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THE STABILITY OF AN AIR-MAINTAINED CAVITY BEHIND A STATIONARY OBJECT IN FLOWING WATER

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In studies made in the Free-Surface Water Tunnel of a projectile running in an air-maintained cavity, the experimental relation between air entrainment rate and cavitation number was determined. The entrainment-rate coefficient $C_Q = Q/V_o d^2$, where Q is the air rate in cfs, V_o the free-stream velocity, and d the disk nose diameter, was plotted against cavitation parameter, $K = (p_o - p_k)/q_o$ where p_o is the free-stream pressure at the disk center line, p_k the cavity pressure, and q_o the free-stream dynamic pressure. This experimental relationship for one single disc is shown for three different velocities in Fig. 1. The curves are similar in shape and each has a minimum value of entrainment coefficient which is designated by C_Q^* at a value of K designated as K^*.

Since an air cavity is a two-phase flow phenomenon, Froude number was considered as a significant parameter. A plot of the reciprocal of Froude number based on nose diameter, $F_d = V_o / \sqrt{gd}$, against the value of K^* yields the near-linear relation shown in Fig. 2. Since the slope obtained was nearly constant, a method of plotting all the data together on the basis of C_Q vs. $K F_d$ was suggested. This was done for all the data taken for air entrainment behind a flat-disk nose yielding approximately a single graphical relationship. This relationship could be generalized further since Reichardt (1)[1] showed that if the maximum cavity diameter were taken as a significant dimension, the drag coefficient and the length-diameter ratio of the cavity would be related to K in a manner that is independent of the nose geometry. By using Reichardt's relationship of cavity diameter to disk diameter (d_m/d) vs. K, the entrainment data were plotted on the basis of maximum cavity diameter (C_{Qm} vs. $K F_m$). The resulting entrainment relationship shown in Fig. 3 is independent of the nose shape.

The minimum point and the dip in the curves in Fig. 1 are of interest since two values of K representing two sizes of cavity are indicated for a single entrainment rate in this region. The portion to the right of the minimum point is of particular interest in that it indicates a decreasing amount of air is needed to supply a cavity increasing in size, therefore the cavity configuration is unstable. The portion to the left of the minimum points, indicating an increasing

[1] Numbers in parentheses refer to the references listed at the end of this paper.

