[1] *Z. Physik*, **43,** 658 (1927).
[2] *Naturwissenschaften*, Nov. 20, **1925.**
[3] *Proc. Nat. Acad. Sci.*, Feb., **1926.**
[4] *Z. Physik*, **44,** 292 (1927).
[5] *Ann. Physik*, **83,** 956 (1927).
[6] *Z. Physik*, **40,** 492; **43,** 624 (1927).
[7] *Ibid.*, **43,** 788 (1927).
[8] Eldridge, *Phys. Rev.*, **24,** 234 (1924).
[9] G. W. Kellner, *Z. Physik*, **44,** 91 (1927).
[10] Ellett and Macnair, *Proc. Nat. Acad. Sci.*, Aug., **1927.** The departure from 100% is due to the fine structure components which behave anomalously.
[11] *Z. Physik*, **30,** 93 (1924).
[12] *Phil. Mag.*, **3,** 1306 (1927).
[13] *Proc. Roy. Soc.*, **112A,** 642 (1926).
[14] W. Hanle, *Z. Physik*, **41,** 164 (1927).
[15] *J. O. S. A. and R. S. I.*, **7,** 415 (1923).

THE SYMMETRY OF THE STRESS-TENSOR OBTAINED BY SCHROEDINGER'S RULE

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In the recent developments of the calculus of variations used in the new quantum theory the problem arises of finding a general expression for a world-function such that a symmetrical stress-energy tensor may be derived from it by means of Schroedinger's rule.[1]

We shall consider here the analogous problem for the case of a Euclidean space of three dimensions in which the rectangular coördinates of a selected point P are x, y, z. To simplify matters we shall consider a world-function L which depends only on the first derivatives of the components u, v, w of a single vector q associated with the point P. Applying the rule used by Schroedinger in his discussion of Gordon's equations, but with the necessary modifications appropriate for a space of three dimensions, we may associate with L a tensor T with mixed components[2] of type

$$T_{23} = u_2 \frac{\partial L}{\partial u_3} + v_2 \frac{\partial L}{\partial v_3} + w_2 \frac{\partial L}{\partial w_3} + v_1 \frac{\partial L}{\partial w_1} + v_2 \frac{\partial L}{\partial w_2} + v_3 \frac{\partial L}{\partial w_3}$$

the suffixes 1, 2, 3 being used to denote differentiations with respect to x, y, z, respectively.

In order that this tensor may be symmetric the relations

$$T_{23} = T_{32}, \qquad T_{31} = T_{13}, \qquad T_{12} = T_{21}$$

must be satisfied and these give the partial differential equations

$$(v_2 - w_3)\left(\frac{\partial L}{\partial v_3} + \frac{\partial L}{\partial w_2}\right) + (v_3 + w_2)\left(\frac{\partial L}{\partial w_3} - \frac{\partial L}{\partial v_2}\right)$$
$$+ v_1 \frac{\partial L}{\partial w_1} - w_1 \frac{\partial L}{\partial v_1} + u_2 \frac{\partial L}{\partial u_3} - u_3 \frac{\partial L}{\partial u_2} = 0$$

$$(w_3 - u_1)\left(\frac{\partial L}{\partial w_1} + \frac{\partial L}{\partial u_3}\right) + (w_1 + u_3)\left(\frac{\partial L}{\partial u_1} - \frac{\partial L}{\partial w_3}\right)$$
$$+ w_2 \frac{\partial L}{\partial u_2} - u_2 \frac{\partial L}{\partial w_2} + v_3 \frac{\partial L}{\partial v_1} - v_1 \frac{\partial L}{\partial v_3} = 0$$

$$(u_1 - v_2)\left(\frac{\partial L}{\partial v_1} + \frac{\partial L}{\partial u_2}\right) + (u_2 + v_1)\left(\frac{\partial L}{\partial v_2} - \frac{\partial L}{\partial u_1}\right)$$
$$+ u_3 \frac{\partial L}{\partial v_5} - v_3 \frac{\partial L}{\partial u_3} + w_1 \frac{\partial L}{\partial w_2} - w_2 \frac{\partial L}{\partial w_1} = 0.$$

These equations form a complete system and since there are nine independent variables and three equations, the general solution should be of the form

$$L = f(\alpha, \beta, \gamma, \theta, \phi, \psi)$$

where $\alpha, \beta, \gamma, \theta, \phi, \psi$ are independent solutions.

Writing

$$a = u_1, \qquad b = v_2, \qquad c = w_3,$$

$$2f = w_2 + v_3, \quad 2g = u_3 + w_1, \quad 2h = v_1 + u_2,$$
$$2\xi = w_2 - v_3, \quad 2\eta = u_3 - w, \quad 2\zeta = v_1 - u_2$$

we choose as our independent solutions the quantities

$$\alpha = \xi^2 + \eta^2 + \zeta^2$$
$$\beta = a\xi^2 + b\eta^2 + c\zeta^2 + 2f\eta\zeta + 2g\zeta\xi + 2h\xi\eta$$
$$\gamma = (bc - f^2)\xi^2 + (ca - g^2)\eta^2 + (ab - h^2)\zeta^2 + 2(gh - af)\eta\zeta + 2(hf - bg)\zeta\xi + 2(fg - ch)\xi\eta$$

$$\gamma = a + b + c$$
$$\phi = bc - f^2 + ca - g^2 + ab - h^2$$
$$\psi = abc + 2fgh - af^2 - bg^2 - ch^2$$

