A Three-Dimensional Analysis of Rotordynamic Forces on Whirling and Cavitating Helical Inducers

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1 Introduction
The combined effects of destabilizing rotordynamic fluid forces and cavitition represent the main fluid dynamic phenomena that adversely affect the dynamic stability and performance of high power density turbopumps (Brennen, 1994). This can lead to very serious problems ranging from fatigue failure to sudden destructive damage of the machine (Jery et al., 1985; Franz et al., 1990). Rotordynamic fluid forces under cavitating conditions have long been known to play an important role in promoting the development of self-sustaining latal motions (whirl) of the impeller (Rosenmann, 1965). Recent experiments carried out in the Rotor Force Test Facility at the California Institute of Technology by Franz et al. (1990) and Bhattacharyya (1994) showed that cavitition significantly affects the rotordynamic fluid forces on axial flow inducers. In general, cavitition has been found to have a destabilizing effect on the whirl motion. In the present context, it is important to note that cavitition replaces the characteristic quadratic behavior of the noncavitating rotordynamic fluid forces with a more complex dependence on the whirl speed, thereby undermining the traditional expansion of the rotordynamic fluid forces in terms of stiffness, damping and inertia matrices. Bhattacharyya (1994) tentatively correlated these changes to the development of reverse (possibly oscillatory) flow at lower flow coefficients, implicitly postulating some form of interaction between cavitition, backflow and whirl motion.

The present research aims at obtaining some fundamental understanding of the basic fluid dynamic phenomena responsible for the observed behavior of the rotordynamic fluid forces in whirling and cavitating inducers. In particular, the main purpose of this study consists of investigating to what extent this behavior results from the dynamic response of the bubbles in the cavitating flow through the inducer under the excitation provided by the whirl motion. The flow is studied using the linear perturbation approach used by the authors in their previous dynamic analyses of bubbly liquids (d'Agostino and Brennen, 1983, 1988, 1989; d'Agostino, Brennen and Acosta, 1988; Kumar and Brennen, 1993; d'Auria, d'Agostino and Brennen, 1994, 1996), extending earlier two-dimensional results (d'Auria, d'Agostino and Brennen, 1993) to account for the presence of the inducer blades and the occurrence of significant tangential components of the rotordynamic fluid forces. By introducing suitable simplifications, this approach leads to a fully three-dimensional boundary value problem for a linear Helmholtz equation in the complex amplitude of the pressure perturbation. Solution to this equation can be efficiently obtained by separation of variables. Despite its intrinsic limitations, the results of the theory are consistent with the general features of the available experimental data. Hence, it appears that the present analysis correctly captures some of the fundamental fluid dynamic phenomena in whirling inducers under cavitating conditions and can usefully contribute to the understanding of the rotordynamic fluid forces and instabilities in a number of important turbomachinery applications.
A number of simplifying assumptions are introduced in order to obtain a soluble set of equations that still reflects the dynamics of a whirling inducer in a bubbly mixture. The relative motion of the two phases, whose dynamic role is insignificant in the present linearized approximation (d’Agostino and Brennen, 1989), is neglected. Viscous effects are also neglected, except in the bubble dynamics where they contribute to the damping. As shown in Fig. 1, an infinite helical inducer is considered, with \( N \) radial blades, zero blade thickness, hub radius \( r_h \), tip radius \( r_r \), tip blade angle \( \beta_r \), and constant pitch:

\[
P = 2\pi r_r \tan \beta_r = 2\pi r' \tan \beta
\]

The mean flow conditions are specified by the flow coefficient, \( \phi \), and the cavitation number, \( \sigma \), assuming fully-guided forced-vortex flow with axial velocity \( u_0 = \phi \Omega r_r \), angular velocity \( \Omega_0 = \Omega(1 - \phi \cot \beta_r) \), zero radial velocity \( u_r \), and uniform mean pressure \( p_{0} = p_{in} + \rho \Omega^2 r_r^2 \) (neglecting centrifugal effects), where \( p_{in} \) is the vapor pressure of the liquid.