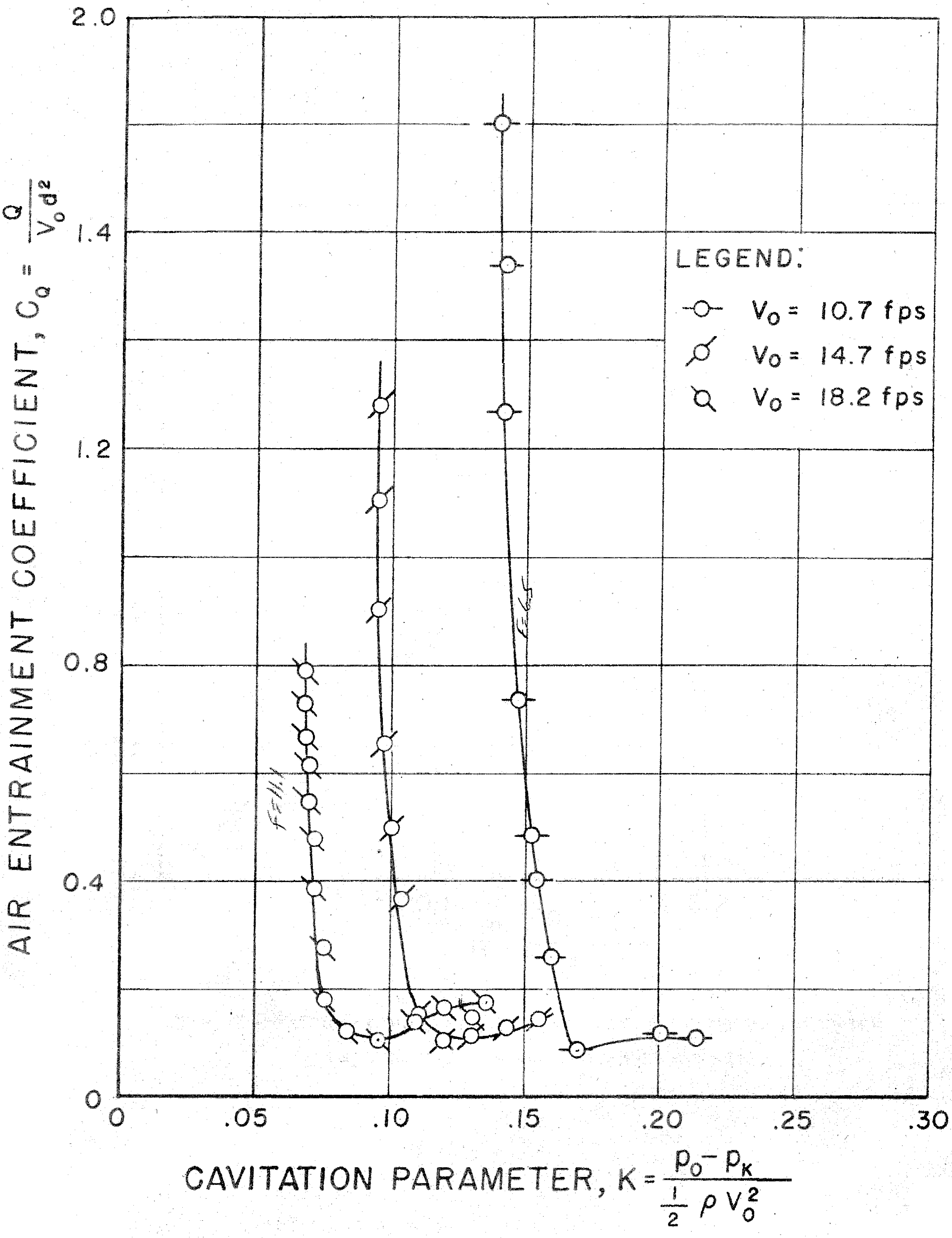

Fig. 1 - Effect of velocity and cavitation parameter on the entrainment of air in a cavity produced by a 1-in. diameter disk nose

