is a slightly more restrictive constraint:

\[ R^*_c = \sigma^2 \begin{bmatrix} 1 & \rho_0 & \rho_0 \rho_0 & \rho_0 \\ \rho_0 & 1 \\ \rho_0 & \rho_0 & \rho_0 \rho_0 & \rho_0 \\ \rho_0 & \rho_0 & \rho_0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & \rho_0 \\ \rho_0 & 1 \\ \rho_0 & \rho_0 & \rho_0 \rho_0 & \rho_0 \\ \rho_0 & \rho_0 & \rho_0 & 1 \end{bmatrix} \]

(2)

We note that this latter case corresponds to the direct separable extension of the results reported in [1], since the 2 × 2 DFT can be constructed from the outer product of horizontal and vertical one-dimensional sum and difference operators. The ability of the 2 × 2 DFT to decorrelate the texture images is usually excellent and is sufficient for most practical purposes. For instance, it was shown in [4] that the 2 × 2 DFT as applied to each of 12 Brodatz textures of an experimental set almost identical to that used by Cohen et al., reduces the nondiagonal energy of the covariance matrix to less than 1% (with one exception at 2.2%) of its initial contribution.

REFERENCES

On Coefficient-Quantization and Computational Roundoff Effects in Lossless Multirate Filter Banks

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Abstract—FIR lossless transfer matrices have found recent applications in multirate analysis/synthesis systems having the perfect reconstruction property. It is shown in this correspondence that these lossless systems can be implemented such that regardless of coefficient quantization, the lossless property (and hence perfect reconstruction) can be retained. Such a result was shown to be true in the past only for two-channel filter banks. This correspondence also presents a noise gain result for lossless systems which finds application in analyzing roundoff noise in multirate filter banks.

I. INTRODUCTION

Multirate filter banks with the perfect reconstruction property have received considerable attention recently (see [1]-[7] and references therein). In this correspondence we are concerned with the technique reported in [3]-[7], in which the polyphase matrix of the set of analysis filters is FIR and lossless. Our purpose here is to present two results, one concerning coefficient quantization and the other concerning roundoff noise propagation.

Unless otherwise mentioned, the notations used are the same as those in any one of the [3]-[7]. As a review, \( H_k(z) \) and \( F_k(z) \), \( 0 \leq k \leq M-1 \) represent the \( M \) analysis and synthesis filters of an \( M \)-band maximally decimated filter bank [6, fig. 1]. The analysis and synthesis banks are associated with two \( M \times M \) matrices \( E(z) \) and \( R(z) \) called polyphase component matrices [6, fig. 2]. The methods employed in [3]-[7] are such that \( E(z) \) is (FIR and) lossless. This means that \( E(z) \) satisfies

\[ E(z) E(z) = c^2 I, \quad \forall z, \quad c \text{ real.} \]  

(1)

With \( E(z) \) satisfying this property, the maximally decimated system has perfect reconstruction if and only if the synthesis filters are chosen as \( F_k(z) = \alpha z^{-k} H_k(z) \) where \( \alpha \neq 0 \) and \( L \) is an arbitrary integer.

If \( c^2 = 1 \) in (1), we say that \( E(z) \) is normalized-lossless. Note that losslessness can also be defined for rectangular matrices [5]. For \( M \times 1 \) stable systems, losslessness is same as the power-complementary property [7].

In order to design and implement filter banks with lossless \( E(z) \), it is necessary to obtain a structure for \( M \times M \) FIR lossless systems. Structures for arbitrary real-coefficient FIR lossless systems were presented in [5] based on a state-space approach, with planar rotations as building blocks. More recently, a method was developed in [6] which leads to a different representation of lossless systems. This method does not involve rotations, but is entirely in terms of diadics (i.e., matrices of the form \( uv^\dagger \) where \( u \) and \( v \) are column vectors). This is simpler in terms of derivation as well as implementation. Also, it covers both real and complex coefficient FIR lossless systems. In this correspondence, we shall show that the diadic based structure retains losslessness in spite of coefficient quantization (for arbitrary \( M \)), whereas such a property is not true for the rotation based structure except in the \( M = 2 \) case.