Cavitation is modeled by a uniform distribution of small spherical bubbles of unperturbed radius, \( R_0 \), and void fraction, \( \alpha \approx 1 \). The dynamics of vapor-gas bubbles is modeled as proposed by Nigmatulin et al. (1981), assuming uniform internal pressure, equal gas and vapor temperatures, and linear subsurface bubble oscillations. For assigned values of the pressure, temperature, and surface tension of the surrounding liquid it is possible to determine the amount of non-condensable gas stabilizing a bubble of given radius (d’Auria et al., 1997). The effects of compressibility, inertia, and energy dissipation due to the viscosity of the liquid and the transfer of heat and mass between the two phases are included in the model. In this model, the vapor-gas bubbles, when excited at frequency \( \omega_L \), behave as second-order harmonic oscillators:

\[
(-\omega_L^2 - i\omega_L 2\lambda + \omega_B^2)\hat{R} = -\frac{\beta}{\rho_0 R_0} \hat{p}
\]

where \( \hat{R} \) and \( \hat{p} \) are the complex amplitudes of the bubble radius and liquid pressure perturbations:

\[
\hat{R} = R - R_0 = \text{Re}(\hat{R} \exp(-i\omega_L t)) \quad \text{and} \quad \hat{p} = p - p_0 = \text{Re}(\hat{p} \exp(-i\omega_L t))
\]

Assuming that the gas and vapor densities are negligible when compared to the liquid density and solving the energy equation at \( R = R_0 \), the damping coefficient \( \lambda = \lambda(\omega_L) \) and the bubble natural frequency \( \omega_B = \omega_B(\omega_L) \) are obtained as:

### Nomenclature

\[\begin{align*}
a & = \text{sound speed} \\
b & = \text{boundary equation} \\
e & = \text{unit vector} \\
E & = \text{bubble thermodynamic function} \\
P & = \text{force} \\
i & = \text{imaginary unit} \\
J & = \text{Bessel function of the first kind} \\
k & = \text{wave number, thermal conductivity} \\
l & = \text{hub excitation mode index} \\
m & = \text{blade excitation mode index} \\
n & = \text{cross-flow helical coordinate} \\
N & = \text{number of blades} \\
p & = \text{pressure} \\
P & = \text{blade axial pitch} \\
r & = \text{radial coordinate} \\
R & = \text{bubble radius} \\
z & = \text{streamwise helical coordinate}
\end{align*}\]

\[\begin{align*}
t & = \text{time} \\
u & = \text{velocity vector} \\
u, v, w & = \text{velocity components} \\
x, y, z & = \text{Cartesian coordinates} \\
Y & = \text{mass fraction, Bessel function of the second kind} \\
\alpha & = \text{void fraction} \\
\beta & = \text{blade angle} \\
\gamma & = \text{specific heat ratio} \\
\epsilon & = \text{whirl eccentricity} \\
\theta & = \text{azimuthal coordinate} \\
\lambda & = \text{bubble damping coefficient} \\
\mu & = \text{eigenvalue} \\
\nu & = \text{kinematic viscosity} \\
\rho & = \text{density} \\
\sigma & = \text{cavitation number} \\
\tau & = \text{bubble volume} \\
\phi & = \text{flow coefficient} \\
\omega & = \text{frequency, whirl angular speed} \\
\Omega & = \text{rotational speed}
\end{align*}\]

### Subscripts and Superscripts

\[\begin{align*}
B & = \text{bubble, blade} \\
G & = \text{gas} \\
H & = \text{hub} \\
I & = \text{inducer} \\
L & = \text{liquid, Lagrangian} \\
M & = \text{bubble mixture} \\
p & = \text{pressure} \\
R & = \text{radial} \\
T & = \text{tangential, blade tip} \\
v & = \text{volume} \\
V & = \text{vapor} \\
VG & = \text{vapor-gas mixture} \\
o & = \text{unperturbed or reference value}
\end{align*}\]
Fig. 3 "Cold bubble." Radius response, $|\delta R_p/R_0|$, of a 1 mm radius gas-vapor bubble in water at $p_w = 5 \times 10^4$ Pa, $T_w = 293$ K, as a function of the excitation frequency, $\omega_0$, for different values of the bubble vapor content: $Y_v = 0$ (noncondensible gas bubble), $Y_v = 0.5$, $Y_v = 0.7$, $Y_v = 0$, and $Y_v = 0.9$ (from d’Auria et al., 1997).