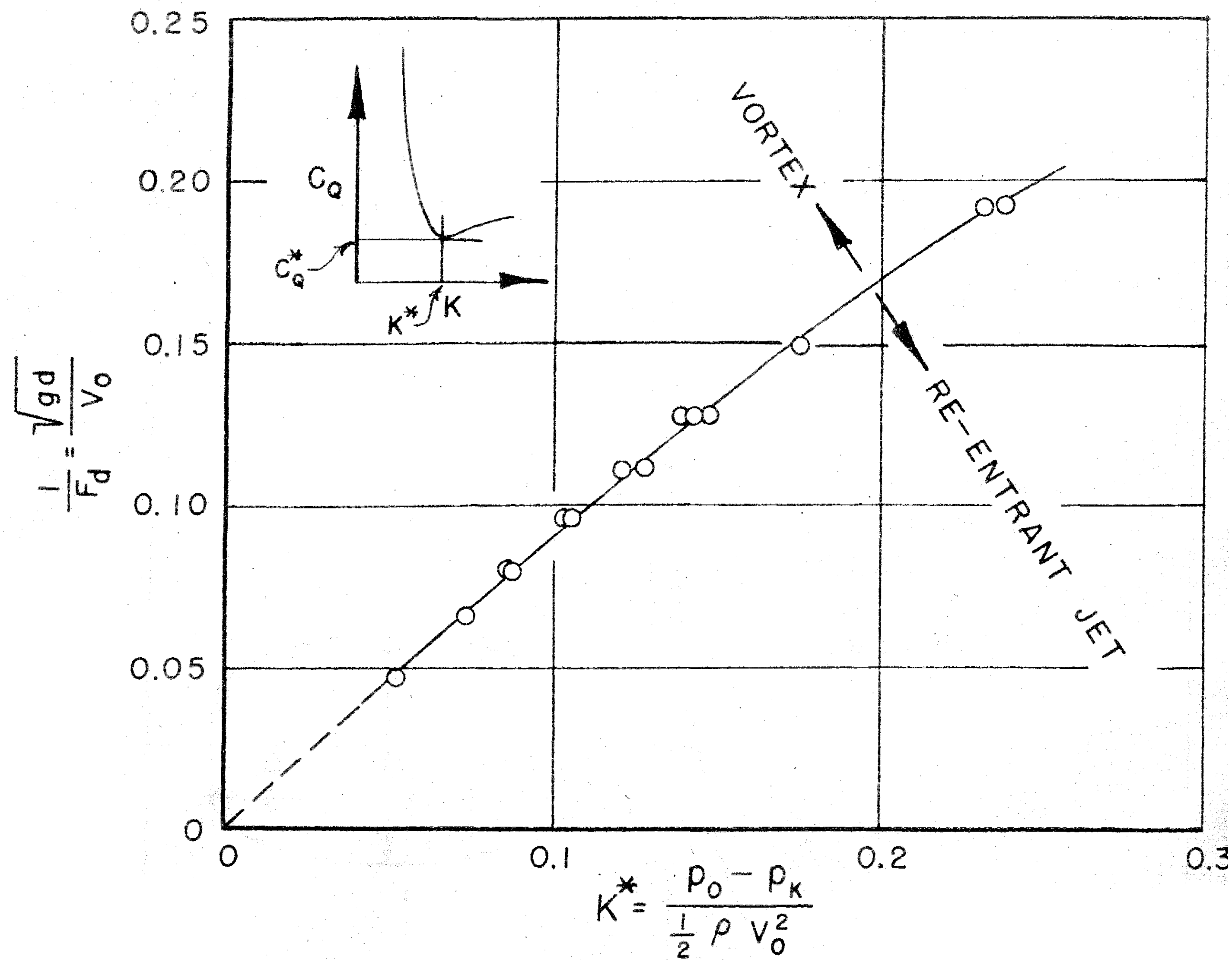

Fig. 2 - Influence of gravity on the type of cavity as indicated by the point of minimum air entrainment

