ON THE MEANING OF THE PREPONDERANCE TEST IN JUDICIAL REGULATION OF CHEMICAL HAZARD

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Law and Contemporary Problems forthcoming

SOCIAL SCIENCE WORKING PAPER 515
February 1984
ABSTRACT

As usually defined, the preponderance test is a standard of proof which directs the jury to accept the plaintiff’s version of the disputed facts if they are more probably true than not. But what happens when the most important disputed "facts" are judgments about probability? This paper offers an interpretation of the preponderance test which can be applied to this situation.

In the example of the paper, B is the benefit of a drug, C is the health cost if it is a teratogen, and p is the probability of teratogenicity. The contested "fact" is the magnitude of p, the probability of harm. In the interpretation considered by the paper, the jury finds in favor of the plaintiff if the jury decides that it is more likely than not that p is greater than B/C. This definition of the preponderance test does not quite minimize expected costs, and compared with expected cost minimization it is likely to be biased toward under-protection when the health costs are high compared with the benefits. But when the mean and median of the second order probability of p are the same, the definition coincides with expected cost minimization. It is also shown that under a criterion of expected cost minimization, contrary to Posner, judicial error costs are not in general the same and the number of erroneous judgments favoring undeserving plaintiffs is not likely to be the same as the number of erroneous judgments favoring undeserving defendants.

I. Introduction

In his Economic Analysis of Law (2) Posner suggests that the criterion of economic efficiency provides a basis for burden of proof rules, and in particular a basis for the preponderance test. This paper also examines the preponderance test in light of the criterion of economic efficiency, but with results that differ from Posner’s in some ways. The example and focus, in this paper, is on the appropriate control of chemical hazards; or more precisely the design of burden of proof rules with appropriate incentives, stemming from past decisions, for future precautionary controls in similar cases. The analysis is sufficiently general to apply to other rules besides the preponderance test, much as clear and convincing evidence, and beyond a reasonable doubt, and, of course, to apply to other problems besides chemicals.

For the paper, we assume that general regulatory frameworks -- judicial, legislative, or administrative -- are in place, and that their chief purpose is to maximize the social benefits, or minimize the social costs, of the substance or activity under consideration as a candidate for regulation. In any given case, cost-minimization might dictate doing nothing—not finding liability or leaving (say) a chemical unregulated because its benefits exceed its costs, and what

(1) This research was supported by the National Science Foundation and the Mellon Foundation. The paper owes much to Jim Krier, who generously contributed much of the first two sections.
(2) R. Posner, Economic Analysis of Law 432 (2nd ed.)
costs there are could only be avoided at undue expense; it might call for finding liability or for regulating the chemical in some way and to some degree, because a carefully designed regulation will yield benefits net of regulatory costs; it might command that the chemical be banned, because it has no benefits, or because its benefits are smaller than adverse effects that could be avoided only by foregoing use of the chemical.

There is no great difficulty in choosing among these alternatives if all relevant costs and benefits are known; there is considerable difficulty, however, if they are not. And in the case of chemical hazards and related risks, benefits and especially costs are typically uncertain. For example, we might be pretty sure of the direct costs of manufacturing and of controlling a given chemical, of the good things the chemical can do for us, and of the probability that it will do those things. On the other hand, we will commonly be unsure of the range of possible adverse effects the chemical might entail, and of the probability that any of those effects will in fact occur. Hence subjective judgments about uncertainty are necessary, and our chief concern here is the rules—of burden of proof—by which they should be made. We explore some implications of a simple criterion—that burden of proof rules should be designed to minimize the expected costs of the decision process. We shall focus most closely on the costs of error, the undesirable consequences that result when regulatory objectives fail to be met. If error costs are to be kept to a minimum, it follows that the standard of proof must vary with the hazards in question.

While this is not a novel proposition, its implications for a wide range of contemporary problems—of which synthetic chemicals are a good example—have received little attention.

II. The Conventional and Economic Wisdom

We shall begin by considering judicial regulation of hazards through such substantive common law rules as strict liability, negligence, and nuisance. We start with the courts and the common law not because these are the best institutions for controlling the problems we have in mind—probably they are not—but because it is in judicial common law decisions (and the literature about them) that one finds the most explicit and canonical recognition of various burden of proof rules.

Suppose first a substantive standard of strict liability on the part of producers of a certain chemical for any injuries the chemical happens to cause. For our purposes, strict liability means liability whether or not the producer is careless. In other words, the fact that measures to reduce the hazards of the chemical (including foregoing its production) would cost more than the increased safety resulting from the measures is irrelevant; if the chemical did in fact cause the injury, its producer is accountable.

Turn now from a strict liability regime to a regime based on negligence (or on related substantive doctrines, such as nuisance, that (3) Irrelevant for the particular case, but under an economic efficiency approach these costs are relevant to the decision whether to impose a strict liability or a negligence rule in similar cases.
incorporate a standard of reasonableness). On one view, an economic one, negligence, nuisance, and other rules of reason are designed to promote cost-minimization. Judge Learned Hand’s definition of negligence, set out in United States v. Carroll Towing Co., illustrates the proposition: negligence exists where the cost of an accident, multiplied by the probability of the accident occurring, exceeds the cost of measures the injurer could have taken to avoid the accident. This version of the negligence standard aims to minimize costs by deterring activities in all those cases (but only those cases) where the activities give rise to expected costs larger than the costs of avoidance.