If

$$A = u^2_1 + v^2_1 + w^2_1 \qquad F = u_2u_3 + v_2v_3 + w_2w_3$$
$$B = u^2_2 + v^2_2 + w^2_2 \qquad G = u_3u_1 + v_3v_1 + w_3w_1$$
$$C = u^2_3 + v^2_3 + w^2_3 \qquad H = u_1u_2 + v_1v_2 + w_1w_2$$

it is easily seen that the following quantities are also solutions

$$\rho = A + B + C$$

$$\sigma = BC - F^2 + CA - G^2 + AB - H^2$$
$$\tau = ABC + 2FGH - AF^2 - BG^2 - CH^2$$
$$\omega = A\xi^2 + B\eta^2 + C\zeta^2 + 2F\eta\zeta + 2G\zeta\xi + 2H\xi\eta$$

$$\Delta = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$X = v_2w_3 - v_3w_2 + w_3u_1 - w_1u_3 + u_1v_2 - u_2v_1$$

These quantities can, however, be expressed in terms of α, β, γ, θ, ϕ, ψ for we have the relations

$$\begin{aligned} 2\rho &= \alpha + 2\theta^2 + 4\phi \\ \sigma &= (\alpha - \phi)^2 + 4\gamma + 2\theta(\beta - \psi) \\ \tau &= \Delta^2 \\ \omega &= \gamma + \theta\beta - \phi\alpha \\ 4\Delta &= \beta + 4\psi \\ X &= \phi + \alpha. \end{aligned}$$

Many of the solutions which have been written down represent quantities of importance in the theory of finite displacements and the theory of elasticity.[3] The three-dimensional problem was, in fact, studied with the aim of finding a general type of world function which can be used in the theory of elasticity. For the purpose of extending the present results to a space of n dimensions and in particular to space-time geometry it will be useful to note a very general type of solution of our partial differential equations. Let p, q, r, s be arbitrary parameters then the partial differential equations are satisfied by the determinant

$$L = \begin{vmatrix} pu_1 + qa + rA + s & pu_2 + qh + rH & pu_3 + qg + rG \\ pv_1 + qh + rH & pv_2 + qb + rB + s & pv_3 + qf + rF \\ pw_1 + qg + rG & pw_2 + qf + rF & pw_3 + qc + rC + s \end{vmatrix}$$

Another point which may be mentioned is that the differential equations are unaltered in form if we replace u, v, w by u', v', w' where

$$u' = u + kx, \qquad v' = v + ky, \qquad w' = w + kz$$

and k is an arbitrary parameter. Using primes to denote quantities derived from u', v', w', we have

$$\begin{array}{lll} u_1^1 = u_1 + k & u_2' = u_2 & u_3' = u_3 \\ v_1^1 = v_1 & v_2' = v_2 + k & v_3' = v_3 \\ w_1^1 = w_1 & w_2' = w_2 & w_3' = w_3 + k \\ \xi^1 = \xi & \eta^1 = \eta & \zeta^1 = \zeta \\ \alpha^1 = \alpha & \beta^1 = \beta + k\alpha & \gamma^1 = \gamma + k(\alpha\theta - \beta) + k^2\alpha \\ \theta^1 = \theta + 3k & \phi^1 = \phi + 2k\theta + k^2 & \psi^1 = \psi + k\phi + k^2\theta + k^3 \end{array}$$

It is clear that if any solution of our equations is expressed in terms of the derivatives of u', v', w' and then expanded in powers of k the coefficients of the different powers of k will all be solutions of the equations.

[1] E. Schroedinger, *Ann. Physik*, **82** (1927), 265.

[2] We call T_{11} a pure and T_{23} a mixed component. The expression for T_{11} is found by a somewhat similar rule except that there is an additional term—L.

[3] See, for instance, Love's Treatise, Ch. I, Appendix. L. Brillouin, *Ann. Physique*, (10) **3** (1925), 251.

SOME EXTENSIONS OF THEORY AND MEASUREMENTS OF SHOT-EFFECT IN PERIODIC CIRCUITS

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Introduction.—The paper published last year by Williams and Vincent[1] contained computations of the expected magnitude of the effect in an aperiodic circuit of the probability fluctuations of anode current in a vacuum tube. The measurements submitted were made with a system of this type. There appeared there also a discussion of the problem of the tuned circuit in which a method was suggested to simplify the necessary measurements. Since this paper is concerned with some additions to this method and measurements made thereon, a brief résumé seems desirable here.

Simplification of the Tuned Circuit Problem.—The power factor of the circuit varies with the thermionic current, but it may be maintained constant by the addition of suitable resistances. Let this be done.

If now we take the expression given by Hull and Williams[2] for the mean square output voltage due to the k-component of the thermionic current, we have

$$\overline{E_k^2} = {}^1/_2 L^2 \omega_0{}^2 A_0^2 C_k^2 \frac{x^2 f(x)}{(1 - x^2)^2 + r^2 x^2},$$

where

L is the inductance in the shot-circuit,

ω_0 is 2π times the resonance frequency of the amplifier,

A_0 is the voltage amplification at the resonance frequency,

C_k is the coefficient of the k-component of the Fourier series into which the thermionic current was originally developed by Schottky[3]

$x = \omega/\omega_0$