$$2\lambda = \frac{3p_{bo}}{p_rR_0^2\omega_0} \text{Im} \left[ \frac{\gamma v_\text{o}}{E(\omega_0)} \right] + \frac{4\nu_L}{R_0^2}$$

(2)

$$\omega_0^2 = \frac{3p_{bo}}{p_rR_0^2} \text{Re} \left[ \frac{\gamma v_\text{o}}{E(\omega_0)} \right] - \frac{2S}{\rho L R_0^3}$$

(3)

which are similar to the expressions of Chapman and Plessner (1972) and Prosperetti (1984, 1991). Here $v_\text{o}$ is the specific heat ratio of the vapor-gas mixture inside the bubble, $\nu_L$ and $S$ are the kinematic viscosity and the surface tension of the liquid, and $p_{bo} = p_r + 2S/R_0$ is the bubble internal pressure, which is the sum of the partial pressure of the vapor, $p_{bo}$, and of the non-condensible gas, $p_{ro}$. The quantity $\dot{u} = E(\omega_0)$ accounts for the bubble compressibility, interfacial phase changes, heat transfer, mass diffusivity, and inertial effects (Nigmatulin et al., 1981).

It is important to note that the dynamic behavior of vapor-gas bubbles changes dramatically when the temperature is such that interfacial heat transfer limits the bubble dynamics (Brennen, 1995). This is illustrated in Figs. 3 and 4, where the response of bubbles with different vapor content is plotted as a function of the excitation frequency for $p_w = 5000$ and $40,000$ Pa, respectively corresponding to temperatures of $20^0$ and $70^0$ Celsius. In both cases the normalized amplitude of the bubble response increases rapidly with the vapor content. However, vapor-gas bubbles at $20^0C$ are much more compliant than those at $70^0C$. This has important consequences for the forces acting on the inducer.

The perturbation velocities must satisfy the kinematic conditions on the hub, $D(r - r_h)/Dt = 0$, on the outer casing, $D(r - r_r)/Dt = 0$, and on the blade:

$$\frac{D}{Dt} \left[ \dot{\theta} + \frac{z}{r_r} \cos \theta \right] = 0$$

where $\dot{\theta} = 2\pi(j - 1)/N$ identifies the angular position of the $j$th blade ($j = 1, 2, \ldots, N$). It is convenient to analyze the flow in body-fitted Lagrangian helical coordinates:

$$r_L = r, \quad z_L = \frac{\theta - \Omega t}{2\pi} \cos^2 \beta - \frac{z - \omega d}{p} \sin^2 \beta,$$

$$n_L = \frac{N}{2\pi} (\theta - \Omega t) + \frac{N}{p} (z - \omega d)$$

moving with the mean flow at axial velocity, $w_\text{av}$, and angular speed, $\Omega$, and normalized so that $s_L$ and $n_L$ are incremented by 1 for one rotation about the axis (see Fig. 5). Then, the linearized kinematic conditions for the Lagrangian velocity perturbations $(\ddot{u}, \ddot{v}, \ddot{w})$ on the solid surfaces are found to be:

$$\ddot{u} = \epsilon v_\text{w} \left( 2\pi s_L + \frac{2\pi}{N} n_L \sin^2 \beta - \omega t \right)$$

on the hub \quad $r_L = r_h$

$$\ddot{u} = 0 \quad \text{on the casing} \quad r_L = r_r$$

and:

$$\ddot{w} = \epsilon \omega v_\text{w} \cos \beta \cos \left( 2\pi s_L + \frac{2\pi}{N} n_L \sin^2 \beta - \omega t \right)$$

on the blades \quad $n_L = 1, 2, \ldots, N$

where $\omega = \omega - \Omega$ is the frequency of the bubble excitation in the Lagrangian frame. Finally, appropriate boundary conditions at the inlet and exit of the inducer are needed. Here, for simplicity, we impose periodic conditions $\beta(s_L) = \beta(s_L + 1)$, consistent with the original assumption that the inducer is long in the axial direction.