Just as we said in our earlier discussion that there is no great difficulty in choosing an efficient decision if all relevant costs and benefits are known, so too there is no great difficulty in applying a strict liability or negligence or nuisance standard if one knows the relevant facts. In the case of strict liability, this means knowing that an injury occurred, knowing that the defendant’s activity caused it, and knowing what damage resulted. In the case of negligence, it means knowing, in addition, the defendant’s avoidance cost, the cost of injuries of the sort threatened by the defendant’s activity, and the probability that the threat will be realized.

Typically, of course, some or all of these elements are contested (uncertain), such that the trier of fact must, based on the evidence presented, make a guess as to the true state of affairs. But these guesses are not made in the abstract; burden of proof standards establish and allocate the risk of uncertainty by determining who wins if the factfinder is undecided, and by determining what "undecided" means (a judge may find evidence convincing if the applicable standard of conviction is low, but unconvincing if it is high). In ordinary civil cases like those we have been discussing, the plaintiff bears the burden of persuasion (that is, the risk of indecision is allocated to him), and carries it only if he can convince the factfinder that his version of the facts is supported by a preponderance of evidence. This is generally taken to mean that the plaintiff must show that the assertions on which his case turns are more likely true than not.

The traditional interpretation of this standard in a strict liability regime is that the plaintiff must convince the court that the probability that the defendant’s activity caused the plaintiff’s injury, and to the extent of SX, is more than 50 percent. If it does not, the preponderance test implies that the plaintiff loses. The negligence case is more complicated, partly because one has to deal with probabilities of probabilities. According to the Hand formula, negligence liability turns on a comparison of the defendant’s avoidance cost to the expected cost of an accident. To make out a case, then, the plaintiff must in principle prove not only all the "facts" required under strict liability (identity of the injurer, cause and effect relationship, extent of injury), but also the additional "facts" regarding defendant’s avoidance cost, the cost of an accident of the sort threatened, and the probability that such an accident would result from the defendant’s operations. The preponderance test requires that

(4) 159 F 2d 169 (2d Cir 1947)
the plaintiff's evidence must convince the factfinder that the expected accident cost is sufficiently high, relative to the defendant's avoidance cost, to establish negligence. This does not mean that the probability of an accident must exceed 50 percent. Suppose, for example, that the plaintiff proves, by a preponderance of the evidence, that the defendant engaged in an operation that injured the plaintiff, that defendant's avoidance cost was $1000, and that the gravity of the threatened harm was $5000. To win his case (contributory negligence aside), the plaintiff must also convince the judge that the probability of injury exceeded 20 percent without the defendant's precautionary actions and so the expected accident cost would exceed the defendant's avoidance cost. One possible interpretation of the preponderance test is that "convince" here means to show that a greater than 20 percent probability is more likely true than not, and the plaintiff wins if the factfinder concludes from the evidence that the probability that the 20 percent figure is "true" or "accurate" or a "fact" exceeds 50 percent.

We take the foregoing to represent the conventional wisdom--to the extent there is agreement on a conventional view at all--regarding the burden of proof (more particularly, the burden of persuasion) in ordinary civil litigation. Thus, a standard treatise on evidence states that in such cases--a suit in tort for personal injuries, say--"the party who has the burden of pleading a fact will have the burdens of producing evidence and of persuading the jury of its existence as well."(5) The party carries the burden of persuasion of a fact by proving it "by a preponderance of evidence," which by its "most acceptable meaning...seems to be proof which leads the jury to find that the existence of the contested fact is more probable than its nonexistence."(6) On this conventional view, then, "a lawsuit is essentially a search for probabilities. A margin of error must be anticipated in any such search. Mistakes will be made and in a civil case a mistaken judgment for the plaintiff is no worse than a mistaken judgment for the defendant."(7)

There is a marked congruity between the foregoing and Posner's view regarding burden of proof. In his treatise Economic Analysis of Law, Posner considers

\[(A)\] the preponderance-of-the-evidence standard that governs civil cases. This standard directs the trier of facts to find in favor of the party who has the burden of proof on an issue if that party's version of the disputed facts is more probably true than the other party's version; thus, to prevail, the party bearing the burden of proof need only establish the validity of his claim by a probability infinitesimally greater than 50 percent. This implies that of cases decided erroneously, about half will be won by undeserving plaintiffs and about half lost by deserving plaintiffs. Whether this result is economically sound depends on whether the costs of each type of error--in favor of

\[(B)\]\[(C)\]\[(D)\]


(6) Id. 339 at 793, 794. This definition of the preponderance test will be referred to as "point A" below.