It should be noticed that the above discussion does not take into account errors due to quantization of signals (such as state variables) in the structure. The noise due to this must be separately analyzed; thus in Section IV we present a result on noise amplification by lossless systems, and indicate applications in filter bank structures.

II. COEFFICIENT QUANTIZATION IN ROTATION-BASED STRUCTURES

First consider a degree-0 lossless system \( R \) (i.e., a constant unitary matrix). The unitary property is equivalent to

\[ R_k R_m = 0, \quad k \neq m \]  

(2)

\[ R_m R_m = I, \quad \forall m \]  

(3)

where \( R_m \) is the \( m \)-th column of \( R \). Such matrices can be expressed in terms of planar rotations [8]. For example, in the 3 × 3 real-

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coefficient case

\[
R = \sqrt{d} \begin{bmatrix}
\mu_0 & 0 & 0 & 1 & 0 & 0 \\
0 & \mu_1 & 0 & 0 & c_0 & s_0 \\
0 & 0 & \mu_2 & 0 & -s_0 & c_0 \\
0 & c_1 & 0 & 0 & 0 & c_1 \\
0 & 0 & 0 & s_2 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( c_0 = \cos \theta, s_0 = \sin \theta, \) and \( \mu = \pm 1. \) When the multipliers \( c_0 \) and \( s_0 \) are quantized to \( Q[c_0] \) and \( Q[s_0], \) then \( Q[c_0] + Q[s_0] \neq 1 \) any more. So each nondiagonal factor in (4) is such that it has unequal column norms. In other words, the factor satisfies (2) but not (3). As a result, one can verify that the product (4) satisfies neither (2) nor (3), i.e., it is not unitary. One could consider replacing the entry ‘-1’ in each nondiagonal factor of (4) with the norm of the remaining columns after quantization, but norms involve square roots which usually require infinite precision.

For similar reasons, structures for lossless systems of arbitrary degree based on rotations [5] do not retain the lossless property when the cosines are quantized. (An exception to this is the 2 \( \times \) 2 case, as elaborated in [4]. In this case there is no difficulty because the column-norms of 2 \( \times \) 2 orthogonal matrices are equal even after quantization.)

### III. Coefficient Quantization in Dyadic-Based Structures

First consider constant unitary matrices with possibly complex entries. It is well known [9], [10] that such a matrix can be factorized as

\[
R = \sqrt{d} G = G_u \cdot D
\]

where \( G_u \) are Householder matrices, i.e., \( M \times M \) matrices of the form

\[
G_u = I - 2 u_u^T u
\]

where \( u_u \) is a unit-norm column vector. It is easily verified that \( G_u \cdot G_v = I. \) The quantity \( D \) in (5) is diagonal with \( D = e^{i\phi}. \) If we implement a unitary matrix in factored form as in (5), then the quantization of the components of \( u_u \) results in loss of the unit-norm property of \( u_u \) so that \( G_u \) (and hence \( R \)) does not remain unitary. However, a simple trick can be used to overcome this difficulty. For this, note that the modified matrix \( H_u = u_u^T u - 2 u_u^T u_u \) satisfies \( H_u^T H_u = u_u^T u \) i.e., is unitary for any nonzero vector \( u. \) We can therefore express any unitary \( R \) as

\[
R = H \cdot \ldots \cdot H_{M-1} \cdot D
\]

where \( D = \sqrt{d}/\alpha, \alpha = e^{i\phi}. \) When the components of \( u_u \) are quantized, the matrices \( H_u \) remain unitary and so does the product \( H_1 \cdot \ldots \cdot H_{M-1}. \) But we have to be careful about the diagonal matrix \( D \) (which can be complex). Under unquantized conditions the diagonal elements have equal norm (\( = \sqrt{d}/\alpha \)). Upon quantization of the diagonal entries, the norms can become unequal. So the product (7) after quantization satisfies (2) but not necessarily (3). In other words, the columns of \( R \) are still pairwise orthogonal but may have unequal norms. This is, of course, better than losing both the properties (2) and (3) as in the rotation-based structure. Finally, for real matrices the diagonal elements of \( D \) are real and have the form \( \pm \sqrt{d}/\alpha \) and have equal norm even after quantization so that both (2) and (3) hold.

