THE TAX COMPLIANCE GAME: TOWARD AN INTERACTIVE THEORY OF LAW ENFORCEMENT*

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ABSTRACT

The existing paradigm for the economic analysis of tax compliance provides an inadequate theory of the revenue collection process. Even as a purely economic model, its exclusive focus on individual taxpayers' decision making promotes an unduly restrictive vision of the compliance problem and potential responses to it. In this paper we outline a more comprehensive theoretical basis for analyzing tax compliance, and illustrate it with a simple model. We believe our approach to be a significant improvement in the economic theory of law enforcement because it views the noncompliance problem as an interactive system. In our theoretical construct, individual decision making not only depends upon and responds to the detection and punishment structure, but, unlike prior models, we also explicitly include the law enforcement agency -- in this case the Internal Revenue Service -- as an important interactive element.

Initially we outline our general approach and its differences from the existing economic law enforcement paradigm. We then detail a simple model and its results and compare these results both to the prior literature and to some of our ongoing research in an effort to illustrate how our theoretical construct may affect predictions. Finally, we describe potential extensions of the model, examine its robustness with respect to various underlying assumptions and offer suggestions for further research, including possible applications to other law enforcement contexts.
1. INTRODUCTION

Tax noncompliance is now widely recognized to be an extremely serious problem; recent estimates suggest that at least $90 billion of taxable income goes unreported annually — an average of 10 to 15 percent of total taxable income. Commissioner of Internal Revenue Roscoe Egger reported in 1982 that the "income tax gap" in the legal sector grew from $29 billion in 1973 to $87 billion in 1981, and he projected a gap of $120 billion for 1985. Shocking estimates are also offered of income tax evasion in the illegal sector, and some observers regard declining compliance as posing a clear and present danger to the income tax.

Concern about declining tax compliance has produced an outburst of government, tax bar, and academic analyses, but both the causes and the appropriate responses to the problem remain controversial. Tax noncompliance suggests at least three reasons for concern. First, revenue losses from noncompliance become particularly significant at a time of substantial budget deficits. Second, tax noncompliance may be creating a nation where citizens' disrespect for the tax laws will expand disrespect for other laws. Third, compliance is desirable because fairness in taxation requires that equals should pay equal taxes.

Congress has responded to the perceived tax compliance crisis by enacting legislation in 1981, 1982, and 1984 that provides new and increased penalties, expanded taxpayer information reporting requirements and novel procedures. Examples include penalties for substantial understatements of tax liabilities, for aiding and abetting understatements of tax liabilities, for the filing of frivolous returns; for failure to supply taxpayer identification numbers, for failure to file information returns, for extended failure to file tax returns, and for the promotion of abusive tax shelters. Criminal fines also were increased and additional information reporting was required. New requirements for registering tax shelters with the IRS and for maintaining lists of tax shelter investors were enacted and the IRS was given authority to seek injunctions against the promoters of abusive tax shelters. Compliance measures were estimated to raise one-third of the total revenues to be raised by the 1982 Tax Equity and Fiscal Responsibility Act, and additional compliance legislation seems virtually certain in 1986.

In adopting these measures, Congress explicitly embraced an economic approach to the noncompliance problem — an approach that has seemed to dominate policy discussions, notwithstanding important contributions from a variety of other disciplines, including law,
sociology and psychology. The congressional committee reports on the 1982 Act, for example, describe the tax collection process as a "tax or audit lottery," which presumes that citizens endeavor to maximize their own narrowly-perceived financial self-interests.

Unfortunately, the existing paradigm for the economic analysis of tax compliance provides an inadequate theory of the revenue collection process. Even as a purely economic model, its exclusive focus on individual taxpayers' decision making promotes an unduly restrictive vision of the compliance problem and potential responses to it. An inadequate theoretical construct may both distort empirical analyses and imply unhelpful policy responses. Our mission here is to outline a more comprehensive theoretical basis for analyzing tax compliance. We believe our approach to be a significant improvement in the economic theory of law enforcement because it views the noncompliance problem as an interactive system. In our theoretical construct, individual decision making not only depends upon and responds to the detection and punishment structure, but, unlike prior models, we also explicitly include the law enforcement agency -- in this case the Internal Revenue Service -- as an important interactive element.

In the next section of this Article we outline our general approach and its differences from the existing economic law enforcement paradigm. We then detail a simple model and its results and compare these results both to the prior literature and to some of our ongoing research in an effort to illustrate how our theoretical construct may affect predictions. Finally, we describe potential extensions of the model, examine its robustness with respect to various underlying assumptions and offer suggestions for further research, including possible applications to other law enforcement contexts.

2. THE NEED FOR AN INTERACTIVE THEORY

More than fifteen years ago, Gary S. Becker (1968) established the basic economics-of-crime methodology. Generally this approach treats criminal activity as a rational individual decision that depends upon probabilities of detection and conviction and levels of punishment. Becker explicitly suggested that this methodology was applicable to tax evasion and avoidance (Becker, pp. 170 and 172) and it frequently has been so applied. The classic treatment can be found in Allingham and Sandmo (1972), where the authors derive conditions under which an increase in the penalty rate or the probability of imposition of sanctions increases the reported taxable income of a rational maximizing taxpayer, who is subject to a proportional tax schedule and certain about both the penalty schedule and the probability of sanction imposition.7

Although the utilization of economic methodology in this context seems natural -- tax evasion, after all, is an economic crime -- most of the theoretical results to date do not seem particularly useful for either policy analysis or empirical study. The models consider only the actions of taxpayers and ignore those of others
involved in the revenue collection process. In particular, they fail to take into account the interrelationships between flexible IRS policy instruments and noncompliance; in prior work, IRS actions and policies are treated as exogenous parameters.8

The exclusive focus on taxpayer behavior of the dominant theoretical model means that individual-linked variables largely determine the outcomes. Results, for example, are particularly sensitive to assumptions about individuals’ attitudes toward risk. In addition, omitting the law enforcement agency from the theoretical paradigm makes it natural to ignore important legal, political and institutional factors that in some cases, enhance the IRS’s ability to enforce the tax laws and, in others, inhibit the government’s ability to apply what might otherwise seem to be theoretically desirable remedies.9

To be fair, Becker’s primary inquiry was explicitly normative -- an effort to determine the "optimal" choice of punishment strategies. As he put it, his purpose was to answer the question "how many resources and how much punishment should be used to enforce different kinds of legislation" (Becker, p. 170). To answer this question, he needed a positive theory of criminal behavior, which he provided in the form of a rational utility-maximizing decisionmaker; this has become the most influential aspect of his analysis. However, we contend that it is also necessary to integrate any positive description of individual behavior with a positive theory of the law enforcement agency to address properly the relevant policy issues.10

Our analysis of the tax compliance problem here introduces the IRS as a strategic actor in a game-theoretic approach that allows the IRS to condition its audit rules on the reports it receives from taxpayers.11 Although we believe our methodology to be a significant improvement in the economic theory of law enforcement generally, the behavioral framework of tax noncompliance makes it an especially appropriate case for a theoretical construct that explicitly takes into account the interactions and responses of the cognizant law enforcement agency. The tax law, unlike many other laws, requires the taxpayer to submit a report, or tax return, that directly conveys a significant amount of information to the law enforcement agency. The IRS has broad authority to demand that taxpayers supply information on tax returns under penalties of perjury and to adjust its audit and other detection programs in light of the reports that it receives. In addition, the IRS has great power to demand information relevant to the taxpayer’s tax liability from a variety of third parties.12 The IRS also conducts a special series of intensive line-by-line audits of a randomly selected group of taxpayers (called the Taxpayer Compliance Measurement Program or TCMMP); this program demonstrates the extent to which careful audits will detect tax understatements (and overstatements) and guides the allocation of IRS audit resources.13

It is a major weakness of the existing theoretical literature to ignore this central feature of the tax system -- a preliminary round of information transmission -- and to treat the IRS either as a completely exogenous element of the revenue collection process or, at
best, as constrained to audit all taxpayers with the same frequency. In fact, the IRS audit selection process clearly turns on the information it receives from taxpayers, and the evidence suggests that the IRS is more likely than taxpayers to act in a rational, optimizing manner. Thus, the revenue collections process is better seen as a "game" with levels of noncompliance, auditing and penalty assessment determined by the interaction between taxpayers and the IRS.

Two of us have previously analyzed a so-called principal/agent model of income tax evasion in which the IRS is designated the principal and the taxpayer the agent (Reinganum and Wilde, 1985). This formulation permits the IRS to adjust its audit and enforcement strategy in light of the information contained in a taxpayer's report, and thereby treats IRS behavior as endogenous to the model. The principal/agent model used, however, requires the IRS to announce and commit to an audit policy before receiving taxpayers' reports. This audit policy will typically prove suboptimal once taxpayers' reports have been received; thus, the IRS will have incentives to deviate from its announced policy once the reports are submitted. Taxpayers will anticipate such deviations and will not believe (or be influenced by) the previously announced policy. In the game-theoretic approach employed here, we therefore restrict the set of strategies available to the IRS to eliminate such "empty threats."