(7) Id. 341 at 798. The claim that the costs of the two errors are the same will be referred to as "point C" below.
undeserving plaintiffs, and against deserving plaintiffs—are about the same; and in general it would seem that they are the same. (8)

As the last line of this quotation suggests, the preponderance test is justified, on the economic view, as a cost-minimizing measure (with regard to the costs of error in particular); Posner makes this point slightly more explicit in another work of his. (9) Surprisingly, perhaps, conventional writers appear to agree with this view, at least in part; the language of "cost-minimization" might be foreign to them, but the idea is reflected in McCormick's observation, quoted above, that "in a civil case a mistaken judgment for the plaintiff is no worse than a mistaken judgment for the defendant." By the presumption of equal error costs in either direction, the correct approach in terms of minimizing those costs appears to be a standard of proof favoring neither side while still serving the necessary function of resolving cases where the mind of the factfinder is in equipoise. The preponderance test is precisely such a standard.

Why should cases be resolved in favor of the defendant when the factfinder is in equipoise? Here it is difficult to judge the degree of agreement between the economic and the conventional views. Posner's answer is straightforwardly concerned with efficiency: "since no allocative purpose would be served by shifting a loss in a case where the defendant's liability was indeterminate, the rule [placing the risk of nonpersuasion of the plaintiff] economizes on litigation expenditures." (10) Or, as he puts it elsewhere, "an error in plaintiff's favor involves a cost not incurred when the error goes the other way—the cost of actually collecting a legal judgment." (11)

Perhaps conventional analysts like McCormick mean much the same thing as this when they say that the "burdens of pleading and proof with regard to most facts have been and should be assigned to the plaintiff who generally seeks to change the present state of affairs and who therefore naturally should be expected to bear the risk of failure of proof or persuasion." (12) Other factors conventionally mentioned as bearing on allocation of the risk of nonpersuasion are obscure "policies" of various sorts—handicapping disfavored contentions, convenience in following the natural order of storytelling, consideration of which party has readier access to knowledge about the fact in question, consideration of the extent to which a party's contention departs from what would be expected in the course of ordinary experience (the more unusual your claim, the more likely you will have to prove it), and, finally, a catchall of "fairness." (13) Each of these factors could be construed as consistent with the economic view, or inconsistent. Their obscurity defies generalization along these lines, as does the fact that none of the

(8) R. Posner, Economic Analysis of Law 432 (2d ed.)
(10) Id at 408.
(11) R. Posner, supra note 8 at 433.
(12) McCormick, supra note 2, 337 at 786 (emphasis added).
(13) Id. at 786-89.
factors governs regularly. James and Hazard conclude that "[t]here is no satisfactory test for allocating the burden of proof . . . ." (14) Posner (and we along with him) would argue that there is—if the objective is cost-minimization.

III. Points at Issue

It appears to us that there are several confusions in the conventional wisdom concerning the preponderance test and expected cost minimization. We begin by setting out four points. The first is not in itself a confusion, but leads to a problem in interpretation. We believe, however, that the next three contain erroneous statements. Further, we believe that disentangling the latter three confusions will suggest a new, and more economically sound, interpretation of the preponderance test.

First, (Point A) both McCormick and Posner provide a fairly clear definition of the preponderance standard itself. In the reference marked (A) Posner states that the preponderance test "directs the trier of facts to find in favor of the party who has the burden of proof on an issue if that party's version of the disputed facts is more probably true than the other party's version." (15) However, it is not altogether clear what this definition means when the "fact" is a disputed probability. The difficulty here is not so much in the definition but in its application to probabilities of probabilities.

Second, (Point B) Posner states "that of the cases decided erroneously, about half will be won by undeserving plaintiffs and about half lost by deserving plaintiffs."

Third, (Point C) both McCormick (note 7) and Posner state that the costs of the two types of error are about the same.

Fourth, (Point D) Posner states that a necessary condition for the preponderance test to be economically sound (we take this to mean minimizing expected cost), the costs of the two types of error must be about the same.

In our argument that the last three statements are erroneous, we shall develop our own interpretation of the preponderance test, one based on the criterion of expected cost minimization. In doing so, we do not wish to suggest that this interpretation, or other standards of proof, can (or should!) be applied as precisely or elegantly in practice as it can be discussed in principle. As James and Hazard observe in a slightly different context, burden of proof tests "formulated in terms that suggest objective, scientific precision" are in fact not really precise or confining at all. (16) Still, the different standards regarding the required degree of conviction of the part of a decisionmaker "probably suffice" to create "a frame of mind to resist persuasion" to a roughly appropriate extent, "and it is doubtful whether anything more can be done where a difference in degree is sought in dealing with factors so subjective and imponderable." (17)

(14) F. James and G. Hazard, Civil Procedure 7.8 at 249.

(15) McCormick, cited on note 6, says much the same thing. See RAJI, also Oregon

(16) Id. 7.11 at 273-73.

(17) Id. 7.6 at 245.
IV. Social Costs

The key to our argument lies in the distinction between the private costs in a single negligence case and the social costs associated with a particular standard of proof. The distinction is easiest to draw in terms of a specific example.

