reach by an appropriate choice of agenda? For decision making under the amendment procedure\(^1\) with complete information, the answers to these questions are now known: see, in particular, Farquharson (1969), McKelvey and Niemi (1978), and Moulin (1979) on the first problem, and Miller (1980), Shepsle and Weingast (1984), and Banks (1985) on the second.

In the wake of the theoretical advances, empirical work on legislators' voting behavior has become increasingly directed toward trying to discover whether legislators do in fact vote strategically (Bjurulf and Niemi, 1978; Enelow and Koehler, 1980; Enelow, 1981; Denzau, Riker and Shepsle, 1985; Krehbiel and Rivers, 1985). For if they do, then any legislator's voting behavior cannot be used unequivocally as an instrument to reveal his or her (legislative) preferences. The findings are mixed: this is at least partly because, as Krehbiel and Rivers remark, "to assess the voting strategies of [legislators] one needs to know [their] preferences" (1985, p.3), and preferences are private information. Bjurulf and Niemi, Enelow and Koehler, and Enelow find evidence of strategic voting, Denzau et.al. examine a case in which some legislators appear to vote strategically and others do not, and Krehbiel and Rivers are unable to reject the hypothesis of sincere voting.

To account for legislators not voting strategically to promote their prima facie interests, Denzau et.al. extend the domain of individual preferences to "home style" constituency concerns. With this wider framework, they show that there are situations in which it is rational not to vote strategically: the cost of doing so in terms of the probability of reelection is too high. Because preferences and
motives are private information, constituents are unable to disentangle the reasons why a legislator voted in any particular way. Since they can never be sure that their representative was voting strategically or simply against their interests, legislators face a risk, when deciding to vote strategically, that constituents will believe the latter. The evidence that "home style" considerations are important in voting is strong (Kingdon, 1973). But the Denzau et.al. explanation is problematic in that it makes it even more difficult to test any hypothesis of strategic voting. If a legislator is observed to vote sincerely when there exists an opportunity to be strategic, it is prima facie impossible to reject the hypothesis of sophisticated behavior (relative to the broader domain of preferences): the unobserved "home style" costs may be too high.

Krehbiel and Rivers estimate legislators' preferences and find that sophisticated and sincere voting coincide for the issue they study. Consequently, "failure to reject sincere voting does not automatically imply rejection of sophisticated voting" (p.26). They conjecture that, since the issue was one-dimensional and a median most-preferred outcome existed, a process of amendments and counter-amendments resulted in this median being proposed. Although the existence of a median on the agenda does not, in and of itself, guarantee that sincere and sophisticated voting will coincide, the suggestion is that the locus of strategic behavior may be more at the agenda-setting stage of the process than at the subsequent voting stage.

In this note I show that this suggestion generalizes. If decision-making is by the amendment procedure, if the agenda is set endogenously, and if information is complete, then sophisticated voting over the resulting agenda is observationally equivalent to sincere voting. The voting strategies which support the observed voting behavior, however, are sophisticated in the classical sense. The intuition behind the result is straightforward. Given an arbitrary agenda, a legislator knows that rational committee members will vote strategically. So the legislator takes these voting strategies into account when selecting a proposal or amendment to put on the agenda: proposing is "sophisticated". And given the structure of sophisticated voting on binary agendas, this results in
choosing proposals and amendments which defeat previously proposed alternatives under sincere voting. The class of agenda-setting procedures explicitly covered by the result is quite broad. Effectively, any mechanism satisfying the following three criteria fall within it: (1) legislators are given at most one opportunity to offer an alternative; (2) proposals are made openly within the committee; and (3) the agenda is set sequentially -- i.e. the $s^{th}$ proposal on the agenda can be offered only after the first $(s-1)$ proposals have been fixed and before the $(s+1)$-to-final proposals are volunteered.

In the next section, the model and the result, which is technically quite trivial, are developed formally. A final section offers some concluding remarks.