The model described in the next section of this paper neither permits nor requires precommitment; instead, it follows the natural temporal sequence of decisions. First, the taxpayer reports his or her income; then the IRS decides (on the basis of the taxpayer's reported income) whether to perform an investigative audit. If the taxpayer is not audited, the taxpayer's report determines his or her final tax liability; if the taxpayer is audited, his or her tax liability is computed on the basis of true income (which we assume to be discovered in the audit process), plus any applicable fines, penalties or interest.

Explicitly including the law enforcement agency in an economic analysis of noncompliance requires us to make specific assumptions about characteristics of the agency's behavior in order to specify the actions that it will take in response to actions taken by individuals. Needless to say, any particular specification of an agency's preferences over such decisions is likely to be controversial and may have an important impact on the predictions of the theory. In our basic model, we assume that the IRS in its enforcement strategy attempts to maximize total government revenue, including taxes, interests and penalties, net of audit costs. We regard this as a reasonable assumption, and, to date at least, have found none other that we find more convincing, although we do not mean to suggest that others might not be equally reasonable or, perhaps, even better. This question of the IRS's preferences is explored further in Part 5, below.

A number of policymakers, including Commissioner Egger, seem to perceive the current noncompliance problem to be one of eroding ethics -- that more and more people are refusing to comply voluntarily
with the tax laws and instead are responding "strategically" to the enforcement structure of the income tax laws. Commissioner Egger believes that the proportion of people who routinely and habitually comply with income tax rules is declining, perhaps at a rapid pace.  

In an effort to begin to analyze this problem, our model includes two types of taxpayers: (1) the "strategic noncompliers" -- the type of individuals posited by the standard economics of crime literature, who maximize results by weighing rationally the potential costs and benefits of understating tax liability; and (2) the "habitual compliers" -- individuals who endeavor to report taxable income correctly without regard either to the costs and benefits of playing the audit lottery or to their perceptions about the compliance behavior of others. By developing a model that has the potential to consider the impact of such a variety of types of taxpayers, we may eventually be able to distinguish among classes of taxpayers depending upon their relative opportunities to avoid taxes as well as their attitudes. Habitual compliers, for example, may be thought of as persons who have little or no real opportunity not to comply, such as wage-earners in the legal sector without time to moonlight who do not itemize deductions. 

The model treats income levels, tax rates and fines as exogenous. This treatment of fines differs somewhat from their typical treatment in the economics of law enforcement literature, but follows naturally from our decision to use a model that treats the law enforcement agency as an endogenous interactive actor. The level of fines, after all, is typically set by the legislative branch and, although the law enforcement agency may have considerable discretion in determining how to apply statutory penalties, it will be constrained by the statutory requirements. The statutory level of fines, as well as the necessary conditions for their application -- for example, the requirement that the taxpayer's tax understatement be determined to be willful in order to apply fraud penalties -- in turn may depend on a variety of legal, moral and political constraints. In contrast, the economics of crime literature standardly treats fines as a variable that can be adjusted to maximize a specified social welfare function and, principally because of the costs of detection, often yields the prediction that fines should be set at the maximum level with corresponding probabilities of detection at the minimum. (One wag has described the resulting policy prescription as one that typically recommends hanging with probability zero.) As the following discussion will make clear, our treatment of the level of fines as an exogenous variable does not produce any difference in their impact on individual decisionmaking, nor does it eliminate potential tradeoffs between changes in the level of fines and probabilities of detection. Our model, however, does require us to address explicitly the question of how the existence and level of fines should be treated in describing both the preferences and the actions of the law enforcement agency. 

For mathematical tractability, in the model set forth here, income has one of two values, high or low. The IRS does not observe
true income: instead, it must rely initially on the report made by the taxpayer. The basic model assumes the IRS can audit as many taxpayers as it wants; initially we impose no budget constraint (but other of our work including budget constraints is discussed in Part 5).\(^{19}\)

The model is considered to be solved when a so-called Nash equilibrium is reached. Such an equilibrium involves a probability of audit, chosen by the IRS, and a probability of noncompliance, chosen by those taxpayers who act strategically. A Nash equilibrium in this context is a pair of strategies by taxpayers and the IRS that are best responses to each other; at a Nash equilibrium, neither the IRS nor potential noncompliers have an incentive (unilaterally) to change their strategies.\(^ {20}\) The purpose of using such an equilibrium concept in a theoretical model -- such as the use of the Nash equilibrium here -- is that it will be suggestive of directions and tendencies of the relevant parties' responses in the context of a complex dynamic process. In analyzing these responses, several seemingly counterintuitive results emerge -- for example, an increase in tax rates on high income is found to decrease noncompliance.\(^ {21}\) These results occur largely because in our model the audit policy of the IRS is codetermined with the reporting behavior of potential noncompliers. The details of these results will be discussed in Parts 4 and 5 below.

3. A SIMPLE INTERACTIVE MODEL

This section presents our formal model, translating the concepts discussed in the last section into mathematical terms, and characterizing the equilibrium levels of auditing and noncompliance that emerge from the interaction of the IRS and taxpayers. Our model is relatively simple as models go and we will endeavor to explain each step to the nontechnical reader as we proceed. A more concise presentation which presumes familiarity with the technical economics literature can be found in Graetz, Reinganum and Wilde (1983).

Suppose that some taxpayers are “habitual compliers”; that is, they report their income truthfully regardless of their pecuniary interest. The remaining taxpayers act strategically, they examine their incentives carefully and act so as to maximize expected utility, taking as given the probability of audit associated with the income they choose to report. We denote the proportion of strategic taxpayers by \(\rho\) and the proportion of habitual compliers by \(1 - \rho\), where \(0 < \rho < 1\).

For simplicity, we assume there are only two income classes — high and low, denoted \(I_H\) and \(I_L\), respectively, where \(I_L < I_H\).\(^ {22}\) The IRS does not directly observe income; thus the taxpayers, regardless of their income, actually may report either high or low income.\(^ {23}\)

Denote by \(T_H\) and \(T_L\) the individual tax payments owed by high- and low-income taxpayers, respectively. We assume that \(T_L < I_L\), \(T_H < I_H\) and \(T_L < I_H\). A taxpayer who is discovered to be underreporting income is fined in addition to being assessed the tax on high income; this fine is denoted by \(F\), where \(F > 0\). We assume that the IRS treats as fixed tax levels and the fine for underreporting.\(^ {24}\)
Auditing, of course, is costly to the IRS; we denote the cost of a single audit by \( c \), where \( c \geq 0 \). We further suppose that \( T_H + F - T_L > c \); that is, the increment to revenue associated with uncovering a noncomplier exceeds the audit cost. If this were not true, then even if noncompliers could be identified a priori, it would not pay to audit them and collect the additional taxes and fines. We also assume that \( T_L + F \leq I_L \) and \( T_H + F \leq I_H \); the sum of taxes plus fines cannot exceed a taxpayer's income.

We assume taxpayers who report truthfully are never fined, and suffer no other costs if they are audited (this assumption can be relaxed; see Part 5). Finally, \( q \) represents the probability that a randomly chosen taxpayer has high income, where \( 0 < q < 1 \). Since each taxpayer is viewed as drawing high income with this probability, auditing one taxpayer provides no information about the income of any other taxpayer.25 With probability \( p \) the taxpayer is a strategic taxpayer, with probability \( 1 - p \) the taxpayer is a habitual complier and simply reports his or her income truthfully.

A simple "game tree" (Figure 1) describes the information and actions available to each player. The dotted lines in Figure 1 represent so-called "information sets." Points in the game tree enclosed by the same information set are indistinguishable to the agent whose turn it is to make a decision at that stage.

[Figure 1 approximately here]

Since we assume initially that taxpayers who report truthfully suffer no cost by being audited, taxpayers with low income will always report truthfully. Thus a strategy for a strategic taxpayer is simply a probability of reporting low income when he or she actually has high income. We denote this probability by \( a \). Any probability between 0 and 1 is a feasible strategy for strategic taxpayers so the only constraint on their behavior is \( 0 \leq a \leq 1 \).

Since the IRS cannot distinguish a priori between compliers and noncompliers (that is, from a report of low income alone), in order to evaluate the benefits of auditing it must compute the distribution of honest and dishonest taxpayers among those who report low income. Let \( \mu(a) \) be the probability that a taxpayer who reports low income actually has high income, given that strategic taxpayers report low income when they really have high income with probability \( a \). A routine statistical calculation known as Bayes' rule implies

\[
\mu(a) = \frac{pqa}{pqa + 1 - q}.
\] (1)

Equation (1) has a simple explanation. Taxpayers report low income either when they really have low income (this happens with probability \( 1 - q \)) or when they have high income and lie (this happens with probability \( pqa \)).

A strategy for the IRS, meanwhile, is simply a probability of auditing a taxpayer who reports low income, since, in this model, it would have no reason to audit a taxpayer who reports high income. We denote this probability by \( p \). Again, any probability between 0 and 1 is a feasible strategy for the IRS so the only constraint on IRS behavior is \( 0 \leq p \leq 1 \). Expected net revenue to the IRS when it
observes a report of low income, strategic taxpayers who observe high income report low with probability $\alpha$, and it audits low income reports with probability $\beta$ can be written formally as

$$\Pi(\alpha, \beta) = \beta [\mu (T_H + F - c) + (1 - \mu) (T_L - c)] + (1 - \beta) T_L.$$  

The first term in this expression is the expected return from an audit, net of audit costs, and the second term is the certain revenue received if no audit is performed.