Suppose a woman takes an anti-nausea drug during pregnancy, and her child is born without arms. She believes that the drug was teratogenic and sues the drug manufacturer for $1 million. To make matters simple, let us assume that the defendant stipulates that $1 million is a good estimate of the pain and suffering, lost income, etc., associated with the teratogenic effect, but argues that its drug was not the cause of the effect.

If the plaintiff is deserving, but liability is not found, the private cost of this judicial error is $1 million to the plaintiff. If the plaintiff is undeserving, but liability is found, the private cost of this judicial error is $1 million to the defendant. Thus it would seem that the costs of the two types of error are the same as affirmed by McCormick and Posner.

However, these error costs have to do with a private transfer, which may or may not occur between the defendant and the plaintiff. They relate to a decision already made (the decision to produce and market the drug) and a harm already sustained. As such these error costs have little to do with economic efficiency or cost minimization. Economic efficiency focuses upon the consequences of future decisions which are affected by the precedent in today’s particular case.

Suppose that liability is found in this particular case. Then there will be an incentive for manufacturers to not produce or restrict drugs in future but otherwise similar situations, where the evidence of toxicity is equally strong and the costs and benefits are about the same as they are in the case at hand. If liability is found for the particular case at hand, then society is likely to forego the benefits of drugs with similar benefits and risks in the future.

Suppose that liability is not found in this particular case. Then there will be an incentive for manufacturers to produce drugs in future but otherwise similar situations, where the evidence of toxicity is equally strong and the costs and benefits are about the same as they are in the case at hand. If liability is not found for the particular case at hand, then society is likely to bear the risks of drugs with similar benefits and risks in the future.

In our particular case we have so far identified a single victim with a cost of $1 million, the cost which may or may not have been caused by the drug. This $1 million, is, however, only part of the relevant cost comparison for cost minimization. There may be 1000 or 10,000 other victims and this number is also relevant for a decision which attempts to minimize social costs. To be concrete, let us suppose that in the particular example, and in similar cases, there are an estimated 1000 victims, each suffering $1 million in damages, for a total cost of $1 billion. We will call this total cost C.

On the other side of the ledger, the chemical conveys benefits in the form of relief from nausea. Let us suppose that 10 million women take the drug, and the relief from nausea is valued at $30 over
and above the manufacturing, distribution, and retailing costs of the
drug. (18) The total benefit — net production cost — is $30 times 10
million or $300 million. We will call this total net social benefit B.
This $300 million is also what Hand called the avoidance cost.
If liability is found in this case, then in similar ones, manufacturers
can avoid the risk of teratogenicity by not manufacturing the similarly
suspected drugs. The cost of such avoidance is net social benefit
foregone — the $300 million. The costs of manufacturing, distribution
and retailing are transfers which are netted out in the analysis of net
social cost minimization.

We are now ready to consider the expected cost minimizing
standard of proof. Suppose that the court weighs the evidence brought
forward by both plaintiff and defendant and concludes that the
probability of teratogenicity is p. If the court finds liability, it
knows that in this case and in similar cases drugs will not be
manufactured. For each such situation there will be no benefits, but
at the same time no toxic effects of the drugs. With the zero net
benefit from non-manufacture as the benchmark, we can compare the
effects of the other possible decision, a finding of non-liability.
With a finding of non-liability in this case, in similar situations
drugs will be manufactured. For such cases the benefit is B and the
expected cost is pC. When pC is greater than B there are net expected
costs of manufacturing and society will minimize future costs by future
precautionary decisions of non-manufacture. This is achieved by
sending the signal of a finding of liability in the case at hand. When
pC is less than B there are net expected benefits for the drugs and,
compared with the zero benefit benchmark of non-manufacture, society
will do better with the non-precautionary decision encouraging
manufacture. This is achieved by sending the signal of finding no
liability in the case at hand.

Note that pC is greater than B just when p is greater than B/C.
Thus our cost-minimizing standard of proof tells us to find liability
when the court decides that the probability of teratogenicity is
greater than B/C and to find no liability when the probability is less
than B/C. This standard of proof is the same as that proposed by
Learned Hand, with the language recast in terms of toxic torts:
negligence exists where the cost of the toxicity multiplied by the
probability that the chemical is toxic, exceeds the benefits of the
chemical (or the avoidance costs of the manufacturer).

For the particular case at hand B is $300,000,000 and C is
$1,000,000,000. Thus B/C is 0.3 and the cost minimizing standard of
proof is to find liability if the court concludes that the probability
of teratogenicity is greater than 0.3. Let us restate the matter --
the costs being minimized are not those of the present case at hand.
For this case, the decision to manufacture is already taken and the
injury (whether or not caused by the drug) already sustained. The
costs to be minimized are those associated with future decisions of
whether or not to manufacture drugs with similar benefits and similar

(18) It might cost a woman $20 for the prescription, but she ob­
tains nausea relief which she values at $50. This $30 of net
benefit is also valued as the benefit over and above the next
best alternative treatment for the nausea.
suspicion of toxic risk. Note that "similar" need not be restricted to situations where B is $300 million and C is $1 billion. The standard of proof of 0.3 applies for any situation where the ratio of B to C is 0.3. In other words the precedent of the 0.3 standard of proof in this case should apply to all future cases where the ratio of B to C is 0.3.

