Magnetic Moments of Dirac Neutrinos
Nicole F. Bell*, V. Cirigliano*, M. J. Ramsey-Musolf*, P. Vogel* and Mark B. Wise*
*California Institute of Technology, Pasadena, CA 91125, USA

Abstract. The existence of a neutrino magnetic moment implies contributions to the neutrino mass via radiative corrections. We derive model-independent "naturalness" upper bounds on the magnetic moments of Dirac neutrinos, generated by physics above the electroweak scale. The neutrino mass receives a contribution from higher order operators, which are renormalized by operators responsible for the neutrino magnetic moment. This contribution can be calculated in a model independent way. In the absence of fine-tuning, we find that current neutrino mass limits imply that \( \mu_\nu < 10^{-14} \) Bohr magnetons. This bound is several orders of magnitude stronger than those obtained from solar and reactor neutrino data and astrophysical observations.

Keywords: Neutrino magnetic moments, neutrino mass.

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Current advances in uncovering the pattern of neutrino mass and mixing, lead naturally to questions about more exotic neutrino properties, such as the magnetic moment, \( \mu_\nu \). In this paper, we describe how the smallness of the neutrino mass may be used to set a strong model-independent limit on the size \( \mu_\nu \). Neutrino magnetic moments are reviewed in [2], and recent work can be found in [3]. In the Standard Model (SM), extended to contain right-handed neutrinos, \( \mu_\nu \) is non-zero but unobservably small, \( \mu_\nu \approx 3 \times 10^{-19} (m_e / 1 \text{ eV}) \) [4]. Current limits are several orders of magnitude larger, so a magnetic moment anywhere near the present limits would certainly be an indication of physics beyond the SM. The best laboratory limits arise from neutrino-electron scattering. The weak and electromagnetic contributions to \( v - e \) scattering are comparable if

\[
\frac{\mu_\nu^{\text{exp}}}{\mu_\nu^{\text{exp}}} \approx \frac{G_F m_e}{\sqrt{2\pi}a} \sqrt{m_eT} \sim 10^{-10} \sqrt{\frac{T}{m_e}},
\]

where \( T \) is the kinetic energy of the recoiling electron. The present limits derived from solar and reactor neutrino experiments are \( \mu_\nu \lesssim 1.5 \times 10^{-13} \mu_B \) [5] and \( \mu_\nu \lesssim 0.9 \times 10^{-10} \mu_B \) [6] respectively. A more stringent limit can be derived from bounds on energy loss in stars, \( \mu_\nu \lesssim 3 \times 10^{-15} \mu_B \) [7].

The presence of a non-zero neutrino magnetic moment will necessarily induce a correction to the neutrino mass term. (The problem of reconciling a large magnetic moment with a small mass has been recognized in the past, and possible methods of overcoming this restriction through the use of symmetries are discussed in [8]i.)

Assuming that \( \mu_\nu \) is generated by physics beyond the SM at a scale \( \Lambda \), its leading contribution to the neutrino mass, \( \delta m_\nu \), scales with \( \Lambda \) as

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\delta m_\nu \sim \frac{\alpha \Lambda^2 \mu_\nu}{32\pi m_e \mu_B},
\]

(2)

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where $\Delta m_\nu$ is a contribution to $m_\nu$ arising from radiative corrections at one-loop order. The $\Lambda^2$ dependence arises from the quadratic divergence of the dimension four neutrino mass operator, $\sigma_\nu^{(4)} = (\bar{\nu} \nu) \nu B_{\nu
u}$, although the precise value of this term cannot be calculated in a model-independent way, we can estimate that for $\Lambda \gtrsim 1$ TeV and $\Delta m_\nu \lesssim 1$ eV, we require $\mu_\nu \lesssim 10^{-14} M_\nu$. Given the $\Lambda^2$ dependence, this bound becomes considerably more stringent for $\Lambda$ well above the electroweak (EW) scale. However, if $\Lambda$ is not significantly larger than the EW scale, higher dimension operators are important, and their contribution to $m_\nu$ can be calculated in a model independent way.

We start by constructing the most general operators that could give rise to a magnetic moment operator, $\bar{\nu} \sigma^{\alpha \nu} F_{\mu \nu} \nu B_{\nu
u}$. Demanding invariance under the SM gauge group $SU(2)_L \times U(1)_Y$, we have the following 6D operators

$$\sigma_\nu^{(6)} = \frac{8}{3} L \bar{\nu} \sigma^{\alpha \nu} \nu B_{\nu
u}, \quad \sigma_W^{(6)} = \frac{8}{3} L e^\alpha \phi \sigma^{\alpha \nu} \nu W_{\nu
u}. \quad (3)$$

After spontaneous symmetry breaking, both $\sigma_\nu^{(6)}$ and $\sigma_W^{(6)}$ contribute to the magnetic moment. Through renormalization, these operators will also generate a contribution to the 6D neutrino mass operator

$$\sigma_\nu^{(6)} = \frac{1}{\Lambda^2} L \phi \nu \sqrt{\phi} \nu \left( \phi \sqrt{\phi} \right) \nu. \quad (4)$$

The three operators, $\{ \sigma_\nu^{(6)}, \sigma_W^{(6)}, \sigma_M^{(6)} \}$ constitute a closed set under renormalization, so that our effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = C_B(\mu) \sigma_\nu^{(6)} + C_W(\mu) \sigma_W^{(6)} + C_M(\mu) \sigma_M^{(6)}, \quad (5)$$

where the operator coefficients, $C(\mu)$, depend upon the energy scale $\mu$. The magnetic moment and mass are related to the operator coefficients as

$$\frac{\mu_\nu}{\mu_B} = -4\sqrt{2} \left( \frac{m_\nu}{\Lambda^2} \right) C_B(\nu) + C_W(\nu), \quad (6)$$

$$\delta m_\nu = -C_M(\nu) \frac{v^3}{2\sqrt{2}\Lambda^2}. \quad (7)$$

To connect $\mu_\nu$ with $m_\nu$, we thus need to find the relationship between the coefficients $C(\mu)$ at the weak scale, $\mu = \nu$. This requires that we solve the renormalization group equations (RGE) which relate $C(\Lambda)$ to $C(\nu)$.

Figures (1,2) display representative examples of the one-loop diagrams which contribute to the renormalization of the 6D operators. (See Ref. [1] for further details.) Solving the RGE, retaining only the leading logarithms, we find that $\mu_\nu$ and $m_\nu$ are related as

$$\frac{\mu_\nu}{\mu_B} = \frac{G_F m_\nu}{\sqrt{2} \alpha} \left[ \frac{\Delta m_\nu}{\alpha \ln(\Lambda/\nu)} \right] 32\pi \sin^4 \theta_W \left( \frac{9}{2} f \right), \quad (8)$$

where $\theta_W$ is the weak mixing angle,

$$f = (1 - r) - \frac{2}{3} r \tan^2 \theta_W - \frac{1}{3} (1 + r) \tan^2 \theta_W. \quad (9)$$

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