Let the taxpayers' utility function over certain income be denoted by $u(\cdot)$. Then the payoff to a strategic taxpayer who has high income, in expected utility terms, given he or she reports low income with probability $\alpha$ and is thereby audited with probability $\beta$ is

$$U(\alpha, \beta) = \alpha \{\mu u(I_H - T_H - F) + (1 - \mu) u(I_H - T_L)\} + (1 - \alpha) u(I_H - T_H).$$  

The first term in this expression is the expected utility of reporting low income and the second term is the certain utility received if high income is reported. We assume the utility of certain income is increasing in income but at a decreasing rate (that is, the marginal utility of certain income is falling — taxpayers are risk averse). We also assume the IRS is risk-neutral.

A best response for the IRS to a given reporting rule used by potential noncompliers is a strategy which maximizes the IRS's net revenue, given that all the potential noncompliers use the specified reporting rule. Similarly, a best response for potential noncompliers to any given auditing rule is a strategy that maximizes expected utility, given the specified auditing rule. Finally, a Nash equilibrium is a pair of strategies that are best responses to each other; at a Nash equilibrium neither the IRS nor potential noncompliers have an incentive to (unilaterally) change their strategies.

Thus, in formal terms, a best response for the IRS to a given strategy $\alpha$ for potential noncompliers is a strategy $\hat{\alpha}(\beta)$ such that $\Pi(\alpha, \beta(\alpha)) \geq \Pi(\alpha, \beta)$ for all other strategies $\beta$. Similarly, a best response for potential noncompliers to any given auditing policy $\beta$ is a strategy $\hat{\beta}(\alpha)$ such that $U(\hat{\alpha}(\beta), \beta) \geq U(\alpha, \beta)$ for all other strategies $\alpha$. Finally, a Nash equilibrium is a pair of strategies $(\alpha^*, \beta^*)$ such that $\alpha^* = \hat{\alpha}(\beta^*)$ and $\beta^* = \hat{\beta}(\alpha^*)$.

For any given probability of noncompliance used by potential noncompliers, $\alpha$, the IRS wants to choose its audit probability to maximize its expected net revenue; that is, it sets $\beta$ to maximize $\Pi(\alpha, \beta)$. The marginal benefit of auditing a taxpayer who reports low income is

$$\delta \Pi(\alpha, \beta) / \delta \beta = \mu (T_H + F - T_L) - c.$$  

(2)

The marginal benefit of auditing a taxpayer who reports low income increases with (a) the conditional probability that the taxpayer has high income given that he or she reported low income, (b) the tax owed by high-income taxpayers and (c) the fine. Since the conditional probability of high income given a report of low income increases with (a) the probability that a strategic taxpayer reports
low income when true income is high, (b) the proportion of strategic taxpayers relative to all taxpayers and (c) the proportion of high-income taxpayers relative to all taxpayers, the marginal benefit of auditing a taxpayer who reports low income also increases with these variables. The marginal benefit of auditing a taxpayer who reports low income decreases with the tax owed by low-income taxpayers and the cost of audit. Notice, however, that it is independent of the likelihood of audit itself; that is, the benefits of auditing low income reports are linear in the audit probability. In fact, if the conditional probability that the taxpayer has high income given that he or she reported low income (μ) is such that the marginal benefit of auditing is positive, then it always pays to audit and if it is such that the marginal benefit of auditing is negative then it never pays to audit. Thus, if we define μ so that (2) is equal to zero --

\[ \tilde{\mu} = \frac{c}{(T_H + F - T_L)} \]  

-- then we can describe the IRS' best response to any specified reporting rule, \( \hat{\beta}(a) \), by

\[
\hat{\beta}(a) = \begin{cases} 
1 & \text{if } \mu(a) > \tilde{\mu} \\
 \in [0,1] & \text{if } \mu(a) = \tilde{\mu} \\
0 & \text{if } \mu(a) < \tilde{\mu} 
\end{cases}
\]

where \( \mu(a) \) is given by equation (1). But we can say more: since equation (1) describes how \( \mu \) depends on \( a \), we can substitute directly from that expression to obtain

\[
\hat{\beta}(a) = \begin{cases} 
1 & \text{if } a > \bar{a} \\
 \in [0,1] & \text{if } a = \bar{a} \\
0 & \text{if } a < \bar{a} 
\end{cases}
\]

where

\[ \bar{a} = (1 - q)c/pq(T_H + F - T_L - c). \]  

Here \( \bar{a} > 0 \) unless the fine is infinite (assuming that \( 0 < q < 1 \) and \( c > 0 \)). But we have assumed that \( F < I_H - T_H \); that is, the IRS cannot take more than a person's income. Thus the IRS's best response to the potential noncomplier's strategy (a probability of noncompliance) is to audit if and only if the probability of noncompliance is sufficiently high that it exceeds some trigger value \( \bar{a} \). This trigger value is strictly greater than 0; that is, it is never a best response for the IRS to audit whenever the probability of noncompliance is positive. It is possible that \( \bar{a} > 1 \), however, so that it might never pay the IRS to audit (e.g., if the cost of audit is high enough).

For a given probability of audit, \( \beta \), the strategic taxpayer wishes to choose a probability of noncompliance so as to maximize his or her expected utility of net income; that is, he or she sets \( a \) to maximize \( U(a,\beta) \). The marginal gain to reporting low income when a taxpayer actually has high income -- that is, the marginal benefit of noncompliance -- is

\[
\frac{\partial U(a,\beta)}{\partial a} = (1-\beta)[u(I_H - T_L) - u(I_H - T_H)] + \beta[u(I_H - T_H - F) - u(I_H - T_H)].
\]

This gain decreases with (a) the probability of audit, (b) the fine, and (c) the tax owed by low-income taxpayers. The dependence of this gain on the tax owed by high-income taxpayers is ambiguous in
This is because increasing the tax owed by high-income taxpayers not only increases the gain from noncompliance when the taxpayer is not audited but also increases the loss from noncompliance when the taxpayer is apprehended and punished. For risk-neutral taxpayers, the marginal benefit of noncompliance increases with the amount of tax owed by high-income taxpayers. More important, however, is the fact that the marginal benefit of noncompliance is independent of the likelihood of noncompliance itself; that is, the benefits of noncompliance are linear in the probability of noncompliance. Thus, the potential noncomplier will always comply if the probability of audit (p) is such that the marginal benefits of noncompliance are negative and never comply if it is such that the marginal benefits of noncompliance are positive. Hence, if we define \( \bar{p} \) so that (4) is equal to zero, that is,

\[
\bar{p} = \frac{u(I_H - T_L) - u(I_H - T_H)}{u(I_H - T_L) - u(I_H - T_H - p)}. 
\]

then we can describe the taxpayer's best response to any specified audit rule, \( \hat{a}(\beta) \), by

\[
\hat{a}(\beta) = \begin{cases} 
1 & \text{if } \beta < \bar{p} \\
0 & \text{if } \beta = \bar{p} \\
0 & \text{if } \beta > \bar{p} 
\end{cases} 
\]

It is easy to show that \( 0 < \bar{p} < 1 \). Thus the potential noncomplier's best response to the IRS' strategy (a probability of audit) is to comply if and only if the probability of audit is sufficiently high that it exceeds some trigger value \( \bar{p} \). This trigger value lies strictly between 0 and 1; that is, it is not necessary to audit every low income return to induce compliance.

Given the two best response functions \( \hat{a}(a) \) and \( \hat{a}(\beta) \) we are in a position to calculate equilibrium configurations. If we think in terms of graphs, each of these best response functions defines a curve in two-dimensional space (one dimension representing \( a \) and the other representing \( \beta \)). The equilibrium is given where these curves intersect. Since both best response functions take the value 0 or 1 except at their respective trigger values \( \bar{a} \) and \( \bar{\beta} \), the equilibrium strategies will equal those trigger values so long as both are between 0 and 1. It's always the case that \( 0 < \bar{\beta} < 1 \), but it might be that \( \bar{a} > 1 \). Thus combining the best response functions gives two possible equilibrium configurations. In the first configuration, illustrated in Figure 2, all strategic taxpayers underreport but there is no auditing; that is, \( (a^*, \beta^*) = (1, 0) \). In this case, it is not worth auditing any individual who reports low income even if all strategic taxpayers are known to be underreporting, because the IRS still cannot tell (without auditing) whether an individual who reports low income is reporting truthfully (that is, is a habitual complier with low income) or untruthfully (that is, is a strategic taxpayer with high income). This situation, which occurs when the trigger value \( \bar{a} \) exceeds 1, arises for any of several reasons; for example, audit costs may be high, the proportion of strategic taxpayers may be low, or the proportion of low-income taxpayers may be high.
In the second configuration auditing sometimes pays: \( a < 1 \). In equilibrium, some strategic taxpayers with high income actually do underreport, and some taxpayers who report low income actually are audited; in particular, \((a^*,p^*) = (\tilde{a},\tilde{p})\). This case is illustrated in Figure 3.

[Figure 3 approximately here]

The next section of this paper will discuss the implications of this basic model, focusing on how changes in underlying parameters affect \( a \) and \( p \), assuming both are between 0 and 1. Part 5 then will discuss various extensions and modifications of the basic model.