The error costs, from the social point of view, are identified as follows. If liability is found when in fact the chemical is non-toxic, the chemical is erroneously withdrawn from the market and its benefits erroneously foregone. We will call this type of judicial error a false positive. The cost of this false positive is B, or $300 million in the example. If liability is not found when it is appropriate, and the chemical is toxic, the chemical is erroneously left on the market. (19) The toxicity costs are borne, but its benefits are obtained as well. We will call this type of judicial error a false negative. The cost of this false negative is C-B, or $700 million in the example.

As can be seen by the example, when the relevant costs of error are identified — the social costs in similar problems — the costs of the two types of error are not in general equal. They are only equal when C happens to be twice B, a circumstance which must be viewed as exceptional. This is our argument against Point C.

(19) Note that if B is greater than C the cost minimizing decision is not to find liability even if it is known for sure that the drug is teratogenic. Thus it is possible for it to be appropriate not to find liability for a teratogen and for there to be no judicial error.

It is important to observe that minimizing expected costs of future decisions which are affected by the precedent of the case at hand comes to the same thing as minimizing the expected error costs in the case at hand. Recall that the cost of a false positive is B and the cost of a false negative is C-B, and the court's evaluation of the probability of toxicity is p. If the court finds liability, it risks the possible error of a false positive. Since the court's evaluation of the probability of teratogenicity is p, its probability of non-teratogenicity is (1-p), and if the court finds liability the expected cost of a false positive is (1-p)B. If the court does not find liability, it risks the possible error of a false negative, and the expected cost of this error is (p)(C-B). To minimize the expected cost of error the court should find liability when the expected cost of a false positive is larger than the expected cost of a false positive (when (p)(C-B) > (1-p)(B)); and choose to find no liability when the expected cost of a false positive is greater than the expected cost of a false negative. But (p)(C-B) > (1-p)(B) exactly when p > B/C, so the two criteria boil down to the same thing.

We return to our cost minimizing standard of proof: find liability when the court finds the probability of teratogenicity to be greater than B/C. In the exceptional circumstance when the social error costs are equal, we have C - B = B or C = 2B, and the cost minimizing standard becomes: find liability when the court finds the probability of teratogenicity to be greater than B/2B or 1/2. This of course is the preponderance test applied to two conflicting factual claims — the plaintiff claiming that the drug is a teratogen and the
defendant claiming that the drug is not a teratogen. If the only contested fact is whether or not the drug is a teratogen, and the court decides that the plaintiff's version of the fact is more likely true than not, the court's standard of proof is $p > 1/2$.

It is not altogether clear why Posner and McCormick concluded that in general the error costs are equal, nor is it clear why Posner believed that the error costs must be equal for the preponderance test to be "economically sound." However, the above analysis suggests a possible explanation for these views. It is easy to confuse the relevant social costs of errors with the private costs of transfers between defendant and plaintiff. Since the private error costs are equal (both $1 million in the example), this confusion explains the assertion that the relevant error costs -- the social error costs -- are equal. Once it is believed that the error costs are equal, it is easy to interpret the preponderance test as saying "find liability when the court concludes that it is more likely than not that the chemical is toxic ($p > B/C$ or $p > 1/2$)." And once we have this interpretation of the preponderance test, it is easy to conclude that a necessary condition for the preponderance test to be cost minimizing is for the error costs to be the same.

We are suggesting that this whole rationale unravels once the proper distinction between private and social costs is made. Once made, it is seen that the relevant error costs are not in general the same and that the cost minimizing standard of proof is $p > B/C$. But what about the preponderance test? Our distinction between private and social costs leaves open the question of how to interpret the preponderance test and relate it to cost minimization.

V. Probabilities of Probabilities

There are several ways of interpreting the preponderance test. In the example, the plaintiff might assert as a fact that the drug is a teratogen and the defendant might assert as a fact that the drug is not a teratogen. Suppose that the court finds that the weight of evidence on either side is nearly balanced but that the defendant's version is slightly more credible than the plaintiff's. An obvious interpretation of the preponderance test would be for the court to conclude that since the defendant's version is more probable than the plaintiff's, there should be no liability.

But this interpretation is inconsistent with cost minimization. Suppose, for example, in concluding that the evidence from both sides is almost balanced but slightly favors the defendant, the court decides the probability that the drug is teratogenic is about 0.45. Since 0.45 is greater than 0.30, we know for expected cost minimization the appropriate precedent to set is to find liability in the case at hand.

Leading toward another interpretation of the preponderance test, we return to the example but suppose that neither side claims certainty. Instead of claiming that she knows for a fact that the drug is a teratogen, the plaintiff argues that the evidence points toward a conclusion that the drug is a teratogen. She argues that, although there are still some scientific doubts, a reasonable person would conclude there is an 80 percent probability that the drug is a teratogen. The defendant does not claim as a fact that the drug is not
a teratogen. The defendant admits that there is some evidence which suggests a slight probability of teratogenicity. According to the defendant, a reasonable person would conclude that there is only a 20 percent chance of teratogenicity. Again we will assume that the court finds the arguments supporting these conflicting probabilistic evaluations to be almost equally balanced, but again with the defendant's argument slightly more credible. Here the defendant's version of the "facts" is that the evidence of toxicity leads a reasonable person to conclude the probability is 20 percent. The plaintiff's version of the facts is that the evidence leads a reasonable person to conclude the probability is 80 percent.

