Scattering and Absorption of Gamma-Rays

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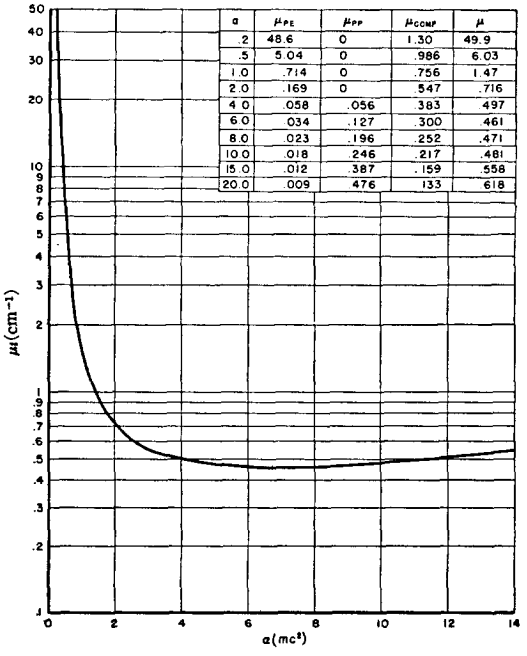
A formulation is presented of the scattering and absorption of gamma-rays in different materials. The range of gamma-ray energies considered is from 1 to 10 mc^2. Results are given for the transmission of gamma-rays through air and lead.

INTRODUCTION

THE release of nuclear energy on a practical scale involves, in general, the emission of large numbers of gamma-rays. The problem of concern in this paper may be stated as follows: for a source of given geometry, intensity, and energy spectrum of the emitted gamma-rays, what are the intensities and energy distributions of these radiations at varying distances from the source? When the intensities and energy distributions are known as functions of distance from the source, the resulting physical and biological effects of the radiations may be determined. Such information is the basis for the determination of the radiological hazards of these radiations and also makes possible the calculation of shielding requirements.

There are three processes which take place as gamma-rays pass through matter: photoelectric effect, pair production, and Compton scattering.



α	μ_{PE}	μ_{PP}	μ_{COMP}	μ
.2	48.6	0	1.30	49.9
.5	5.04	0	.986	6.03
1.0	.714	0	.756	1.47
2.0	.169	0	.547	.716
4.0	.058	.056	.383	.497
6.0	.034	.127	.300	.461
8.0	.023	.196	.252	.471
10.0	.018	.246	.217	.481
15.0	.012	.387	.159	.558
20.0	.009	.476	.133	.618

FIG. 1. Total absorption coefficient *vs* gamma-ray energy, Pb_{82}(lead). [For μ (ordinate and heading of last column) read μ_t.]

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Photoelectric Effect

The gamma-ray, or photon, is absorbed by an atom of the material with the ejection of one of the bound electrons in the atom. This process has a probability which increases rapidly with atomic number. The probability of this process also increases as the energy of the photon decreases down to the threshold energy characteristic of the atom.

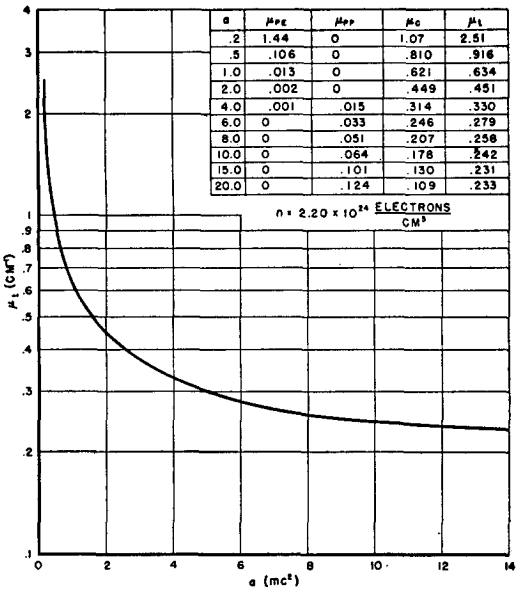
Pair Production

The photon is annihilated in the nuclear field with the creation of an electron and positron pair. The threshold, or minimum, energy of the gamma-ray for this process is the rest energy of the electron pair, 2 mc^2, where m is the rest mass of the electron and c is the velocity of light. The probability of this absorption process increases with the photon energy and with atomic number.

Compton Scattering

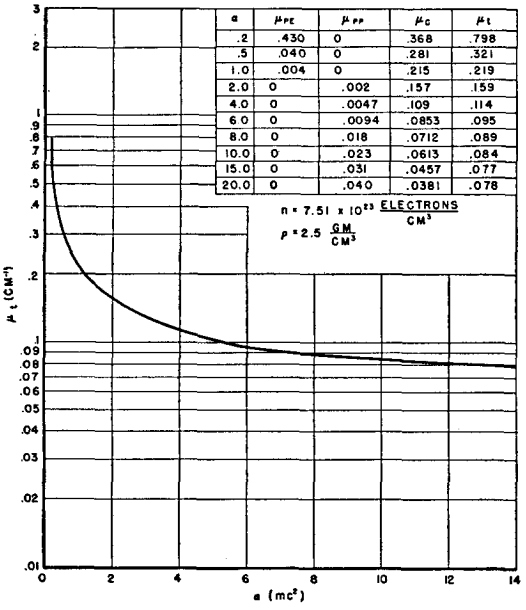
The photon is inelastically scattered by a collision with an electron in the atom. The scattered photon has diminished momentum and energy. The probability of this scattering process decreases with increasing photon energy and is proportional to the number of atomic electrons.