Generalizing the derivation of d’Agostino and Brennen (1988), linearization of the fluid dynamic equations of the bubble mixture in rotating coordinates for time-harmonic fluctuations with frequency $\omega_t$ and irrotational absolute velocity perturbations yields the following Helmhwoltz equation for the pressure field:
where the free-space wave number, \( k \), is determined using the principal branch of the complex square root from the dispersion relation:

\[
\frac{1}{\omega_n^2} = \frac{k^2(\omega_n)}{\omega_n^2} = \frac{1}{\omega_n^2} \left( \omega_n^2 - \omega_n^2 + i\omega_n 2k_n \right)
\]

Here \( \alpha_n(\omega_n) \) is the complex (dissipative) and dispersive (frequency dependent) speed of propagation of harmonic disturbances of angular frequency, \( \omega_n \), in the free bubbly mixture, and \( \omega_n^2 = \) the low-frequency sound speed, given by:

\[
a_n^2 = \frac{\omega_n^2 R_n^2}{3a_n(1 - \alpha)}
\]

where \( \omega_n^2 = \omega^2(0) \) is the natural frequency of oscillation of a single bubble at isothermal conditions (\( \omega_n = 0 \)) in an unbounded liquid. Assuming uniform mean pressure in the inducer, \( k \) is constant and, neglecting Coriolis forces (\( \Omega = 0 \)), to this order of approximation, the complex velocity and pressure perturbations are related, as usual, by \( \omega_n^2 \rho_n(1 - \alpha) \hat{u} = \nabla \hat{p} \).

With the above boundary conditions, the homogeneous Helmholtz equation for the pressure represents a well-posed complex boundary value problem for \( \hat{p} \). If the blade angle \( \beta = \beta(r) \) is approximated by a constant value \( \beta_f \) at some suitable mean radius \( r_m \), the separable solution (Lebedev, 1965) in the blade channel \( j \) = 1 \( \Rightarrow \) \( n_1 \Rightarrow j \) is:

\[
\hat{p} = \sum_{(n,k)} a_{n,k} R_m(r_k) \cos \left[ 2\pi(n_l(j - 1) + 1) \right] e^{i2\pi r_k - \omega_n t}
\]

\[
+ \sum_{m=0}^\infty R_m(r_m) \left[ c_{n,m} \cos \left[ \mu_m(n_l(j - 1) - j + 1) \right] \cos \left[ \mu_m(n_l(j - 1) - j) \right] e^{i2\pi r_m - \omega_n t} \right]
\]

where:

\[
a_0 = c_p(1 - \alpha) \omega_n^2 r_n \int_{r_l}^{r_1} \exp \left( i \frac{2\pi}{N} n_l \sin \beta_n \right) d\ell
\]

\[
a_1 = 2\pi c_p(1 - \alpha) \omega_n^2 r_n
\]

\[
\times \int_{r_l}^{r_1} \left( i \frac{2\pi}{N} n_l \sin \beta_n \right) c_0 \cos \left[ \pi(n_l(j - 1) + 1) \right] d\ell
\]

\[
\text{for } I \neq 0
\]

\[
c_{n,m} = \frac{e^{\pi \mu_m(1 - \alpha) \omega_n^2}}{2\pi \mu_m \sin \mu_m} \int_{r_l}^{r_1} \frac{N_m r_l}{P_l^2 \cos^2 \beta_n} R_m(r_m) \cos \left[ \mu_m(n_l(j - 1) - j) \right] d\ell
\]