A second interpretation of the preponderance test would be to decide that since the court found the defendant's version to be more probable, the court should accept the defendant's version. Thus the court would conclude that the probability of teratogenicity is 20 percent. Since 0.2 is less than the 0.3 required standard of proof, the court would find no liability, under this interpretation.

This second interpretation of the preponderance test is also inconsistent with expected cost minimization. To see the inconsistency suppose that the court decided that the defendant's argument was twice as credible as the plaintiff's. To define what we mean by "twice as credible" we need to talk about probabilities of probabilities, or what are called "second order probabilities of probabilities" in decision theory. To say that the defendant is twice as credible as the plaintiff is to mean that the court assesses a .67 probability that the defendant is right in her probabilistic judgment. With this assessment of the relative credibilities, the court forms its own judgment of the probability of teratogenicity by taking the weighted average $(0.67)(0.2) + (0.33)(0.8) = 0.4$.

This precise definition of "credibility" may be foreign to legal commentators, thus some further illustration may help. If the court found that the defendant's argument was not credible at all, but the plaintiff's argument was highly credible, the court would adopt the plaintiff's judgment of the probability of teratogenicity, 0.8; note $(0)(0.2) + (1)(0.8) = 0.8$. Similarly, in reverse, if the court found the plaintiff not credible at all and the defendant's judgment of 0.2; note $(1)(0.2) + (0)(0.8) = 0.2$. More typically, in weighing the arguments of both sides, the court would find its judgment of the probability of teratogenicity to be somewhere in between the plaintiff's and defendant's conflicting judgments. When the court forms its own judgment of the probability of teratogenicity, it is saying something about the relative credibilities of the two parties. The court's own judgment can be considered a weighted average of the defendant's and plaintiff's judgments, where the weights are defined as the relative credibilities of each version.

In our second possible interpretation of the preponderance test the court does not find liability, because it adopts the defendant's version, which is that a reasonable person would assess probability of teratogenicity as 0.2, and this 0.2 is less than the 0.3 standard of proof. The court adopts the defendant's version because it considers that the defendant's version is more credible than the plaintiff's. At the same time the court believes that the plaintiff has some
credibility and believes that the probability of teratogenicity is actually a weighted average of 0.2 and 0.8. With relative credibilities of 0.67 and 0.33, the court assesses the probability of teratogenicity to be 0.4. Since this is greater than the 0.3 standard of proof, the cost minimizing decision is to find liability. Thus our second interpretation of the preponderance test is inconsistent with expected cost minimization.

Having rejected two possible interpretations of the preponderance test, we are still looking for an interpretation which is consistent with expected cost minimization and has the idea of "something more likely than not" in it. We can get close to our objective by looking a little more closely at second order probabilities. First we assumed that the litigants asserted full knowledge about the toxicity of the drug, the plaintiff asserting that she knew for a fact that the drug is teratogenic and the defendant asserting that he knew for a fact that it was not. Next we assumed that the litigants were a little more humble. They agreed that they did not know for a fact whether or not the drug is a teratogen. Instead each weighed the existing evidence and came up with probability judgments. The plaintiff argued that the probability of teratogenicity was, in fact, 0.8, and the defendant argued the probability was, in fact, 0.2. This time we will allow the litigants to be a little more humble yet.

We imagine the plaintiff's pleading as follows. She agrees that the evidence of toxicity is uncertain and that reasonable people can differ in its interpretation. She still argues that a reasonable
person would assess the evidence as about an 0.8 probability of teratogenicity, but she admits some uncertainty as to this precise estimate. She expresses her uncertainty by "a judgmental probability distribution" (Figure 1a). This judgmental probability allows the plaintiff to state her uncertainty about her probability estimate. If her judgment had been very concentrated around the value of 0.8 (like a spike centered on 0.8, as in Figure 1b) this would mean that she was claiming strong knowledge in her estimate of 0.8. If her judgment had been more spread out (as in Figure 1c) this would mean that she was more uncertain in her probability estimate.

Similarly we can imagine that the defendant also admits that he does not know for sure (for a fact) that the probability of teratogenicity is only 0.2. He too, especially under cross examination, is willing to express some uncertainty about his probability estimate of 0.2. He does this in the form of his own judgmental probability distribution, in Figure 2. His judgment is a little more spread out than the plaintiff's, because he admits to a little more uncertainty about his estimate.

After weighing the evidence brought forward by the two parties, the court forms its own judgment as to the probability of teratogenicity. The court also feels some uncertainty as to its probability judgment, and the court expresses its uncertainty by its own judgmental probability distribution (Figure 3a). As before the court's judgment is in between the versions of the two parties, and its judgment depends on the relative credibilities of the litigants. But this time the court expresses its uncertainty as to the precise value of the probability.