4. RESULTS

As we have indicated, the purpose of the preceding formal model is to illustrate, in a relatively simple framework, some of the effects of introducing the IRS as a strategic actor in the compliance game. The sort of "results" typically yielded by economic models of tax compliance take the form of comparative statics; that is, they consist of predictions concerning the effects of changing underlying parameters such as tax rates or the cost of audits on the equilibrium values of the choice variables of the model. In the classic economic model of tax compliance the principal choice variable is the extent of underreporting.\(^{32}\) In our interactive framework the choice variables are the probability of underreporting, \( \tilde{a} \), and the probability of audit, \( \tilde{p} \) (the former given the taxpayer acts strategically and has high income and the latter given a report of low income has been received by the IRS). Two related variables are the probability that a randomly drawn taxpayer will be a noncomplier (denoted \( P_N \) and given by \( P_N = q\tilde{a} \)) and the probability that a randomly drawn report will be audited (denoted \( P_A \) and given by \( P_A = \tilde{p}(q\tilde{a} + 1 - q) \)).\(^{33}\) The latter two variables represent aggregate noncompliance and aggregate auditing, respectively. Table 1 summarizes the effects of changes in the underlying parameters on these four variables. Some of the effects may initially seem counter-intuitive primarily because they depend upon equilibrium interactions between the taxpayer and the IRS. The discussion following Table 1 describes the results, offers intuitive explanations for them, and compares them to those found in the existing literature (where applicable).

[Table 1 approximately here]

In interpreting our results, the reader should keep in mind that, even though we have presented our model here as if it were a global representation of the noncompliance problem, the model might also be viewed as addressing issues of noncompliance across a relatively small range of income — for example, within a given audit class. As our subsequent discussion in Part 5 of extensions and modifications of the model suggests, this basic model may well have advantages over more mathematically complex models for analyzing this important set of issues.
4.1 **Fines**

We begin with the effects of increases in the fine for noncompliance since, although our methodology is different, these results agree with those found in the existing literature. As in the standard economics of crime and tax evasion literature, an increase in the fine for underreporting reduces both the likelihood that a potential noncomplier actually fails to comply and aggregate noncompliance. In our model, however, an increase in the fine also results in less auditing, both of individual low income reports and in the aggregate. The direct impact of an increase in the fine is to increase the marginal benefit of auditing, but it also increases the marginal cost of noncompliance so that strategic taxpayers respond by increasing their compliance rate. This allows the IRS to reduce the probability of audit in the new equilibrium. But driving equilibrium noncompliance (and the equilibrium probability of audit) to zero would require an infinite fine.

4.2 **Audit Costs**

The existing literature has little to say about the effects of changes in audit costs on noncompliance because the probability of audit is generally taken as given. In our model, an increase in audit costs results in an increase in both individual and aggregate noncompliance. It has no effect on the probability of audit for an individual who reports low income, but it increases the aggregate number of audits.

To understand why this must be so, suppose there is an exogenous increase in audit costs. If potential noncompliers made no adjustment in their behavior then it would no longer pay the IRS to audit an often. But then potential noncompliers would have an incentive to underreport more frequently, which in turn would cause the IRS to increase its audit rate. In the new equilibrium there must be no incentive for either the IRS or taxpayers to make further adjustments of this sort. This happens when potential noncompliers increase the probability of underreporting just enough to compensate the IRS for the increase in audit costs by making the average audit more productive (in terms of discovering actual noncompliers and collecting more taxes and fines). Hence individual noncompliance and aggregate noncompliance increase. This increase in noncompliance perfectly offsets the increase in audit costs, so that the probability of audit for a taxpayer who reports low income is unaffected in equilibrium. In spite of this, because the aggregate number of low income reports increases and because each of these taxpayers is audited with an unchanged probability, the aggregate number of audits increases.

4.3 **The Distribution of Income**

Recall that \( q \) is the probability that a random taxpayer has high income. An increase in this probability thus reflects an upward shift in the distribution of income. In such a case, it will be less likely that a given taxpayer who reports low income actually is a low
income individual, so that the IRS will have a greater incentive to audit. Thus, in the new equilibrium, each strategic taxpayer will respond to an upward shift in the distribution of income by decreasing his or her probability of noncompliance, thereby compensating the IRS for the increase in the marginal benefit of auditing. Again, as with changes in audit costs, the probability of audit among those who report low income is unaffected in the new equilibrium so noncompliance and the probability of audit decrease in the aggregate.

4.4 Tax Rates

An important class of results involves changes in the level of taxes. First, consider an increase in the tax on high income individuals. This is analogous to an increase in the progressivity of the tax rate schedule since the tax on low income is unchanged. (In Part 5 we assume taxes are proportional to income and consider the effects of changes in the tax rate). An increase in the tax on high income increases the marginal gain to underreporting, but also increases the marginal gain to auditing. The latter effect dominates, so in the new equilibrium both individual and aggregate noncompliance fall. An increase in the tax on high income can either increase or decrease audit rates, depending on the risk preferences of taxpayers. Generally speaking, increases in the tax on low income have opposite effects to those stemming from increases in the tax on high income.

4.5 The Percentage of Strategic Noncompliers

As indicated previously, the belief that deteriorating ethics is in part responsible for increases in noncompliance makes the effects of changes in the proportion of strategic taxpayers in the population particularly interesting. The reason for such a change does not matter in our model. The proportion of strategic noncompliers might increase, for example, either because of a change in taxpayer attitudes or because, even without a change in underlying attitudes, the structure of penalties and IRS enforcement policies now makes it more profitable for increased numbers of people not to comply. Of course, many people may have always acted strategically; we may simply have become better able to measure the extent of such behavior.

In any event, in our model, an increase in the percentage of strategic taxpayers has only one effect: to reduce the likelihood that any strategic taxpayer with high income fails to report honestly. This results because with more potential noncompliers (more taxpayers who may falsely report low income), a report of low income is correspondingly more likely to have come from a strategic taxpayer who has failed to comply than from a habitual complier with low income. Thus, although there are more potential noncompliers, in the new equilibrium each is more likely to comply, and these effects exactly cancel out so that the aggregate number of taxpayers who fail to comply is unaffected. Similarly, both the probability of audit for an individual who reports low income, and the aggregate probability of
audit are unchanged.

In addition to having no effect on either auditing or the aggregate level of noncompliance, changes in the percentage of strategic taxpayers also have no effect on expected revenue. This observation yields a provocative conclusion: an exogenous increase in the proportion of strategic taxpayers has no impact on aggregate expected revenues or aggregate compliance, and should consequently have no affect on aggregate audit policy. Not only is no change in audit policy warranted to correct for the increase in strategic behavior, but also, as long as a sufficiently large fraction of taxpayers behaves strategically, increases in the proportion of strategic taxpayers do not account for declining compliance and tax revenues.

5. EXTENSIONS AND MODIFICATIONS OF THE BASIC MODEL

The discussion of our model's predictions in Part 4 shows clearly that models of tax compliance that include the IRS as a strategic actor can yield results both distinct from and richer than models that include only the taxpayer. The model we have delineated to make this point is, by design, very simple. It nevertheless yields a number of interesting results; therefore, it becomes useful to ask how robust are these results to various extensions or modifications of the basic model. In this section we summarize the effects of introducing into the model proportional taxation and fines, taxpayer audit costs, IRS budget constraints, more than two taxpayer income levels, and alternative IRS objective functions.

5.1 Proportional Taxation and Fines

In our basic model we allow tax levels and fines to take relatively arbitrary values (in particular, we require only that $T_L < T_H$). A special case often considered in the literature assumes that taxes are proportional to income. Let the tax rate be denoted by $t$, where $0 < t < 1$. We then have $T_H = tI_H$ and $T_L = tI_L$. Further, since penalties for underreporting are generally proportional to evaded tax, we can assume $F = \pi t(I_H - I_L)$ where $\pi$ is the penalty rate on evaded tax, $\pi \geq 0$. If these expressions for tax levels and the fine are substituted into the equilibrium values for the probability of noncompliance and probability of audit of Part 3, we get,

\[ \bar{\alpha} = \frac{(1-q)\sigma/\rho q(t(I_H - I_L) + \pi t(I_H - I_L) - \epsilon)}{Q(1-t)} \]  
\[ \bar{\beta} = \frac{[u(I_H - tI_L) - u(I_H(1-t))][u(I_H - tI_H) - u(I_H(1-t) - \pi t(I_H - I_L))]}{[u(I_H - tI_L) - u(I_H(1-t))]} \]

Table 2 summarizes how changes in the tax rate, the penalty rate and the difference between high and low income affect individual noncompliance and audit levels ($\bar{\alpha}$ and $\bar{\beta}$ respectively) and aggregate noncompliance and audit levels ($\bar{P}_N$ and $\bar{P}_A$ respectively).