Model and result

The model is fairly standard. A committee $N$ consists of an odd number, $n \geq 3$, of individuals, $N = \{1, 2, \ldots, n\}$. Each individual $i \in N$ has preferences defined on the $k$-dimensional issue space, $R^k$: $i$'s preferences are assumed representable by a continuous and strictly quasi-concave utility function, $u_i : R^k \to R$. The committee makes a choice from the alternative space, $X (R^k \supset X)$, using simple majority voting over an agenda based on the amendment procedure. An agenda of length $t$ is a $t$-tuple $y = (y_1, y_2, \ldots, y_t) \in X^t$. The agenda is based on the amendment procedure if the sequence of votes over the agenda is given by first putting $y_t$ against $y_{t-1}$; putting the winner of this contest against $y_{t-2}$; and so on until $y_1$ is paired against the winning alternative in the $(t-2)^{th}$ contest: the winner of this last contest is the final outcome. Let $\Sigma(y)$ be the voting tree defined by this process. Hereafter, leave the dependency of $\Sigma(\cdot)$ on $y$ implicit, and write $\Sigma(y) \equiv \Sigma$.

The agenda is determined endogenously. The mechanism assumed is as follows. If there is an historically given status quo, labelled $b(0)$, then this occupies the position of $y_1$ in any agenda. Thereafter, alternatives are offered by eligible committee members in a sequence, $\pi$. Although all
committee members have voting rights, it is not assumed that all committee members are necessarily eligible to make proposals: agenda-setting may be confined to a subcommittee. Let $M, N \supseteq M$, be the set of legislators eligible to offer proposals and amendments. Let $|M| = m$.

The first legislator under $\pi$ is chosen by some (possibly degenerate) lottery over $M$. Let $p(j = i_1)$ be the probability in this lottery that legislator $j \in M$ is the first-ranked individual in $\pi$. Assume $p(j = i_1) \in [0, 1], \forall j \in M$, and $\sum_M p(\cdot) = 1$. Once the first-ranked individual is chosen, he or she determines the second alternative on the agenda, $y_2$: call this proposal $b(1)$. Once $b(1)$ is revealed, the second individual under $\pi$ is selected by a second lottery from the remaining $m-1$ eligible legislators, and this individual then determines $y_3$, denoted $b(2)$. In general, given the subsequence $\pi_{s-1} = (i_1, i_2, \ldots, i_{s-1})$, where $i_r$ is the $r$th-ranked individual in $\pi$, and given proposals $\{b(0), \ldots, b(s-1)\}$, the $s$th-ranked legislator is chosen by lottery from legislators in $M\not\{i_1, \ldots, i_{s-1}\}, s = 2, \ldots, m$. For any $j \in M\not\{i_1, \ldots, i_{s-1}\}$, the probability that $j$ is selected to make proposal $b(s)$ is given by $p(j = i_s | \pi_{s-1}) \in [0, 1]:$ by assumption, $\sum_{M\not\{\cdot\}} p(\cdot | \cdot) = 1$ (so that $p(j = i_s | \pi_{s-1}) > 0$ for at least some $j \in M\not\{i_1, \ldots, i_{s-1}\}$). So while the lottery at any stage $s$ can depend on the subsequence $\pi_{s-1}$, it is presumed independent of the proposals $\{b(0), \ldots, b(s-1)\}$.

By convention, write $p(j = i_1) = p(j = i_1 | \pi_0)$, $\forall j \in M$. By definition, $\pi = (i_1, \ldots, i_m)$ and $\forall j \in M, \exists s \in \{1, \ldots, m\}$ such that $j = i_s$.

All the probabilities are common knowledge and can be used at each stage of the process to compute the likelihood of any given sequence, $\pi$, occurring. Let $\mu(\pi | \pi_s)$ be the probability that the de facto sequence of proposers is $\pi$, given the realized subsequence $\pi_s$, $s = 0, \ldots, m-1$. For example, if $M = \{1, 2, 3, 4, 5\}$ and $\pi_2 = (4, 2)$ then:

$$\mu((4, 2, 1, 3, 5) | \pi_2) = p(1 = i_3 | (4, 2)) \cdot p(3 = i_4 | (4, 2, 1)) \cdot p(5 = i_5 | (4, 2, 1, 3))$$

$$= p(1 = i_3 | (4, 2)) \cdot p(3 = i_4 | (4, 2, 1)).$$

Evidently, $\mu(\pi | \pi_{m-1}) \in \{0, 1\}$ for all $\pi$.