By the criterion of expected cost minimization, the court should find liability if its assessment of the expected costs is higher than the benefits. But when we have framed the uncertainties in terms of second order probabilities, how do we assess the expected costs? One of the facts about second order probabilities — if one can assert facts in a paper on the difficulty of asserting facts about probabilities — is that the court's evaluation of expected cost is $\bar{p}$ where $\bar{p}$ is the mean of the court's judgmental probability distribution. It follows that the criterion of expected cost minimization says that the court should find liability if the mean of its second order probability of toxicity is greater than 0.3.

We have still not reached a satisfactory interpretation of the preponderance test, but we are now close. There is a special case where the mean of a probability distribution is the same as the median. This is the case where the distribution is symmetric. Let us consider this case, where $\bar{p}$ is both the mean and the median of the court's second order probability of teratogenicity. The median has a simple interpretation: half the probability weight of the distribution is to the right of the median and half to the left. When the median is greater than 0.3, it is more likely than not, in the court's judgment, that the probability of teratogenicity is more than 0.3. This

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(20) The judgmental probability distribution is a second order probability distribution. It places probability weight on where the probability assessor thinks the probability of toxicity is located.
observation suggests a third interpretation of the preponderance test -- the court should find liability when it is more likely than not that the probability of toxicity is greater than B/C (or in the example 0.3).

This third interpretation of the preponderance test seems, to us, rather natural, once the difficulties of the first two are pointed out. However, it does not always accord with expected cost minimization. The condition for its consistency with expected cost minimization is that the mean and median, of the court's judgmental probability distribution, be the same. A sufficient condition for this is that the court's judgmental probability distribution (its second order distribution) be symmetric.

Further, we can say something about when our third interpretation of the preponderance test is too lax in the sense of providing insufficient incentive for precautionary action. This situation arises when there is weak evidence of toxicity, but where, if there is toxicity, the effects are severe compared with the benefits of the chemical. In other words the situation is likely to arise in the case of the example we have been considering all along. Where B is low relative to C, the ratio B/C is relatively close to zero. For the case to be difficult in judgment the probability of toxicity must be centered somewhere around B/C, as in Figure 3b. If the court has much uncertainty as to its assessment of the probability of toxicity -- as the court is likely to in such cases -- the court's judgmental probability distribution is likely to be spread out. It cannot spread far to the left and the only way it can spread, while still centering around B/C is for it to be skewed to the right, as in Figure 3b. But in this situation the mean is larger than the median. Thus it is possible for the court to find that it is more likely than not that the probability of toxicity is less than B/C (and thus find no liability under our interpretation), yet at the same time for the mean to be greater than B/C (so that there should be a finding of liability under the criterion of expected cost minimization). This is the situation depicted in Figure 3b.

So far we have carried forward the argument in principle. As James and Hazard put it much of what seems "objective, scientific precision" becomes less confining in practice, especially where numerical quantification is not attempted. The main purpose of the above reasoning is to see how the ideas fit together -- or how they do not. If successful, the exercise leads to a "frame of mind" which helps untangle vague and sometimes erroneous qualitative arguments.

In practice we have to choose between two cumbersome criteria. (1) Require liability if the court decides that it is more likely than not that the probability of toxicity is greater than B/C; and (2) Require liability if the court finds that the mean of its judgmental probability distribution is greater than B/C. The latter is consistent with expected cost minimization. But the former appears a little less cumbersome a little easier to grasp. What the former loses in principle it may more than make up for in concreteness and implementability. Nonetheless, it is useful to know that there appears to be some (slight) bias toward laxness, when the risks are high relative to the benefits, and a bias the other way when the risks are
small relative to the benefits. (21)

The above argument leads to an interpretation of the preponderance test which appears to be at least somewhat different from Posner's. It also leads to conditions when the interpretation is "economically sound" which are definitely different from Posner's.

In this frame of mind judgmental probabilities are not "true" or "false" — they are not objective facts in that sense. Instead the plaintiff's 0.8 probability is her evaluation of the likelihood of toxicity; the defendant's 0.2 is his evaluation; and the 0.4 is the court's evaluation based on the arguments brought forward. All three are judgments. When the court comes out closer to the defendant's version of the probability, this does not mean that the defendant's precise number 0.2 has the status of an objective fact which is taken to the "true" or "accurate." it simply means that the court finds the defendant's argument more weighty in the sense that the court's judgment comes out closer to the defendant's than to the plaintiff's. There are some facts, however. It is a fact that the plaintiff's probability is 0.8, another that the plaintiff's evaluation is 0.2, and a third that the court's judgment is 0.4. It is also a fact that each of these evaluations are uncertain, and the second order probabilities express these uncertainties.

VI. The Ratio of Judicial Errors

(21) The likely cutoff in bias is where B/C=1/2.