Now consider lossless systems with arbitrary degree. Every \( M \times M \) FIR lossless system of degree \( N \) (with possibly complex coefficients) can be represented [6] as \( E(z) = V_u(z) \cdot \ldots \cdot V_1(z) S \) where \( V_u(z) \sim I - v_u^T v_u + z^{-1} v_u^T v_u \). Here \( v_u \) is \( M \times 1 \) so that \( V_u(z) \) has degree \( 1 \), with \( \| v_u \|^2 = 1. \) The unit-norm constraint on \( v_u \) ensures that \( V_u(z) \) is lossless. The unit-norm property is usually lost so that \( V_u(z) \) (and hence \( E(z) \)) does not remain lossless. To avoid this difficulty we define

\[
U_u(z) = \| v_u \|^2 I - v_u v_u^T + z^{-1} v_u v_u^T
\]

and rewrite \( E(z) = U_u(z) \cdot \ldots \cdot U_1(z) R \) where \( R = \beta I, \beta = \Pi \| v_u \|^2. \) Equation (8) represents a lossless system for any nonzero \( v_u \). The unit-norm constraint on \( v_u \) is not required any more. When \( v_u \) are quantized, \( U_u(z) \) still remains lossless so that \( U_u(z) \cdot \ldots \cdot U_1(z) \) remains lossless. The matrix \( R \) which is unitary, can be represented as in (7) so that it remains unitary (except that in the complex case the column norms can become unequal). Summarizing, \( E(z) \) continues to satisfy \( E(z) E^* = J \) inspite of quantization, where \( J = \pi I \) is a constant diagonal matrix. In the real-coefficient case \( v_u \) and \( R \) real so that \( J = e^{-i\pi I}, \) i.e., \( E(z) \) remains lossless. As a result one can implement a \( M \)-band FIR perfect reconstruction system which retains the perfect reconstruction property in spite of coefficient quantization. It is true that the multipliers \( \| v_u \|^2 \) require higher precision than the components of \( v_u \) but these precisions remain finite. Notice that the modified structure does not require multipliers involving square roots.

Similar results are easily derived [11] for IIR lossless systems (both \( M \times M \) and \( M \times 1 \)) but are not of immediate relevance here.

### IV. Roundoff Noise Considerations

All structures for lossless systems reported in [5] and [6] have the form shown in Fig. 1 where \( V_u(z) \) are degree-one lossless systems and \( R \) is constant unitary satisfying (2), (3). Assume \( d = 1 \) for simplicity. Let the computation inside each building block \( V_u(z) \) and \( R \) be performed without quantization. Assume that quantizers are inserted only at the outputs of these building blocks so that the noise model is as shown in Fig. 1. Here each \( e_u(n) \) is a vector of \( M \) noise sources. We shall make standard assumptions about the noise model[12]. Thus the quantizers are \( b \)-bit fixed point roundoff devices generating zero-mean white noise with variance \( \sigma^2_u \) and the noise components in \( e_u(n) \) are mutually uncorrelated. The following result helps to simplify noise analysis in lossless systems.

**The Noise Gain Lemma:** Let \( H(z) \) be a system such that \( H^*(z) \) is normalized-lossless. Let the input vector \( u(n) \) be wide-sense stationary, with zero mean and

\[
E[u(n) u^T(n')] \leq \delta \sigma^2(n - n').\]

Then the output \( y(n) \) has zero mean and satisfies

\[
E[y(n) y^T(n')] \leq \delta \sigma^2(n - n').\]

**Proof:** Let \( H(z) = \sum_{k=0}^{m-1} h_k(n) z^{-k} \) so that \( y(n) = \sum_{k=0}^{m-1} h_k(n) u(n - m). \) Losslessness of \( H^*(z) \) implies \( \sum_{k=0}^{m-1} h_k(n) h^*(k + m) = \delta(k)1. \) By using this and (9) it can be verified that \( E[y(n) y^T(n')] \) indeed simplifies to the form (10).