Table 2 approximately here

An increase in the penalty rate decreases equilibrium noncompliance and equilibrium auditing, both at the individual and the
aggregate level. This is consistent with the general results summarized in Table 1 and follows from the same logic given in the discussion of fines following that table. Similarly, the model shows that increases in the tax rate, like increases in the degree of progressivity, decrease individual and aggregate noncompliance. The popular press and many government and academic commentators often assume the opposite, and partial equilibrium models are generally ambiguous on this matter; in the standard models, whether an increase in the tax rate increases or decreases noncompliance depends largely on the risk preferences of taxpayers.42

In our model, equilibrium compliance is enhanced by an increase in the tax rate, regardless of the risk preferences of taxpayers. The intuition behind this result is the same as that which explains why increases in the degree of progressivity increase equilibrium compliance -- both the incentive to underreport and the incentive to audit increase, and the latter dominates in equilibrium. A similar analysis explains why equilibrium compliance increases with income inequality, as measured by the difference between the high and low income levels. In general, the dependence of audit probabilities, both at the level of an individual low income report (\( P_L \)) and at the aggregate level (\( P_A \)), upon the tax rate and the extent of income inequality are ambiguous.43

5.2 Taxpayer Audit Costs

We have heretofore assumed that audits are costless to taxpayers, but an audit can be both costly and time-consuming even if the taxpayer has reported honestly and can demonstrate the accuracy of his or her report. Suppose that all taxpayers suffer a cost of \( k \) when audited. The net expected revenue generated by an audit is unaffected since taxpayer audit costs can be regarded as a deadweight loss rather than a transfer to the government.44 Thus it is clear that it still never pays the IRS to audit individuals who report high income. However, if, for example, taxpayer audit costs are large or the difference in taxes is small, low-income taxpayers might prefer to report high income in order to avoid taxpayer audit costs.

Analysis of this amended model yields the following results (assuming the marginal utility of certain income does not fall too rapidly -- for details, see Graetz, Reinganum and Wilde, 1983). In equilibrium, all low income taxpayers still report low income, and high income strategic taxpayers use the same strategy as when taxpayer audit costs are zero.45 The IRS's equilibrium audit probability for individual low income reports becomes less than the equilibrium audit probability for individual low income reports when taxpayer audit costs are zero.46

Thus, taxpayer audit costs have no effect on equilibrium noncompliance; no low income taxpayer elects to overreport and the same fraction of high income strategic taxpayers elect to underreport. The easiest way to see why no low-income taxpayers overreport is to
recognize that all taxpayers must pay the taxpayer audit cost if they are actually audited. Assume, then, that low income strategic taxpayers report high income in order to avoid the risk of incurring these costs, paying unnecessary taxes (\(T_H - T_L\)) to do so. Generally this implies that a high-income strategic taxpayer will not risk paying the taxpayer audit cost plus the fine for underreporting to save an identical amount in taxes; he or she would also prefer to report high income. But then it would never pay to audit, and in such circumstances, all strategic taxpayers would report low income. So, it must be the case that in equilibrium all low-income strategic taxpayers report honestly. Hence the existence of positive taxpayer audit costs affects only the equilibrium audit probability, which is reduced; the same level of noncompliance is sustained with a lower level of auditing.

Taxpayer audit costs may depend to some extent upon the complexity of tax laws. If an increase in taxpayer audit costs is due to increased complexity, however, it is also likely to be accompanied by an increase in the IRS's audit costs. Thus an increase in the complexity of the law can result in both more noncompliance (because of the increase in the IRS's audit costs) and a lower probability of audit for each taxpayer who reports low income (because of the increase in taxpayer audit costs). However, since the number of low income reports is increased, the net effect on the aggregate number of audits is ambiguous.

5.3 Budget Constraints

All formal models make simplifying assumptions and our basic model is no different from others in this regard. It is, however, important to understand to what extent any model, especially one that may have implications for public policy decisions, is sensitive to these kind of assumptions. Two of the most crucial simplifying assumptions in our basic model are that income only takes one of two values and that the IRS faces no budget constraint. We have examined the effects of relaxing both of these assumptions and the results of those analyses appear to be closely related. The effects of a binding budget constraint seem now, as a formal matter, to depend on whether income can take more than two values. In this subsection we discuss the effects of imposing a budget constraint on the model, while maintaining the hypothesis of only two income levels (and, implicitly, only two possible levels of reported income). In the next subsection we discuss the effects of introducing a range of possible income levels, both without and with a budget constraint.

In the equilibrium of our basic model, the IRS would like to audit, on average, \(\bar{p}\) percent of all low income reports. But a budget constraint may limit the IRS to auditing some fraction of all taxpayers, say \(\lambda\). If \(\lambda < \bar{p}\), the IRS sometimes might wish to audit more taxpayers than it can. In other words, for the budget constraint not to bind, the budget must be large enough that the IRS can afford to audit every taxpayer with probability \(\bar{p}\) (it is potentially the case that all taxpayers report low income, so to eliminate any effects due
to a budget constraint, the IRS must be able in principle to audit them all with the desired probability $\bar{\pi}$). An analysis of optimal auditing and reporting strategies in this case quickly becomes complicated. We have performed this task elsewhere (Graetz, Reinganum and Wilde, 1984) and will only summarize the results in this paper.

Consider first the taxpayer side of the problem. A binding budget constraint may dramatically affect taxpayer behavior. If the IRS faces a binding budget constraint, the probability that any individual taxpayer will be audited depends on the behavior of other taxpayers; the odds that any individual taxpayer will be audited are much lower if everyone with high income reports low income than if everyone reports honestly. Thus, if one looks only at the taxpayer side of the problem, IRS budget constraints produce "congestion" that leads to the existence of multiple equilibrium reporting strategies whenever the budget is neither too high nor too low. In one of these equilibria no strategic taxpayer ever complies and in the other they always comply. This congestion feature of the compliance problem largely has been ignored in the crime and punishment literature generally and appears nowhere in the literature on tax compliance.47 Some authors do consider the effects of budget constraints (e.g., Greenberg, 1984), but none explicitly model the strategic interaction between taxpayers, so that this congestion effect never arises. Some surveys report that noncompliance is more likely as a taxpayer knows more taxpayers who have failed to comply (Spicer and Lundstedt, 1976; Song and Yarbrough, 1978), but usually offer psychological or sociological, not economic, explanations for this phenomenon.

Of course the taxpayers' response to audit rules is only part of the story. Since universal compliance eliminates auditing it can never be part of a "full equilibrium." In fact, in the simple framework of our model, the addition of the IRS as a strategic actor generally produces a unique equilibrium and a binding constraint on the IRS budget typically leads to total noncompliance (by strategic taxpayers with high income). When the budget constraint does not bind, of course, the equilibrium is as described in Part 3 of this paper.

That the introduction of a binding budget constraint into our basic model generally implies total noncompliance by strategic taxpayers is a disturbing result. The reason for this result is, however, clear; as we discussed in Part 3, whether a strategic taxpayer with high income complies depends on whether the audit probability is greater than or equal to a trigger value $\bar{\pi}$. If the presence of a budget constraint means the IRS cannot attain this level of auditing, then no strategic taxpayer will ever report high income. In such an equilibrium, changes in other parameters cannot effect behavior unless they result in lowering the trigger value sufficiently to make the budget constraint nonbinding.

Several points need to be made regarding this result. The first, and most important, is that it now appears largely to be an artifact of the assumption that income is restricted to one of two levels. This assumption reduces the compliance problem simply to a
comply/don't comply choice that is extremely sensitive to the probability of audit. Two of us have examined a related model that assumes income is randomly distributed on some range of values (Reinganum and Wilde, 1984). The analysis of that model is extremely technical (and certain other simplifying assumptions have to be introduced) but, although this is surely not the last word, the qualitative results there are relatively insensitive to the presence of a binding budget constraint, and in other respects are similar to the results discussed in this paper.48

A second, and related point, concerns the assumption implicit in a budget-constraint model that the budget constraint applies separately to IRS audit activity. The IRS, in fact, has the capability of shifting resources among a variety of administrative and enforcement activities. When such shifts are possible, there may, in terms of qualitative effects, be no binding budget constraint.49 In other words, the qualitative effects summarized in Table 1 will continue to hold. When, as we suggested earlier, the results of the basic model are regarded as applying within a relatively small range of income -- for example, within a single audit class or category of taxpayer, such as large corporations -- an identical point may be made about the lack of a binding budget constraint in analyzing that aspect of the noncompliance problem.

Finally, even if there were a separate budget constraint on audit activity, our simple model will still be useful in evaluating how various changes in the underlying parameters affect whether the budget constraint is, in fact, binding. Until we know how much the IRS should want to audit, we cannot tell whether its budget prevents it from auditing that much. This is not merely a matter of the size of the budget; the constraint binds whenever the IRS is unable to audit all of the taxpayers it might want to audit. For example, if taxpayers are indifferent to risk, our model implies the IRS should want to audit the fraction $\bar{p} = (T_H - T_L)/(T_H - T_L + F)$ of those taxpayers who report low income. Thus, in this case, the budget constraint binds whenever $\lambda < (T_H - T_L)/(T_H - T_L + F)$, where $\lambda$ is again the percentage of the total number of taxpayers which the IRS can afford to audit. Thus the budget constraint is more likely to bind when the budget is small, the cost per audit is high, the fine is low, or the difference between the taxes owed by a high-income and a low-income taxpayer is large.

5.4 More Than Two Income Levels

Perhaps the strongest assumption of our basic model is that taxpayers' true incomes must take one of only two values. This greatly simplifies the analysis by reducing the compliance problem to a simple comply/don't comply decision. Reinganum and Wilde (1984) analyze an interactive model of tax compliance in which income can take any value in some range. In such a case taxpayers must decide both whether and how much to underreport based on their true income and the IRS's audit policy. The latter consists of a probability of audit for each reported level of income. The primary analysis in that
paper seeks to characterize the form of taxpayers' equilibrium reporting rules and the IRS's equilibrium audit rule. That analysis is mathematically very complex and extremely technical, but under some plausible assumptions, it shows that, in one equilibrium, at least, within a given audit class, the audit rule is decreasing in the level of reported income and the reporting rule is such that all taxpayers underreport, but by an amount which is decreasing in true income.