In these equations \( R_m(r_m) \) and \( R_m(r_l) \) are the modal solutions corresponding to the hub and blade excitations, given in terms of the Bessel functions of the first and second kind and order \( \nu = \cos \beta_n \) by:

\[
R_m(r_m) = \frac{1}{\mu_m} \int_{r_1}^{r_1} J_\nu(B_m)Y_\nu(B_m r) - Y_\nu(B_m r)J_\nu(B_m r) \ d\ell
\]

\[
R_m(r_l) = \frac{1}{\mu_m} \int_{r_l}^{r_1} J_\nu(B_m r)Y_\nu(B_m r) - Y_\nu(B_m r)J_\nu(B_m r) \ d\ell
\]

where \( B_m \) are the (negative and positive) zeros of the equation:

\[
J_\nu(B_m)Y_\nu(B_m r) - Y_\nu(B_m r)J_\nu(B_m r) = 0
\]

and \( \mu_m = \) the principal square roots of:

\[
\frac{B_m^2}{\lambda_m^2} = \frac{\nu^2}{\lambda_m^2} - \frac{\nu^2}{\lambda_m^2} \frac{N_m^2}{P_l^2 \cos^2 \beta_n} \quad \text{and} \quad \mu_m^2 = \frac{\nu^2}{\lambda_m^2} \frac{N_m^2}{P_l^2 \cos^2 \beta_n} [B_m^2 - B_m^2]
\]

In the presence of bubble dynamic damping, the series for \( \hat{p} \) in Eq. (5) converges rapidly and only the first few terms are needed in the computations.

In the special case of no bubble dynamic damping, \( k^2 \) is real and the boundary value problem for \( \hat{p} \) is self-adjoint with real eigenvalues, \( \mu_m^2 \). If, in particular, \( k^2 \frac{r_l^2}{B_m^2} < \cos^2 \beta_n \), then all eigenvalues \( \mu_m \) are negative. Given the functional dependence of the solution, this condition and the dispersion relation identify the cut-off frequency:

\[
\omega_L^2 = \omega_n^2 - \omega_n \left( 1 + \frac{3a_n(1 - \alpha) r_l^2}{R_n^2 \cos^2 \beta_n} \right)^{1/2}
\]

whereby no wave-like propagation of the blade excitation takes place in the \( n_1 \)-direction. From the above equations it also appears that resonance occurs in response to the hub and blade excitation when \( B_m = B_n \) and \( \mu_m = l \pi \), respectively. This leads to an infinite set of natural frequencies:

\[
\omega_{Lm} = \omega_n - \omega_n \left( 1 + \frac{3a_n(1 - \alpha) r_l^2}{R_n^2 \cos^2 \beta_n} \right)^{1/2}
\]

for \( l > 0 \), \( m > 0 \). Notice that, consistent with the results of previous dynamic analyses of bubbly flows (d'Agostino and Brennen, 1983, 1988, 1989; d'Agostino et al., 1988; d'Auria, d'Agostino and Brennen, 1994, 1995, 1996), \( \omega_L^2 \) and \( \omega_{Lm}^2 \) never exceed the resonance frequency \( \omega_n \) of an individual bubble, and become much lower when a "cloud interaction" parameter, \( \beta \), becomes significantly greater than unity. In the present case, \( \beta = aL^2r_l^2 \) where \( L = r_l \).

The rotodynamic fluid force per unit length on the inducer is:

\[
F = - \frac{1}{\pi \rho_n \omega_n^2 r_l} \int_{S_1} (p - \rho_n u) dS
\]

where \( S_1 \) is the surface of the inducer (hub and blade surfaces) for one interval \( 0 \leq \phi \leq 2\pi \).