According to the Lemma, if the components of \( u(n) \) are zero mean, white, and uncorrelated with equal variance, then so are the components of \( y(n). \) Returning to Fig. 1, each of the noise sources \( e_u(n) \) faces a noise transfer function which is \( M \times M \) normalized-lossless. Since \( e_u(n) \) and \( e_u((n - m) \) are assumed to be uncorrelated for \( m \neq m' \), the total noise \( f(n) \) at the output terminal is zero mean with \( E[f(n) f^T(n')] = (N + 1) \delta(n - n') \sigma^2_f. \) In other words, each component \( y(n) \) of the output \( y(n) \) is contaminated with additive white noise \( \sigma^2_f (N + 1) \).

Note that the noise gain Lemma holds also for rectangular matrices \( H(z). \) This is useful in analyzing the noise gain due to a power-complementary set of \( M \) synthesis filters \( F(z) = \{ F_0(z), \ldots, F_{M-1}(z) \} \) (Fig. 2). Such a set satisfies \( \sum_{k=0}^{M-1} |F_k(e^{*})|^2 \leq 1 \) so that \( F^*(z) \) is lossless. Assume that at the input of each filter \( F_k(z) \) we have a white noise component with variance \( \sigma^2_z \) from some source. If the noise components are uncorrelated, then the noise at the output is white with variance \( \sigma^2_f \).
As a final example, consider Fig. 3 which is a polyphase implementation of the synthesis bank. Here $R(z)$ has noise components at the input (possibly from quantizers at the analysis end). If the noise sources satisfy the usual assumptions and if $R(z)$ is normalized-lossless (as in the perfect reconstruction systems in [3]) then the noise components at the output of $R(z)$ are white and uncorrelated, with equal variance $\sigma^2$. Since the noise component at the output of $R(z)$ is merely a time-multiplexed version of these components, it is white with variance $\sigma^2$. In practice, the noise generated due to quantization inside $R(z)$ should be added to this; it can be analyzed in a similar way.

V. CONCLUDING REMARKS

Note that in many multirate filter bank applications, a major source of "noise" is the coding error, introduced at the output of the analysis bank. This noise does not satisfy the standard assumptions stated above, so the above results do not apply. The noise gain Lemma is primarily useful for analyzing roundoff noise generated in the filter structure.

It can be shown that the lossless nature of building blocks also results in automatic $\mathcal{L}_2$-scaling of internal nodes. Detailed analysis of this can be found in [13].

The fact that unitary matrices can be implemented without loss of unitary property under quantization appears to have escaped the attention of the researchers (even though the usefulness of Householder matrices are very well known [9], [14]). This property might be valuable in implementation of numerical techniques for least squares problems [14].

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Automatic Segmentation of Speech
Jan P. van Hemert

Abstract—This correspondence describes a method for automatic segmentation of speech. First the incoming utterance is split up into more or less stationary parts, then these stationary parts are labelled as phones using the phonetic transcription of the utterance.

I. INTRODUCTION

For the purpose of recognition or synthesis, speech often needs to be segmented into phonetic units. Manual segmentation is tedious and time consuming, and the results lack reproducibility because of the subjective decisions involved. This calls for automatic segmentation methods. In this correspondence we describe such an automatic method. An important application of the method is the automatic generation of segment libraries for speech synthesis. With this method, dipphone libraries for Dutch, German, and British English have been made in only a fraction of the time a manual method would have taken. A perceptual test has shown that isolated Dutch words constructed with diphones generated by this method compare favorably, both in intelligibility and in speech quality, to words synthesized using a manually prepared inventory.

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