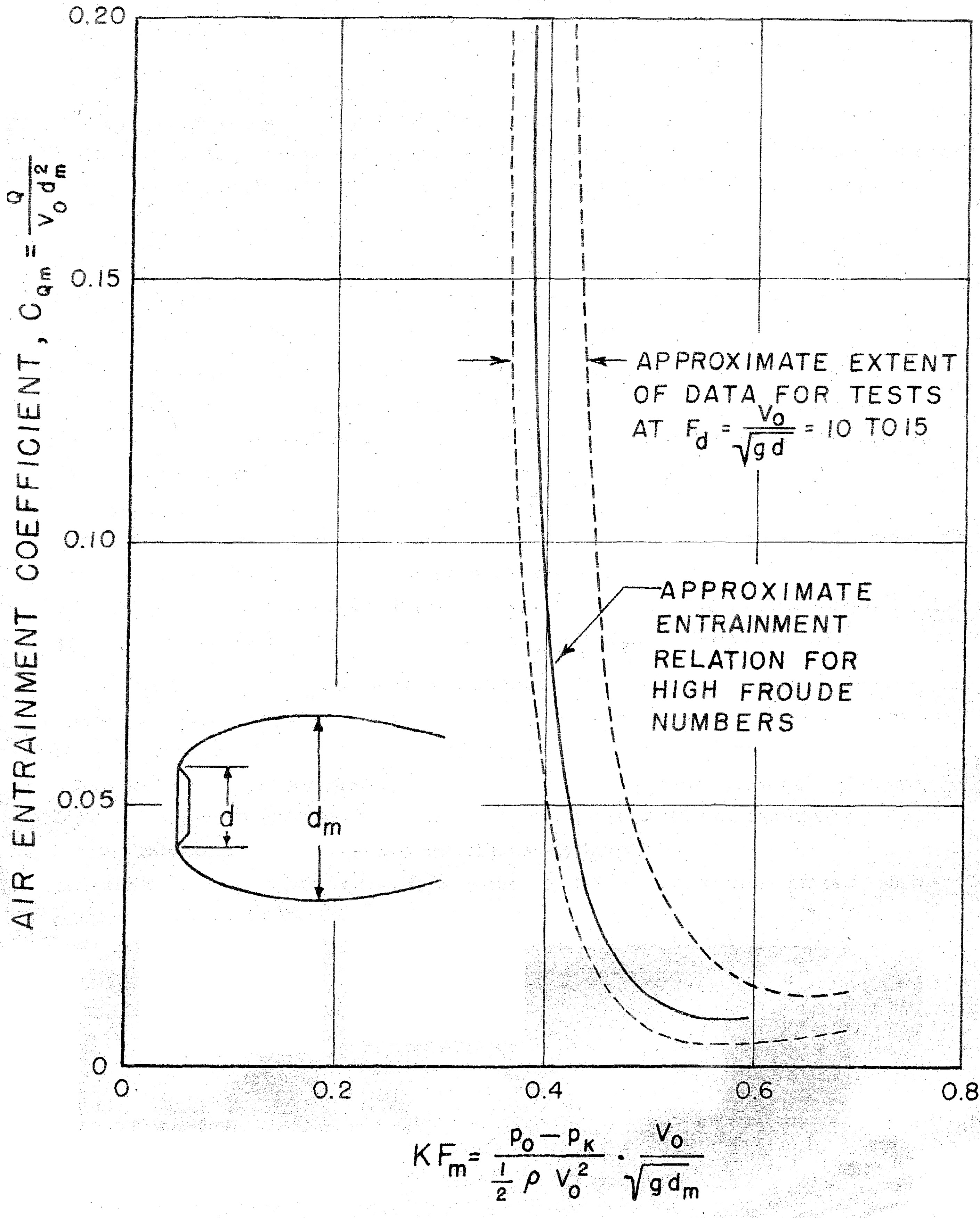

Fig. 3 - Cavity air entrainment coefficient vs. the product of cavitation number and Froude number. The maximum cavity diameter d_m is used as the reference length for the Froude number $F_m = V_o/(\sqrt{g d_m})$.

entrainment with increasing cavity size, appeared more logical. The following is an attempt to qualitatively describe the phenomena responsible for both portions of the curves and to give the corresponding reasons for the shape of the curves.

The experimental apparatus consisted of an arrangement for supporting flat disks in the Free-Surface Water Tunnel. The supporting strut contained lines for injecting air at varying rates immediately behind the disk and for measuring the pressure in the resulting open cavity.

With the tunnel running at constant velocity, a small opening of the air supply valve caused bubbles to be mixed with the water in the wake behind the disk. As the air injection rate was slowly increased, the frothy wake began to be swept downstream. This development of the open cavity required an injection rate equal to or slightly higher than the highest rate indicated at the right hand end of the entrainment curves. Once the cavity began to be formed, the rate could be held constant but the cavity would continue to grow until a stable configuration was reached at the cavitation number corresponding to this entrainment as indicated on the branch of the plots to the left of the minimum point.

The cavity makes considerable change in configuration, aside from its change in length, during the transition stages. The short cavity has a strong re-entrant jet that almost fills the cavity space with a tumbling mixture of jet liquid and cavity gas as shown in Fig. 4. A clean, smooth jet is not ordinarily formed because it strikes the cavity wall and disturbs its own formation. As the cavitation number decreases and the cavity length increases, the re-entrant jet roller decreases and is finally replaced by the stable, double-vortex configuration as shown in Fig. 5.