As suggested previously, the effects of introducing a binding budget constraint on this model are quite different than in the simpler two-income model. If a binding budget constraint is introduced, the audit rate falls for each level of reported income and the extent of underreporting increases, but the basic qualitative properties of the model remain intact. The reason for this is that with a range of possible true incomes, taxpayers can choose a level of noncompliance, not just whether to comply. If the IRS is constrained to audit less often than it would like, compliance levels will fall, but not so drastically as in the two-income model.

5.5 Other IRS Objective Functions

As we discussed in Part 2 of this Article, introducing the law enforcement agency into models of tax compliance requires us to specify in formal terms both the actions available to the IRS and the nature of its preferences with respect to those actions. In our basic model, the actions available to the IRS are probabilities of audit, and we characterize their preferences with respect to different probabilities of audit in terms of expected total revenue (the aggregate of taxes collected plus penalties and fines) net of the costs of audit. This seems a natural starting point, but alternative candidates for IRS objective functions certainly exist.

To illustrate the impact of choosing a different IRS objective function, suppose, for example, that the IRS is interested in maximizing only the amount of understated income discovered by the audit process (again net of audit costs); that is, it does not take into account fines or taxes or any income reported by those taxpayers who are not audited. The expected benefit from auditing a low income report is now the sum of the probability of detecting noncompliance times the difference between high and low income, minus the cost of the audit.

When this new objective function is used, the equilibrium level of auditing is unchanged, but both individual and aggregate noncompliance under the new objective function increase or decrease as the difference between high and low income \((I_H - I_L)\) is less than or greater than the sum of the additional tax collected plus the fine \((T_H + F - T_L)\). In fact, there exists a critical level of the fine such that with the new objective function there will be less noncompliance for any fine less than the critical value and more noncompliance for any finer greater than the critical value. However, the qualitative properties of the new equilibrium will be the same as in our basic model, except that noncompliance now depends on the difference in income between the high-income and low-income
taxpayers rather than the difference in tax levels and is independent of the fine.\textsuperscript{54}

As this brief discussion of one alternative IRS objective function suggests, specifying the IRS’s decisionmaking rule may have a substantial effect on the predictions of an interactive model such as ours. By generally ignoring the law enforcement agency as an actor in the compliance process, this controversial task is avoided in the standard economics-of-crime literature.

We believe that the revenue-maximizing objective function used in our basic model adequately captures both the general and the specific deterrence objectives often attributed to IRS enforcement policy. Many anecdotal descriptions of IRS behavior -- such as identifying and counting the corporate jets at the Superbowl or Kentucky Derby or publicizing tax fraud indictments just before the April 15 tax return filing deadline -- may fit generally with such a revenue maximization objective function. It may well be the case that the IRS would place greater weight on maximizing tax collections than on fines, but we have not yet been able to agree on a weighing that we consider clearly superior to total revenue maximization. In any event, refining and defending descriptions of the objectives and behavior of the relevant law enforcement agency should be an important continuing aspect of tax compliance research, as well as of applications of interactive models such as ours to other law enforcement contexts.

6. CONCLUSION

This article has attempted to illustrate that the introduction of the law enforcement agency into a game-theoretic analysis of tax compliance offers considerable opportunity for insights and predictions that are simply not possible in the standard economic analysis of law enforcement. Relationships that have previously been ignored in the economic literature become clarified by a formal model that explicitly integrates taxpayers and the IRS. Considering the effects on both sides of many important elements of the tax compliance problem -- for example, of legal complexity, potentially declining ethics, and changes in the tax rate or its progressivity -- often implies very different conclusions than would emerge from looking only at the taxpayer side of the compliance problem. In addition, the way in which the law enforcement agency takes the relevant legal structure into account -- for example, how fines and penalties enter into its objective function -- demonstrates explicitly the role of agency behavior and preferences in addressing the compliance problem. Moreover, the game-theoretic approach described here makes it possible to consider the existence of and interactions among a variety of types of taxpayers, who may have differing attitudes or opportunities for noncompliance. In the tax compliance context at least, where the law enforcement agency is known to alter its behavior in light of taxpayers' reports, we believe our theoretical construct to be a major improvement.

The theoretical approach offered here also presents numerous
opportunities for further refinement and extensions. For example, variations in the flow of information between taxpayers and the law enforcement agency may have a significant impact on the results of our models. In addition, it should be possible to introduce important third parties into the analysis; in the tax compliance context, for example, the role of attorneys, accountants and tax return preparers merits exploration.55

Other legal contexts offer opportunities for further exploration of theoretical models along the lines we have described. Many regulatory contexts require initial reporting by the entity that is being monitored; some important examples include securities regulation, employment discrimination regulation, food and drug regulation and environmental regulation. In addition, the administration of many government expenditures, including public welfare and transfer programs, begin with the submission of a claim for relief. Thus, our approach might provide insights into the administration of programs such as those involving public welfare, unemployment compensation and disability insurance.

Moreover, the inclusion of the law enforcement agency as an interactive participant in a formal model of compliance suggests numerous possibilities for analyzing important legal institutions. In addition to government agencies, such as the IRS and the other regulatory and administrative bodies listed above, a variety of actors have significant functions, coupled with considerable discretion, in implementing the mandates of law; prosecutors and juries are but two important examples. Interactive law enforcement models, such as that offered here, should have considerable advantages in exploring the impact on predicted behavior of the differing goals and information of the members of such institutions.

Finally, it seems necessary to subject our approach to empirical testing.56 To date, the rather sparse empirical work on tax compliance considers the important variables on the law enforcement side, such as the probability of audit, to be exogenous, rather than endogenously determined based upon the level of noncompliance. To the extent that these empirical studies take into account IRS responses such as establishing a "Discriminent Index Function" for selecting returns for audit or determining the volume of notices sent to taxpayers, our theoretical work suggests that interrelationships (technically, a simultaneity problem) might affect empirical results.57
FOOTNOTES

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1. Henry (1983) provides a good summary and critique of the techniques used to derive these estimates.

2. Compliance Gap: Hearings before the Subcommittee on Oversight of the Committee on Finance, 97th Congress, 2d session (1982).


4. See, for example, Vitez (1983) at p. 191, who remarks that "the dramatic deterioration in compliance levels witnessed thus far, if not reversed quickly and forcefully, will gain further momentum and eventually erode, beyond repair, the integrity of our present income tax system."

5. Recent surveys of this literature can be found in Witte and Woodbury (1983) and Cowell (1985).


7. Witte and Woodbury (1983) have reviewed various extensions and refinements in the tax compliance context of Allingham and Sandmo's application of Becker's general model. (See, for example, Srinivasan, 1973, Yitzhaki, 1974, or Fishburn, 1981.) Some other authors working in the Becker framework have related the compliance problem to labor supply decisions, and have discussed such questions as the extent to which individuals might shift their labor supply from primary markets, where wage income is reported and taxed, to secondary or "underground" markets, where wage income is not reported. (For example, see Andersen, 1977; Pencavel, 1979; and Sandmo, 1981).

8. Although some empirical work based on Becker's model, in contexts other than tax law, has taken interrelationships between criminals and law enforcement agencies into account, the various theoretical extensions of his model have not (see Pyle, 1983, for an excellent summary of the empirical work). There have been a number of papers in the economics of crime literature which analyze the optimal penalty system using a utilitarian criterion (Becker, 1968; Stigler, 1970; Brown and Reynolds, 1973; Stern, 1975; Polinsky and Shavell, 1979). While these papers incorporate a kind of equilibrium approach, they are neither game theoretic nor do they permit the probability of detection of noncompliance to be sensitive to the actions of the taxpayers (in
particular, to reported levels of income). In one apparent exception to this, Hoeflich (1983) labels his analysis of the tax compliance problem "game-theoretic," but fails to use any game-theoretic methodology as commonly understood.

9. For a general discussion of this problem, see Klevorick (1985).

10. This approach is clearly endorsed by the empirical literature on criminal behavior in contexts other than tax compliance (see footnote 8) and amounts to much more than a minor "twist" on Becker's approach. A theoretical construct that takes account of the structure of the law enforcement process and its relationship to criminal behavior is necessary before normative issues properly can be addressed. It is useless to ask how changes in the level of punishment, for example, effect criminal behavior without taking into account the process by which that punishment is imposed. In fact, our simple model suggests -- in the tax compliance context at least -- that answers to these kinds of questions can depend crucially on the behavior of the law enforcement agency.

11. There have been multi-period analyses of the choice of a penalty system to minimize tax evasion subject to a budget constraint (Greenberg, 1983), to maximize net revenue (Landsberger and Meilijson, 1982), and to maximize an exogenously given social utility function (Rubinstein, 1979). These analyses have essentially treated the probability of audit as independent of a taxpayer's reported income, although the extent of noncompliance depends on detection probabilities. While detection probabilities are treated as uniform across taxpayers, some analysts have explored the optimal (uniform) probability of detection (see footnote 8). In some cases, the probability of audit is made contingent upon factors such as whether the taxpayer had been caught underreporting in the past (e.g., Landsberger and Meilijson, 1982; Greenberg, 1983; and Rubinstein, 1979).