At any stage $s \geq 1$ in the agenda setting process, if the $s$th legislator $(i_s)$ does not wish to offer
any alternative at all or any not already proposed, then \( b(s) = \emptyset \) (where, by an abuse of notation, \( \emptyset \in X \)): under the amendment procedure, this captures the parliamentary rule preventing reconsideration of previously eliminated alternatives. The final agenda, then, is at most of length \( m+1 \leq n+1 \) and is given by \( b = (b(0), b(1), \ldots, b(m)) \). Once \( b \) is determined, voting proceeds according to the amendment procedure to arrive at the committee decision.

To save on notation, given any subsequence \( \pi_s \), \( s \leq m \), relabel individuals in \( N \) so that \( i_r = r \), \( \forall r = 1, \ldots, s \). For every \( s = 1, \ldots, m \), let \( B_s = \{ b(0), \ldots, b(s-1) \} \). A (pure) proposal strategy for individual \( i \in M \) is then a function:

\[
b_i : X^i \rightarrow (X \setminus B_i) \cup \emptyset = B_i.
\]

For legislators \( j \in N \setminus M \), define the (vacuous) proposal strategy, \( b_j \), by setting \( \beta_j \equiv \emptyset \).

Let \( |\Sigma| \) be the number of possible pairwise contests identified by branches of the voting tree, \( \Sigma \). A (pure) voting strategy for individual \( i \in N \) is a function:

\[
v_i : \Sigma \rightarrow \{0, 1\}^{|\Sigma|}.
\]

In any pairwise contest, a value "1" denotes a vote for the alternative proposed later in the agenda (i.e. with the larger index under \( \pi \)), and a value of "0" denotes a vote for the alternative proposed earlier.

Since a legislator will cast at most \( m \) votes during the voting process, \( i \)'s voting strategy can be decomposed; viz. \( v_i = (v_{it} | t = 1, \ldots, m) \), where \( t \) indexes the voting stage (\( t = m \) being the final vote). Legislator \( i \) votes sincerely at stage \( t \), \( t = 1, \ldots, m \), if and only if for any \( j > m-t \):

\[
[u_i(b(j)) > (<) u_i(b(m-t))] \Rightarrow [v_{it}(\{b(j), b(m-t)\}) = 1 \ (0)].
\]

If ever a legislator is indifferent and sincere at stage \( t \), assume he or she votes surely for the alternative with the higher index under \( \pi \) (with endogenous agenda-setting, this turns out to be the unique equilibrium strategy (Banks and Gasmi, 1986)). Legislator \( i \) is said to vote sincerely if and only if \( i \) votes sincerely at every stage \( t = 1, \ldots, m \).

A (pure) strategy for legislator \( i \) is thus an ordered pair, \( \sigma_i = (b_i, v_i) \), \( i \in N \). Let \( \sigma = (\sigma_i)_{N} \) and
Given any list of strategies, \( \sigma \), let \( y(\sigma) \) be the final outcome of the decision making process. The noncooperative solution concept used for the extensive form committee game induced by the structure described above can now be defined. An equilibrium is a list of strategies \( \sigma^* = (\sigma_i^*)_n \) such that, \( \forall i \in N \):

1. \( b(i)^* = b_i^*(-) \) maximizes \( E_{u_i}(y((b, \nu_i^*), \sigma_{-i}^*)) \) over \( \beta_i \), where the expectation operator \( E \) is with respect to the sequence \( \pi \),

2. \( \nu_i^* = (\nu_{it}^*) \) is such that, \( \forall t = 1, \ldots, m \), conditional on \( i \) being pivotal at each stage \( t \), \( \nu_{it}^* \) maximizes \( u_i(y((b_i^*, \nu), \sigma_{-i}^*)) \) over \( \{0, 1\}^{\sum} \).