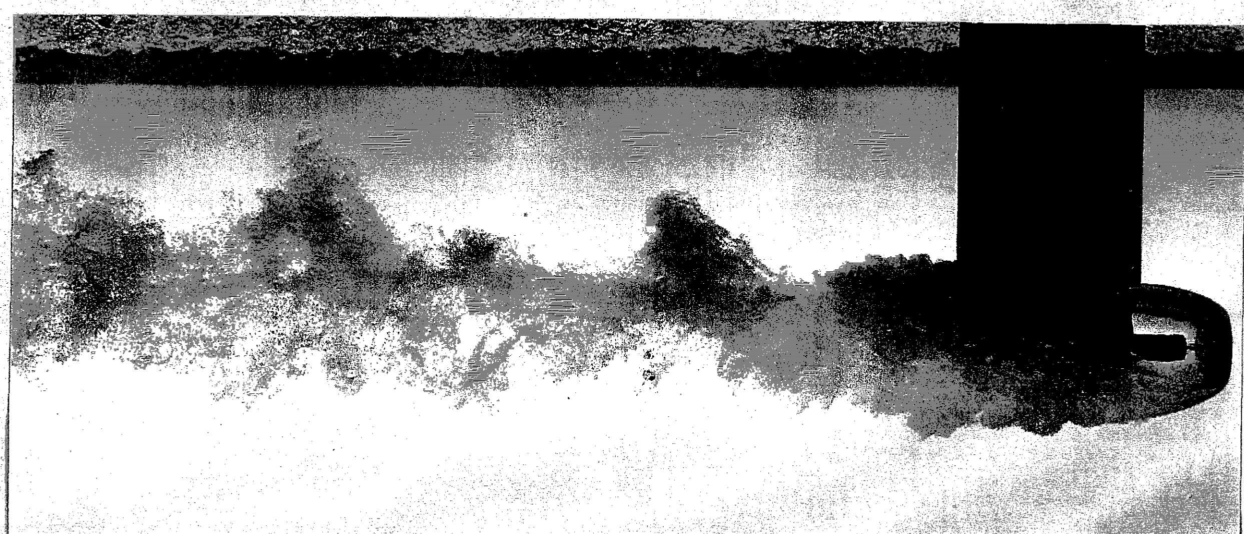

Fig. 4 - A short cavity with a strong re-entrant jet

Fig. 5 - View from underneath cavity showing trailing vorticees in horizontal plane
Nose diameter = 1 inch

Once the stable configuration was established, it was retained for a variation in Q throughout the range represented by the portion of the curves to the left of the minimum point. With the injection rate at or below this minimum, the cavity would begin to collapse and in this process go through the stages producing a re-entrant jet as it did before on opening. The point of minimum entrainment was observed to coincide approximately with the transition from the double-vortex to the re-entrant jet configuration. At this point there was little turbulent mixing of cavity gas in the jet roller and the vortex tubes had not fully developed to rapidly conduct air away from the cavity.

Further consideration of the reasons for the shape of the entrainment curves is in order. The portion to the left of the minimum point has a slope in the direction that was immediately considered logical but the extreme steepness reached was an unpredicted experimental result.

The increased entrainment with decreasing cavitation number appeared logical because this is synonymous with an increase in cavity size and pressure. The extreme slope, however, is produced by the mechanism of the double vortex cavity closure that was discovered in the course of this investigation. The vortex appears to open up passages that very effectively transmit the cavity gas downstream. Once the vortex is well formed, it takes very little increase in cavity pressure to open the passages sufficiently for a rapid increase of the entrainment rate. As a result, there is a limiting low cavitation number that can be reached with air-maintained cavities, if abnormally high injection rates are to be avoided. This limiting condition depends on the full development of the double vortex openings and is a function of the Froude number as indicated by Figures 2 and 3.