12. In subsequent work, we expect to expand our systemic analysis to take into account the impact of third-party agents on the noncompliance problem. The advisory role of attorneys, accountants and tax return preparers, for example, cries for careful theoretical attention.

13. Recent estimates of legal sector noncompliance (and empirical studies of them) almost invariably involve extrapolations from the TCMP. The tax understatements that are detected by the random audits are then extrapolated to the broader universe of taxpayers generally. This is not an occasion for evaluating TCMP in detail, but measuring aggregate noncompliance is not its principal function. The TCMP is principally designed to establish and refine IRS audit selection mechanisms, a task for which it is well-suited, because TCMP provides direct evidence of how much tax understatement (or overstatement) can be detected by
careful audits. It is an excellent mechanism for assessing audit
effectiveness, for guiding the allocation of scarce IRS audit
resources and for predicting how many dollars of additional tax
revenues might be produced from additional dollars spent on tax
audits. Coincidentally, it produces the best available data for
estimating noncompliance, certainly better than is likely to
emerge from surveys. But extrapolations from TCMP data to
estimate aggregate noncompliance are necessarily subject to the
inherent limitations of TCMP. To the extent that taxpayers
selected for TCMP audits are able to conceal, for example,
unreported cash income from the TCMP auditor, extrapolated
estimates of noncompliance will be understated. That TCMP
generally misses nonfilers altogether further restricted its
validity as a mechanism for estimating noncompliance. See Graetz
and Wilde (1985).

14. Border and Sobel (1985) have also examined this problem. Related
principal-agent models have been analyzed in the accounting
literature. For a survey of the latter, see Demski and Krepe
(1982) and the references cited therein.

15. Given the equilibrium concept that we employ, taxpayers are
assumed to the audit strategy used by the IRS. See the text
accompanying footnote 19, infra.

16. The assumption that the IRS maximizes expected net revenue
(rather than some measure of social welfare) reflects our
decision to take a positive rather than normative approach. This
is not to say that government has no normative or
redistributational goals, only that these seem to us more properly
regarded as being embodied in the tax schedule itself or in
programs which make subsequent use of tax revenue. Our
interpretation seems consistent with the current use of expected
"yield" criteria for the selection of returns for audit (e.g.,
the TCMP program). See also Wertz (1979) who discusses the
appropriate IRS objective function in detail and cites several
administrators and observers of tax administration who believe
revenue maximization to be one of the primary goals of the IRS.

17. For example, Commissioner Egger has remarked:

There has always been some resistance in this country, from
colonial times onward, to virtually every form of taxation. As a general rule, with some exceptions, the resistance or
protest was episodic and geographically contained. The system
was never seriously threatened or weakened. From early times,
an de Toqueville observed, most Americans had an unusual
willingness to engage in voluntary activity for the public
good. It can be credited in part to the "frontier mentality"
which required cooperation for survival. That willingness
still exists in large part: most Americans do engage in the
spirit of voluntarism and most Americans do subscribe
voluntarily to and comply with the tax laws to which we are
all subject. . . . Unfortunately, a growing number of what
are otherwise honest citizens are becoming non-persons in the
tax system or are finding various ways to submerge parts of
their income, so as not to have it subject to taxation.
(Egger (1983) p. 5)
18. We believe that there exist at least two other types of taxpayers who should be considered in a complete model of tax noncompliance, but who we do not consider in the model set forth here. These we label: (1) the "coordinating compliers" -- individuals who would feel duped if they thought they were routinely complying with the tax laws while others were not and who therefore approach compliance decisions in a manner consistent with the way they think others are behaving; and (2) the "noncomplying criminals" -- individuals who are engaged in other illegal activities and whose decisions about tax compliance is related to ancillary exposure to other criminal penalties. Al Capone and Spiro Agnew are probably the two best known examples of this latter category. In principle, these distinctions among types of taxpayers could be made endogenous by adding various "costs," psychic or otherwise, into taxpayers utility functions, but there seems little gain in doing so. For further discussion of the implications of including a variety of categories of taxpayers, see below, Part 4.5.

19. As will become clear subsequently, relaxing those two assumptions -- that income can take only two levels and that the IRS is not budget-constrained -- increases dramatically the mathematical complexity of the analysis we are proposing here. Although these assumptions are obviously unrealistic when considering tax noncompliance generally, they may be adequate in evaluating tax enforcement within specific audit classes.

20. In a Nash Equilibrium, each agent's strategy is optimal, given the other agent's equilibrium strategy. In particular, this means that taxpayers know the audit strategy used by the IRS. For a further discussion of the reasonableness of this assumption see Graetz and Wilde (1985).

21. For discussion, see below, Part 4.4.

22. Our model of the interaction between taxpayers and the IRS is a standard two-state, two-action game. P'ng (1983) and Salant and Rest (1982) have used this type of model to analyze the litigation of settlement demands in civil torts cases. Subsequently, Salant (1984) has generalized the analysis to include an interval of possible settlement demands.

23. We implicitly assume taxpayers can only submit reports of $I_L$ or $I_H$ even though, in principle, other reports are feasible. One way to rationalize this assumption, which greatly simplifies the analysis, is to assume taxpayers must report their income on a standard form which only allows them the option of indicating low or high income. At a less general level, the model may also be thought of as dealing with a particular deduction or tax credit, which the taxpayer has only the option of taking or not claiming.

24. Both taxes and the general structure of penalties are fixed by the legislative branch, although the IRS has some control over the choice of penalty (e.g., civil versus criminal). We ignore
the latter distinction in this analysis.

25. Each taxpayer’s income is a random variable which takes the high value with probability $q$ and the low value with the complementary probability $1 - q$. It is important to view $q$ as the ex ante probability that a randomly drawn taxpayer has high income, rather than the realized frequency of high income taxpayers. The IRS is assumed to be unable to observe the realized distribution of income, as well as the income of any individual taxpayer. Under this interpretation of $q$, the realized distribution of reports provides no further information about the veracity of a given report beyond that contained in the report itself.

26. In formal terms, we assume $u(\cdot)$ is twice-differentiable with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

27. This property is not an artifact of any unusual assumptions used in our model but follows directly from the assumption that the IRS maximizes expected revenue. It is a standard feature of the von Neumann-Morgenstern expected utility hypothesis that expected utility is linear in probability. In this case, the returns to auditing will be linear in the probability of audit.

28. The notation $\hat{p}(a) \in [0,1]$ means the IRS is indifferent to any probability of audit between 0 and 1.

29. As with auditing, that the marginal benefits of noncompliance are linear in the probability of noncompliance is a standard consequence of the assumption that taxpayers maximize expected utility.

30. The notation $\hat{q}(a) \in [0,1]$ means the taxpayer is indifferent to any probability of noncompliance between 0 and 1.

31. The knife-edge case of $a = 1$ has a continuum of equilibria. In this situation, all strategic taxpayers underreport, and the IRS is indifferent regarding the probability with which it audits taxpayers who report low income.

32. As footnote 7, above indicates, some authors have extended the Allingham and Sandmo model to include the labor/leisure choice of taxpayers in addition to the choice of how much to underreport income (Andersen, 1977; Pencavel, 1979). Others have introduced a distinction between “regular” and “irregular” labor markets in which income from the former is automatically reported (Isachsen and Strom, 1980; Sandmo, 1981). In these latter models the taxpayer must decide how much labor to allocate to each market as well as how much irregular income to report.

33. The probability that a randomly drawn taxpayer will be a noncomplier is given by the product of the probability of being a strategic taxpayer ($\rho$) times the probability of having high income ($q$) times the probability of not complying ($a$). Hence
\( P_N = qP \). The probability that a randomly observed report will be audited is given by the product of the probability that the report will be low income \((qP + 1 - q)\) times the probability that it will be audited \((\bar{\beta})\). Hence \( P_A = \bar{\beta}(qP + 1 - q)\).

34. See above, Part 2.

35. That increases in the tax rate might increase compliance regardless of the risk preferences of taxpayers is an important result. It illustrates dramatically the importance of an interactive model, and it also demonstrates the need for formal models generally. It is a fairly obvious observation that an increase in the tax rate should increase both the incentives to underreport and the incentives to audit. But intuition alone cannot determine the ultimate effects of these conflicting forces. A formal model often can, and we use the term “dominate” to refer to the incentive which ultimately prevails in the new equilibrium which results from a parametric change.

36. As noted in Table 1, if taxpayers are indifferent to risk, an increase in the tax on high income will increase the probability that a given low income report will be audited. Whether aggregate auditing increases depends on the size of the fine relative to the audit cost; that is, aggregate auditing increases or decreases as the fine is greater than or less than the audit cost.

37. Unlike increases in the tax on high income, an increase in the tax on low income yields an unambiguous prediction regarding the probability that a given low income report will be audited: it decreases. The effect on aggregate audits again depends on the size of the fine relative to the audit cost; that is, aggregate auditing increases or decreases as the fine is less than or greater than the audit cost.