Posner states "that of the cases decided erroneously, about half will be won by undeserving plaintiffs and about half lost by deserving plaintiffs." Here we will suggest that the numbers of the two types of errors are not in general about the same. The ratio of the two types of errors — the ratio of false positive to false negatives in our terms — depends on several circumstances. One circumstance is the standard of proof for the similar cases being considered together, B/C. Another circumstance is the strength of evidence brought forward.

Consider, as in our example, a 1000 cases which are similar in that for each B/C = 0.3. Let us suppose that in 300 cases weak evidence of toxicity is brought forward and the court concludes, after weighing arguments from both sides that the probability of toxicity is only 0.1; in 500 cases suggestive evidence is brought forward and the court concludes the probability of toxicity is 0.4; and in 200 cases strong evidence is brought forward and the court concludes the probability of toxicity is 0.8.

Under the criterion of expected cost minimization, liability is found for the cases where strong and moderate evidence is brought forward, (since for these 700 cases the p > 0.3), but not for the cases of weak evidence. In the 300 cases for which no liability is found, the court expects some false negatives, namely 10 percent of the 300 or 30 false negatives. In the 500 cases where there is suggestive evidence, and liability found, there will be some false positives. For these cases the court found only a 0.4 probability of toxicity, or a 0.6 probability of non-toxicity. Thus the court's expectation is that
60 percent of the 500, or 300 cases, will be false positives. And in the 200 cases where strong evidence is brought forward and liability found, there will be additional false positives. Here the court found an 0.8 probability of toxicity, or a 0.2 probability of non-toxicity. Hence the court's expectation of the number of false positives for this group of 200 cases is 40 false positives. Altogether, under the criterion of cost minimization, the court expects 30 false negatives and 340 false positives. The ratio of false positives to false negatives is 340 to 30, far different from the 1 to 1 ratio posited by Posner (Point B).

VII. Conclusion

In this paper we have suggested, contrary to Posner, that under a criterion of expected cost minimization, judicial error costs are not in general the same, the number of erroneous judgments favoring undeserving plaintiffs is not likely to be the same as the number of erroneous judgments favoring undeserving defendants, and equality of the two error costs is not a necessary condition for the preponderance test to be "economically sound." In developing our interpretation of the preponderance test, we explored three possible interpretations, rejecting the first two as inconsistent with economic efficiency. The third as well is not, in general, consistent with economic efficiency either, but it comes much closer than the others. Moreover, when the practical problems of administration are taken into account, this third interpretation of the preponderance test may come closer to minimizing costs than other operational rules a court might attempt to follow.

This third interpretation of the preponderance test is derived in terms of second order probabilities, or probabilities of probabilities. Under this interpretation the court finds liability when it decides that it is more likely than not that the probability of toxicity is greater than B/C. This interpretation is close, but not in general quite the same, as the criterion which is consistent with economic efficiency -- the court should find liability when it decides that the mean of its second order probability distribution is greater than B/C. While this latter criterion will be foreign to many, fortunately second order probabilities resolve themselves into simple first order probabilities. For the court to decide that the mean of its second order probability distribution is greater than B/C boils down to the court deciding that the probability of toxicity is greater than B/C. The only difference is that with the second order probabilities the court can express its uncertainties in its assessment of the probability of toxicity.

The latter criterion (that the court find liability if it decides that the mean of its second order probability distribution is greater than B/C) is consistent with economic efficiency but it is cumbersome. The former criterion (that the court find liability if it decides that it is more likely than not that the probability of toxicity is greater than B/C) has the notion of preponderance in it (the notion of more likely than not). The latter criterion is not quite consistent with economic efficiency, but it is easy to state the condition when it is. The condition is for the mean and the median of the second order distribution to be the same. In practice they are
likely to be close. However, for cases where there are presumed small probabilities of severe harms, there is likely to be some bias toward too little precaution in our interpretation of the preponderance test, compared with economic efficiency; and some bias toward over-precaution for cases where there are presumed high probabilities of toxicity but relatively unsevere effects.

The difficulties in dealing with uncertain probabilities, or probabilities of probabilities, are real. It is not surprising that there are occasionally confusions with this subject — we hope that we have resolved more confusion than we have added. The language of second order probabilities, or probabilities of probabilities, is useful in providing a frame of reference and syntax for the various probabilities which appear in legal decision making. Hopefully such a language and syntax may prevent such gross confusion as that evidence by the 8th courtin the Reserve Mining Case. There, the court claimed that the lower court had resolved "all" the uncertainties in favor of the plaintiffs. To improve matters the court resolved all the uncertainties in favor of the defendant. The language of probabilities provides a means for making decisions consistent with expected cost minimization, without "throwing" the very real uncertainties all to one side or the other.

Once familiar with the language and syntax, it can be seen that our interpretation of the preponderance test is a kind of stop gap. Once the court becomes familiar with the meaning of criterion "more likely than not that the probability of toxicity is greater than B/C" it may have greater ease in assessing its first order probabilities.

In doing so, it can move directly to the criterion "find liability if the probability of toxicity is greater than B/C" at the same time knowing that its assessment of this first order probability is, in itself, uncertain. This latter criterion has the advantages of simplicity, consistency with economic efficiency, and direct lineage with Learned Hand's formula.