Upon integration, the radial and tangential components, \( F_r \) and \( F_T \), of the rotodynamic force on the inducer are more compactly represented in complex form by:

\[
F = F_r - iF_T
\]

Due to the linear nature of the solution, it is possible to synthesize the rotodynamic forces by examining the separate contributions from the hub and blade motion. With the notation of Eq. (9), the rotodynamic force on the hub generated by the hub motion is:

\[
P_h^{(m)} = P_h^{(m)} - iP_h^{(m)}
\]

\[
= - \frac{1}{\pi \rho_n \omega_n^2 r_l} \sum_{j=1}^{N} \sum_{l=0}^{N} \int_{S_j} a_r R_m(r_n) I_{1l}
\]

where:

\[
I_{1l} = \int_{r_l}^{r_1} \exp \left( -i \frac{2\pi}{N} n_l \sin \beta_n \right) \cos \left[ \pi(n_l(j - 1) + 1) \right] d\ell
\]

Similarly, the rotodynamic force on the hub generated by the blade motion is:
\[ F^{(m)} = F^{(m)}_R - iF^{(m)}_T = - \frac{1}{\pi \epsilon \rho \Omega^2 r_1^2} \]
\[ \times \sum_{j=1}^{N} \sum_{m=0}^{N} \frac{P_{j}}{N} \left( c_m A_{m,j} - c_{m-1,j} B_{m,j} \right) \]  \hspace{1cm} (11)

where:
\[ A_{m,j} = \int_{1}^{j} \exp \left( -i \frac{2\pi}{N} n_{L} \sin^2 \beta_0 \right) \cos \left( \mu_m (n_L - j + 1) \right) d n_L \]
\[ B_{m,j} = \int_{1}^{j} \exp \left( -i \frac{2\pi}{N} n_{L} \sin^2 \beta_0 \right) \cos \left( \mu_m (n_L - j) \right) d n_L. \]

The complex representation of the rotodynamic force on the blades generated by the hub motion is:
\[ F^{(H)} = F^{(H)}_R - iF^{(H)}_T = - \frac{1}{\pi \epsilon \rho \Omega^2 r_1^2} \]
\[ \times \left\{ \sum_{j=1}^{N} \sum_{l=0}^{N} \frac{P_{j}}{2} \int_{r_1}^{r} R_{m} e^{-i(2\pi/N)(l-1)\sin^2 \beta_0} d r_L \right\} \]  \hspace{1cm} (12)
and, finally, the rotodynamic force on the blades generated by the blade motion is:
\[ F^{(B)} = F^{(B)}_R - iF^{(B)}_T = - \frac{1}{\pi \epsilon \rho \Omega^2 r_1^2} \]
\[ \times \left\{ \sum_{j=1}^{N} \sum_{l=0}^{N} \frac{P_{j}}{2} \left( c_{m,j} - c_{m-1,j} \cos \mu_m \right) \right. \]
\[ \times \int_{r_1}^{r} R_{m} e^{-i(2\pi/N)(l-1)\sin^2 \beta_0} d r_L \]
\[ \left. - \sum_{j=1}^{N} \sum_{l=0}^{N} \frac{P_{j}}{2} \left( c_{m,j} \cos \mu_m - c_{m-1,j} \right) \right\} \]  \hspace{1cm} (13)
\[ \times \int_{r_1}^{r} R_{m} e^{-i(2\pi/N)(l-1)\sin^2 \beta_0} d r_L \]

Then, the rotodynamic forces on the inducer are simply synthesized as:
\[ F = F_{R} - iF_{T} = F^{(H)} + F^{(B)} + F^{(H)} + F^{(B)} \]  \hspace{1cm} (14)
The modes \( R_{m} \) due to the hub motion, are analogous to the earlier two-dimensional flow solution (d'Auria et al. 1995), except for relatively minor modifications introduced by the helical nature of the flow. These mostly contribute to the radial component of the rotodynamic force. On the other hand, the modes \( R_{m} \) due to the blade motion are essentially three-dimensional and generate significant contributions to the tangential forces.