The portion of the entrainment curves to the right of the minimum point indicates an increasing entrainment with increasing cavitation number and decreasing cavity size and pressure. This is brought about by the changing configuration due to the action of gravity in altering the strength of the re-entrant jet.

At some high cavitation number where the cavity is short, there is relatively little gravitational action distorting the streamlines from the axially symmetrical configuration. The re-entrant jet is consequently well-formed and it brings into the cavity sufficient liquid to form a large, tumbling roller that becomes well mixed with cavity gas, thereby causing the entrainment to be high. At a lower cavitation number the cavity length is increased and there is consequently a longer time for the gravity forces to bring about a distortion of the axially symmetrical re-entrant jet cavity. This distortion is such that it begins to oppose and decrease the re-entrant jet. The turbulent roller and its air-entraining action is consequently reduced.

The reduction of the re-entrant jet is accomplished as follows: With decreasing K, the cavity becomes long enough for the combination of the varying hydrostatic pressure at different levels in the liquid surrounding the cavity and the constant pressure from top to bottom at the cavity wall to cause the bottom streamlines to rise relative to those higher up along the sides. The curved bottom surface of the cavity tends to be flattened out by this gravitational action; but the bottom streamlines overshoot and form a central ridge. This ridge or hump is similar to the common "fountain" or "roach" that forms in the wake of planing bodies. The "fountain", in rising to the central regions of the cavity, impinges on and opposes the portion of the re-entrant jet formed by the streamlines midway up and on opposite sides of the cavity. Such opposite streamlines are now the only ones that meet at the same axial position downstream. So it is seen that bottom streamlines rise and oppose the re-entrant jet that might be formed by the side streamlines while the top streamlines are of no help in forming the jet since they are brought to the closure position still farther downstream. In this manner the re-entrant jet is finally completely stopped by this manifestation of the gravity action and in its place the rising "fountain", now higher than the side streamlines, creates the double-vortex type of cavity closure.

The rise in the cavity floor relative to the sides might also be considered as the effect of the varying cavitation number from top to bottom of a finite cavity under the influence of gravity. When the difference in free-stream pressure p_o from top to bottom of the cavity is sufficient to alter the cavitation number enough to make a significant difference in cavity length, the bottom streamlines will be brought in for a cavity closure sooner than those on the sides and top, and the "fountain" will result as before.

A secondary factor which might produce a slight tendency toward decreased entrainment with a decrease in cavitation number is independent of the gravity effect and dependent only on the characteristics of the re-entrant jet as a function of K. Relations derived by Birkhoff (2) and O'Neill (3) show that

$$V_j = V_o (1 + K)^{1/2},$$

as obtained by application of the Bernoulli equation to a cavity boundary streamline, and the approximate relation

$$A_j \approx \frac{A_D C_{D0}}{4} (1 + K/4)$$

results from momentum considerations. Here, V_j and A_j are the velocity and area of the re-entrant jet and A_D is the separation area of the nose having a drag coefficient C_{D0} at $K = 0$. Since both the velocity and the area of the jet are decreased with decreasing cavitation number, the disturbance and air mixing might be expected also to decrease. This is, of course, contrary to our first expectation that the larger cavity would have greater entrainment. This variation in jet velocity and area could produce only a very small effect since the rate of variation and the range in cavitation number involved is small. The gravitational action, on the other hand, effects the entire elimination of the re-entrant jet.

The shape of the entrainment curves, as characterized by a point of minimum entrainment with an increase for either higher or lower cavitation numbers, is thus seen to be brought about by the action of gravity in eliminating one entrainment process--the turbulent, re-entrant jet roller, and initiating another--the double-vortex cavity closure. The cleanest cavity closure, without forward splash or without open passages for conducting away cavity gas, occurs at the cavitation number and Froude number at which the transition occurs.

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