Because there is complete information and decision making is by the amendment procedure, in equilibrium legislators will adopt sophisticated voting strategies in the sense of Farquharson (1969), and McKelvey and Niemi (1978). So, taking this as given, it is enough to consider the agenda-setting stage of the game.

For any policy \( x \in X \), define the win-set of \( x \), \( W(x) = \{ y \in X \mid |\{ i : u_i(y) > u_i(x)\}| > n/2 \} \), and let \( W^c(x) \) be its closure. If \( W(x) = \emptyset \) for any alternative \( x \), \( x \) is a Condorcet winner. By the assumptions on individual preferences, for any alternative \( x \), \( W(x) \) is open. Therefore, if ever an individual is constrained to choose an alternative in \( W(x) \) and \( W(x) \) is nonempty, \( i \)'s maximization problem is not well-defined. However, by the observation made earlier regarding equilibrium voting strategies for pairs of alternatives over which an individual is indifferent, the statement "maximize over \( W(x) \)" can always be replaced (without loss of generality) by the statement "maximize over \( W^c(x) \)"; which is well-defined. Hereafter, this is left implicit.

**Theorem:** Suppose there is no Condorcet winner in \( X \). Then there exists a unique equilibrium \( \sigma^* \) to the game, and the observed voting behavior of all legislators in equilibrium is sincere.

**Proof:** To save on notation (and without loss of generality), set \( M = N \). For any legislator \( j \),
define,
\[ Q(j) = [\bigcap W(b(s))] \cup \emptyset, \]
where the intersection is taken over all \( b(s) \in B_j \). Given \( \pi_{n-1} \) and \( B_n = \{ b(0), \ldots, b(n-1) \} \), legislator \( i_n \) offers the final proposal. Without loss of generality, set \( i_n = n \). Hence,
\[ b_n^*(B_n) \in \{ b(n) \in \beta_n \mid b(n) \in \text{argmax}_Q(n)u_n(b) \} \]
is, given sophisticated voting, a best response for legislator \( n \). Given \( \pi_{n-2} \), the individual offering the penultimate proposal knows surely the identity of the final proposer, \( i_n (= n, \text{by assumption}) \). Moreover, \( n \)'s proposal strategy is uniquely defined. Again without loss of generality, label \( N \) so that \( i_{n-1} = n-1 \). Then legislator \( n-1 \) can do no better than to adopt,
\[ b_{n-1}^*(B_{n-1}) \in \{ b(n-1) \in \beta_{n-1} \mid b(n-1) \in \text{argmax}_Q(n-1)u_{n-1}(b_{n}^*(\{ b \} \cup B_{n-1})) \}. \]
Given \( \pi_{n-3} \), there are two possible de facto sequences \( \pi \) that can occur, depending on the outcome of the final lottery over \( \{ n, n-1 \} \) to determine who proposes the penultimate alternative on the agenda. But whichever sequence occurs, the proposal strategies of the remaining proposers are well-defined. Therefore, individual \( i_{n-2} \) (identified by assumption with \( n-2 \)) has a best response to adopt,
\[ b_{n-2}^*(B_{n-2}) \in \{ b(n-2) \in \beta_{n-2} \mid b(n-2) \in \text{argmax}_Q(n-2)E_{u_{n-2}}(\cdot) \}, \]
where,
\[ E_{u_{n-2}}(\cdot) = \sum_{\pi} \mu(\pi \mid \pi_{n-3})u_{n-2}(b_{m(\pi)}^*(\{ b \} \cup B_{n-2})) \]
and \( m(\pi) \) is the final proposer under \( \pi \), etc.. Proceeding iteratively in the obvious manner, define analogous proposal strategies for all \( i_{n-3}, \ldots, i_1 \). By construction, this set of proposal strategies is well-defined, constitutes an equilibrium, and is the unique such set. However, by sophisticated voting, for some \( Q(j) \), \( j \)'s best payoff can be invariant across several feasible proposals, or none. Therefore there may exist several equilibrium agendas induced by \( \sigma^* \).