38. See above, Part 2.

39. Expected revenue net of audit costs is

\[
R = (1 - q)[\bar{\beta}(T_L - c) + (1 - \bar{\beta})T_L] + q[(1 - \bar{\beta})T_H + \rho(1 - \bar{\beta})T_H] + qP\bar{\beta}(\bar{\beta}(T_H + F - c) + (1 - \bar{\beta})T_L).
\]

Using the definitions of \( P_N \) and \( P_A \), this can be rewritten as

\[
R = ((1 - q) + P_H)T_L - P_A^c - P_H(T_H - \bar{\beta}(T_H + F - T_L)).
\]

Since \( P_N \), \( P_A \), and \( \bar{\beta} \) are all independent of \( \rho \), so is expected revenue net of audit costs.

40. In order to guarantee that the probability of noncompliance for a strategic taxpayer with high income is less than one \((\alpha < 1)\) we require \((1 - q)\rho / pq[T_H + F - T_L - c] < 1\) or \(\alpha > (1 - q)\rho / q[T_H + F - T_L - c]\). If \( \rho \) falls below this level the equilibrium involves no auditing and no compliance by strategic taxpayers. In this case increases in the proportion of strategic taxpayers effect nothing except expected revenue, which falls.
41. While this conclusion is indeed provocative, we are not suggesting that it be used as the basis for policy decisions. The model we present in this paper is, as we have emphasized, designed to illustrate the kind of results one can get from an interactive model. We view it as an important first step, but no more than that, and caution should therefore be used in applying the results. The result that an increase in the proportion of strategic noncompliers has no effect on aggregate noncompliance or aggregate revenues may, for example, not be true where such an increase has the effect of changing a nonbinding budget constraint into a binding one, even if the results described in the text would hold in cases where either there was no binding budget constraint or where the budget constraint was already binding. A model with a binding budget constraint is discussed below, Part 5.3.

42. Allingham and Sandmo (1972) found that when the fine is proportional to unreported income (that is, \( F = \pi(I_H - I_L) \)), an increase in the tax rate has a both an income and a substitution effect. Since the substitution effect is negative, while the income effect is positive (negative) if absolute risk aversion is decreasing (increasing), the net effect of an increase in the tax rate is ambiguous in the (presumed most likely) case of decreasing absolute risk aversion. Yitzhaki (1974) has noted, however, that penalties for evasion are most often proportional to evaded tax, not unreported income, so that the income effect is spurious. Instead he finds that if absolute risk aversion is a decreasing function of income, then an increase in the tax rate unambiguously enhances compliance. If absolute risk aversion is increasing with income, then Yitzhaki’s result also is ambiguous.

43. In the case of risk-neutral taxpayers, \( \bar{\pi} = 1/(1 + \pi) \), which is independent of both tax rates and the income differential, while \( P_A \) decreases with an increase in either of these parameters.

44. Taxpayer audit costs may include fees paid to attorneys, accountants, and other third-party agents involved in the revenue collection process. These agents are outside our model so the fact that these costs are a transfer rather than a pure deadweight loss is irrelevant. Revenue will generally not be increased by such transfers because amounts that will be included in the recipients’ taxable incomes are typically deductible to the payor (under Internal Revenue Code Section 212(3)). We ignore any revenue effects that might result because the payor does not itemize deductions or because payors and recipients are in different marginal tax brackets.

45. That is \( \bar{\alpha} = c(1 - q)/(T_H - T_L + F - c) \).

46. The IRS’s equilibrium audit probability for low income reports becomes:
\[ \bar{\beta} = \frac{u(I_H - T_L) - u(I_H - T_H)}{u(I_H - T_L) - u(I_H - T_H - F - k)} \]

47. For an exception see Nagin (1978) who points out that the observed negative relationship between index crime rates and sanction levels may be explained by a constraint on total prison populations.

48. The next section will discuss these results in more detail.

49. See Wertz (1979) for a model which illustrates this point.

50. In formal terms we can express these preferences as

\[ \Pi(a, \beta) = \beta[\mu(I_L - I_H) + (1 - \mu)0 - c] = \beta[\mu(I_H - I_L) - c] \]

where the variables are the same as in Part 3. In particular, \( \beta \) is the probability of audit given a report of low income, and \( \mu \) is the probability that a taxpayer who reports low income is actually a strategic taxpayer with high income who has underreported.

51. Audits generated by the Taxpayer Compliance Measurement Program (TCMP) are apparently based on the amount of additional tax revenue that is likely to be generated by an audit, not on fines or the tax revenue received from those taxpayers who are not audited. Our alternative objective function is based only on the amount of income that is likely to be discovered and thus emphasizes accuracy over revenue (see Wertz, 1979). If the IRS acted so as to maximize the total expected income known to the government net of audit costs, it would use the objective function

\[ \Pi(a, \beta) = \beta[\mu(I_H + (1 - \mu)I_L - c] + (1 - \beta)I_L. \]

This would yield precisely the same audit and noncompliance rates as the alternative objective function discussed in the text and specified in footnote 50, supra.

52. The probability of noncompliance by strategic taxpayers with high income is now

\[ \bar{\alpha} = \frac{(1 - q)c}{pq(I_H - I_L - c)}. \]

The textual assertion may be tested by comparing this expression for the equilibrium probability of audit given a report of low income to the analogous expression in our basic model.

53. The critical value of the fine is defined by

\[ F^* = I_H - I_L - T_H + T_L. \]

Recall that we assume \( F \leq I_H - T_H \), that is, the IRS cannot take more income from noncompliers than they have. But \( F^* \leq I_H - T_H \) if and only if \( T_L \leq I_L \), which we also assume. In other words, there always exist feasible levels of the fine both above and below the critical value.

54. If the IRS ignores tax rates and fines in determining its audit policy, those factors, at least in our simple model, will have no effect on the probability of compliance. This is because taxpayers behave in a way that responds to IRS audit incentives,
in equilibrium generating neither too much or too little auditing. Tax rates and fines, however, may still affect the equilibrium level of auditing because auditing responds indirectly to strategic taxpayers' incentives to underreport, and these incentives will still be affected by tax rates and fines.

55. For further discussion, see Graetz and Wilde (1985).

56. For a discussion of the existing empirical literature on tax compliance see Graetz and Wilde (1985).

57. Levels of noncompliance depend on a number of factors, including, for example, the likelihood of audit. At the same time, the likelihood of audit depends on the level of noncompliance. Thus it is inappropriate to use the likelihood of audit as an exogenous variable to estimate an equation meant to explain the causes of noncompliance. Technically, the results in such a case are said to be subject to "simultaneity bias." As an empirical matter this problem can be overcome by including an additional equation which explains the probability of audit as a function of levels of noncompliance and other factors such as IRS resources. This, however, has not yet been done. An example of the potential problem with simultaneity bias is provided by Witte and Woodbury (1985). These authors, working with 1969 TCMP data, include audit rates and sanction levels as explanatory variables in an empirical analysis of compliance, but fail to deal with the simultaneity problem. They, in fact, find that for some taxpayer classes the probability of a civil fraud penalty is negatively related to voluntary compliance. This may be explained by simultaneity bias, but may also be explained by the failure of audit rates to accurately proxy the likelihood of detection.
REFERENCES


Rubinstein, Ariel, "Offenses that may have been Committed by Accident -- A Policy of Retribution," in J. Brams, A. Shotter and G. Schwodiauer, eds., Applied Game Theory (Wurzburg: Physical-Verlang, 1979), 406-413.


Table 1
Direction of change in the endogenous variables (row entries) given an increase in the exogenous parameters (column entries): the basic model. No change is denoted by a zero.

<table>
<thead>
<tr>
<th>Endogenous Variable</th>
<th>Exogenous Parameter to be Increased</th>
<th>Fine (F)</th>
<th>Cost of Audit (c)</th>
<th>Probability of High Income (q)</th>
<th>Tax on High Income (T_H)</th>
<th>Tax on Low Income (T_L)</th>
<th>Percentage of Strategic Taxpayers (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual probability of noncompliance (a)</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Aggregate probability of noncompliance (p_H)</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Individual probability of audit (q)</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>*</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Aggregate probability of audit (p_L)</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>**</td>
<td>**</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

* ambiguous in general; when taxpayers are risk-neutral, this entry is +.

** ambiguous in general; when taxpayers are risk-neutral, \( \frac{\partial P_A}{\partial T_H} > 0 \) if \( F > 0 \) as \( F \geq c \) (\( c = 0 \)).

Table 2
Direction of change in the endogenous variables (row entries) given an increase in the exogenous parameters (column entries): proportional tax and fines.

<table>
<thead>
<tr>
<th>Endogenous Variable</th>
<th>Exogenous Parameter to be Increased</th>
<th>Penalty Rate (( r ))</th>
<th>Tax Rate (( t ))</th>
<th>Income Differential (( I_H - I_L ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual probability of noncompliance (a)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Aggregate probability of noncompliance (p_H)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Individual probability of audit (q)</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Aggregate probability of audit (p_L)</td>
<td>-</td>
<td>**</td>
<td>**</td>
<td></td>
</tr>
</tbody>
</table>

* ambiguous in general; when taxpayers are risk-neutral, these entries are 0.

** ambiguous in general; when taxpayers are risk-neutral, these entries are -.
Nature determines income

Taxpayer reports

IRS audits or not

Figure 1
Figure 2: Best response functions in the basic model when $\alpha \geq 1$.

Figure 3: Best response functions in the basic model when $\alpha < 1$. 