The entire flow has therefore been determined in terms of the material properties of the two phases, the geometry of the impeller, the nature of the excitation, and the assigned quantities \( \phi, \sigma, \alpha, \) and \( R_{m}. \)

3 Results and Discussion

The calculated rotodynamic forces acting on the inducer as a consequence of the whirl motion are strongly dependent on the propagation of pressure disturbances through the blade channels. The general features of this propagation (behavior in space and time, dependence on the flow parameters, resonances and

Fig. 6 Experimentally measured radial rotodynamic force on the test inducer as a function of the whirl speed ratio \( \omega/\Omega \) under cavitating conditions at \( \phi = 0.049 \) and \( \sigma = 0.106 \) (adapted from Bhattacharyya, 1994)

Fig. 7 Experimentally measured tangential rotodynamic force on the test inducer as a function of the whirl speed ratio \( \omega/\Omega \) under cavitating conditions at \( \phi = 0.049 \) and \( \sigma = 0.106 \) (adapted from Bhattacharyya, 1994)
theory and experiment (notice, for example, that the magnitude of the forces is off by one order of magnitude), the main qualitative features of the forces are correctly reproduced. Note that, consistent with experimental results, the radial force, $F_r$, is essentially negative for $\omega/\Omega > -0.3$. In addition, the zero crossing for positive whirl ratios and the minimum at $\omega/\Omega \approx 0.2$ have been reproduced by the computations. More important, in view of its implications for rotodynamic stability, Fig. 8 shows that the strong positive peak in the tangential force and its location ($\omega/\Omega \approx 0.2$) are also reproduced by the theory.

Figure 9 presents results for various void fractions. As the void fraction increases, the magnitude of the peak in the tangential force increases. This is physically consistent with the experimental observations of Bhattacharyya (1994), who observed the same increase as the cavitation number, $\alpha$, decreased. Figure 10 presents results for various flow coefficients and demonstrates that a similar effect occurs as $\phi$ decreases.

These results are strongly influenced by bubble dynamics effects. The corresponding value of the cloud interaction parameter $3\alpha(1-\alpha)r^2/R_s^2$ (as defined by d’Agostino and Brennen, 1989) is much larger than unity, thus indicating that extensive bubble dynamic and resonant phenomena are likely to occur in the inducer flow.

Finally, we turn our attention to the influence of thermal effects on the rotodynamic forces. Figure 11 shows the behavior of the tangential force for different values of the temperature in the liquid. It may be observed that, as the temperature in the flow increases, the peak decreases in magnitude and eventually disappears. This behavior results from the different bubble response at different temperatures, as exemplified by Figs. 3 and 4.

A few brief remarks on the discrepancies between the present theory and experimental results are appropriate. First, the magnitude of the rotodynamic forces is systematically underestimated. A plausible cause for this discrepancy is the variation of cavitation in the radial direction in the blade channels. Also, the (rather unrealistic) assumptions that all bubbles have the same radius and therefore the same resonant dynamic behavior results in more localized peaks than those observed. Finally, the present theory necessarily neglects the secondary flows that are inevitably present in cavitating inducers (Brennen, 1994).

4 Conclusions

The results of this study show that bubble dynamics cause major modifications of the rotodynamic forces on cavitating inducers.

The propagation of the whirl-induced disturbances within the inducer is significantly modified by the large reduction of the sonic speed in the bubbly cavitating flow. The spectral response of the rotodynamic fluid forces is strongly correlated to the cloud interaction parameter, $3\alpha(1-\alpha)r^2/R_s^2$, and the relative magnitude of the excitation and bubble resonance frequencies. Most of the paper focuses on cavitation that is uninhibited by thermal effects though we also demonstrate the damping effects introduced when thermal effects become important.

The computations show that the rotodynamic fluid forces on the inducer in bubbly cavitating flows no longer vary quadratically with the whirl frequency, as in noncavitating conditions.
The theory qualitatively reproduces the main characteristics of the rotodynamic forces (though there are significant quantitative discrepancies). In particular, the occurrence of a strong destabilizing peak in the tangential force is correctly predicted, including the frequency at which it occurs and the evolution of the magnitude with cavitation number and flow coefficient. These results provide the first real insight into the complex physical phenomena observed experimentally.

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