In the equilibrium, voting strategies are sophisticated. Consider voting behavior along the equilibrium path for any agenda \( b^* \), derived from the specified proposal strategies. At the final
nonvacuous voting stage of this path -- i.e. $b(s)$ vs. $b(0)$, $n \geq s \geq 1$ and $b(s) \neq \emptyset$ -- all legislators surely vote sincerely. Because there is no Condorcet winner, $W(b(0)) \neq \emptyset$ and such a stage exists. By construction, $b(s) \in W(b(0))$, so $b(s) = y(\sigma^*)$ is the final outcome. Consider the penultimate voting stage in which $b(1)\) is put against an alternative $b(j) \neq \emptyset$, $n \geq j \geq 2$. Then, $y(\sigma^*) \in \{b(j), b(1)\}$. If there is no such alternative, we are done. So suppose there is a nonvacuous alternative. If $y(\sigma^*) = b(1)$, then, by the proposal strategies, $W(b(1)) = \emptyset$ implying $b(1)$ is a Condorcet winner: contradiction. Hence, $y(\sigma^*) = b(j)$. By construction, $b(j) \in Q(j)$. Clearly, any individual $i \in N$ for whom $u_i(b(j)) > \max\{u_i(b(1)), u_i(b(0))\}$ will vote sincerely at this stage. Similarly, since $b(1)$ and $b(j)$ are in $W(b(0))$, any individual $i$ for whom $\min\{u_i(b(1)), u_i(b(0))\} > u_i(b(j))$ has sincere voting as a weakly dominant strategy here. Suppose $i \in N$ is such that $u_i(b(1)) > u_i(b(j)) > u_i(b(0))$. Then because $b(1) \in W(b(0))$ by construction, $i$ has a weakly dominant strategy to vote sincerely. Finally, suppose $i \in N$ is such that $u_i(b(0)) > u_i(b(j)) > u_i(b(1))$. By construction, $b(j) \in W(b(1)) \cap W(b(0))$ and $b(1) \in W(b(0))$. So it is again a weakly dominant strategy for $i$ to vote sincerely at this stage. Hence sincere voting over $\{b(j), b(1)\}$ is a weakly dominant strategy for all individuals. By sincere voting at the final stage, definition of $b(j)$ implies all legislators vote sincerely at this penultimate stage. Now apply an obvious inductive argument to complete the proof. \( \square \)

**Corollary:** Suppose there exists a Condorcet winner, $x^*$, in $X$. Then there exists an equilibrium $\sigma^*$ to the game which is unique up to the proposal of $x^*$. The observed voting behavior of all legislators in equilibrium is sincere, except possibly at voting stages up to and including the first vote involving $x^*$.

**Proof:** For any legislator $j \in M$ such that $Q(j) \neq \emptyset$, the proposal strategy specified in the proof of the Theorem is the best response. Suppose there exists $i \in M$ and $b(j) \in B_i$ such that $W(b(j)) = \emptyset$. 

Then \( Q(i) = \emptyset \). Set \( b(j) \equiv x^* \). By a theorem of McKelvey and Niemi (1978), \( x^* \) will be the final outcome under sophisticated voting whatever alternative \( i \) elects to propose. Hence, proposal strategy is a best response for \( i \). In particular, if \( b(i) \neq \emptyset \) and \( b(i) \not\in \bigcap \{ W(b(t)) \mid b(t) \in B_i \) and \( W(b(t)) \neq \emptyset \} \), then sophisticated voting along the equilibrium path need not be sincere. Moreover, by the argument for the Theorem, a necessary condition for voting not to be sincere at some stage for some legislator is that there be at least one such proposal \( b(i) \). The Corollary now follows from the Theorem. \( \Box \)

The Corollary identifies the logically possible exception to the claim that, with endogenous agenda-setting, sophisticated voting is always observationally equivalent to sincere voting. Having said this, three points should be emphasised. First, the conclusion of the Theorem goes through if a Condorcet winner \( x^* \) is not proposed, even though it exists (see below). Second, if \( x^* \) is placed on the agenda then making no further proposals is equilibrium behavior, as is offering alternatives which can defeat (under sincere voting) all those previously proposed other than \( x^* \). In both these cases, voting along the entire equilibrium path will be observationally equivalent to sincere voting. Thus, even when there exists a Condorcet winner \( x^* \), there always exist equilibria to the game in which sincere and sophisticated voting coincide along the equilibrium path; and these equilibria are unique up to the proposal of \( x^* \). Third, in the spatial context at least, the existence of a Condorcet winner is generically confined to one-dimensional issues.

Finally, note that if there exists a Condorcet winner \( x^* \) in \( X \), and if all legislators are eligible to propose alternatives (\( M = N \)), then \( x^* \) is surely the outcome to the agenda game above. To see this, simply note that by Plott's (1967) theorem and \( n \) odd, there exists an individual \( j \in N \) with ideal point \( x^* \). Therefore \( x^* \) will be placed on the agenda.
Conclusion

If committee decision-making is by the amendment procedure, and the agenda is determined endogenously (either by a subcommittee or by the Committee-of-the-Whole), then the Theorem offers an explanation for why legislators are observed to vote sincerely. Furthermore, along the equilibrium path no individual will vote against his or her own proposal should this be put against an alternative offered earlier in the agenda-setting sequence.\(^3\) This explanation does not rest on widening the domain of legislator preferences.\(^4\) Nevertheless, it is worth reiterating that the observed behavior is supported by sophisticated voting strategies. In empirical work attempting to test for strategic voting, therefore, it is important to determine whether or not agenda-setting is endogenous. If it is, then, ceteris paribus, the Theorem predicts that a hypothesis of sincere voting will not be rejected.

As remarked in the Introduction, the key to the result is that sophisticated voting over an agenda induces legislators to confine their proposals to alternatives that can beat -- under sincere voting -- the proposals offered earlier in the agenda-setting process. Since the formal structure of sophisticated voting is invariant across the class of binary agendas (McKelvey and Niemi, 1978), this suggests that the Theorem holds not only for the amendment procedure, but also for any binary agenda and (binary) agenda-setting mechanism.
Footnotes

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1. Formal definitions of the amendment procedure and related concepts are given in the next section.

2. For related models of endogenous agenda-setting, see, for example, Banks and Gasmi (1986), McKelvey (1986), Miller (1985), and Miller, Grofman and Feld (1986).

3. Unfortunately, the observation that legislators behave as if constrained to vote for their own proposals against those offered previously has not been formally documented. It is nevertheless a commonplace among students of Congress. In the case where a Condorcet winner, x*, has been placed on the agenda, a legislator can insure against having to vote for alternatives previously placed on the agenda against his or her own proposal -- if this is made after the appearance of x* on the agenda -- only by proposing nothing, or by offering an alternative which beats all previously volunteered proposals other than x*. (Both constitute equilibrium strategies here.) Thus, if legislators, for whatever reasons, do feel obliged to vote for their own proposals against those offered previously, the logically possible exception to the claim that sophisticated and sincere voting observationally coincide with endogenous agenda-setting, identified in the Corollary to the Theorem, vanishes.

4. A referee has rightly observed that, just as the model here excludes "home style" reasons for voting behavior, it excludes similar ("position taking") reasons for proposal behavior. These may well prove important, giving rise to a distinct model of strategic agenda-setting. Having said this, there is in fact some room for position taking in the model here, as the Corollary makes clear. If there is an underlying Condorcet winner (x*) and if this is put on the agenda before all eligible committee members have made a proposal, then subsequent proposers can "position take", since whatever they offer will lose to x* under sophisticated voting.
References