For a quantitative discussion of these effects, it is necessary to define the terms which are customarily used. The probability for gamma-ray absorption or scattering is given in terms of a cross section. For example, the cross section (per atom) for photoelectric absorption, σ_{PE}, is defined as the probable number of photoelectric absorptions divided by the number of incident photons and divided by the number of atoms per cm^2 of the medium. The atomic cross section, σ_{PP}, for pair production is defined in a similar manner. For Compton scattering, the cross section (per electron) is defined as the probable number of inelastic scatterings divided by the number of incident photons and divided by the number of electrons per cm^2 of the medium. The photoelectric absorption coefficient is defined as the product of the atomic cross section in cm^2 by the number of atoms per cm^3 and has the dimension cm^{-1}. It is seen, from the definition of the atomic cross section, that the photoelectric absorption coefficient gives the probable



α	μ_{PE}	μ_{PP}	μ_C	μ_t
.2	1.44	0	1.07	2.51
.5	.106	0	.810	.916
1.0	.013	0	.621	.634
2.0	.002	0	.449	.451
4.0	.001	.015	.314	.330
6.0	0	.033	.246	.279
8.0	0	.051	.207	.258
10.0	0	.064	.178	.242
15.0	0	.101	.130	.231
20.0	0	.124	.109	.233

FIG. 2. Total absorption coefficient *vs* gamma-ray energy, Fe_{26}(iron).



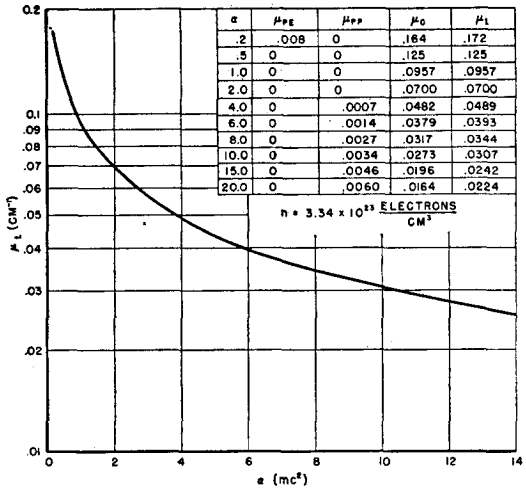
α	μ_{PE}	μ_{PP}	μ_C	μ_t
.2	.430	0	.368	.798
.5	.040	0	.281	.321
1.0	.004	0	.215	.219
2.0	0	.002	.157	.159
4.0	0	.0047	.109	.114
6.0	0	.0094	.0853	.095
8.0	0	.018	.0712	.089
10.0	0	.023	.0613	.084
15.0	0	.031	.0457	.077
20.0	0	.040	.0381	.078

FIG. 3. Total absorption coefficient *vs* gamma-ray energy concrete (SiO_2).

number of photoelectric absorptions per incident photon and per cm of path. The mean free path is defined as the reciprocal of the absorption coefficient. The same definitions apply to the pair production absorption coefficient and the mean free path for this process.

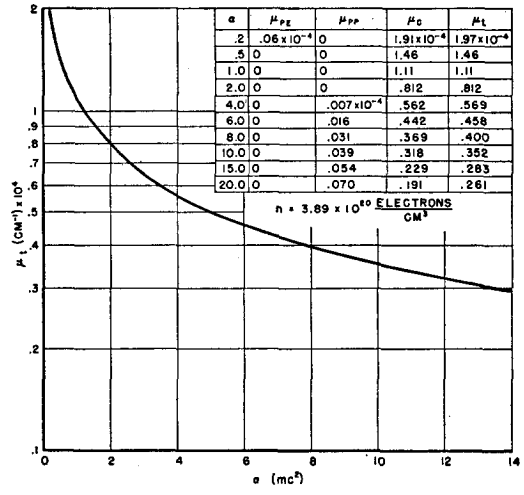
The scattering coefficient for Compton scattering is defined, similarly, as the product of the scattering cross section per electron in cm^2 by the number of electrons per cm^3; and the mean free path is the reciprocal of the scattering coefficient and represents the probable distance traveled by the photon between scatterings.

One may define the total "absorption" coefficient for



α	μ_{PE}	μ_{PP}	μ_C	μ_t
.2	.008	0	.164	.172
.5	0	0	.125	.125
1.0	0	0	.0957	.0957
2.0	0	0	.0700	.0700
4.0	0	.0007	.0482	.0489
6.0	0	.0014	.0379	.0393
8.0	0	.0027	.0317	.0344
10.0	0	.0034	.0273	.0307
15.0	0	.0046	.0196	.0242
20.0	0	.0060	.0164	.0224

FIG. 4. Total absorption coefficient *vs* gamma-ray energy, H_2O.



α	μ_{PE}	μ_{PP}	μ_C	μ_t
.2	$.06\times10^{-4}$	0	1.91×10^{-4}	1.97×10^{-4}
.5	0	0	1.46	1.46
1.0	0	0	1.11	1.11
2.0	0	0	.812	.812
4.0	0	$.007\times10^{-4}$	.562	.569
6.0	0	.016	.442	.458
8.0	0	.031	.369	.400
10.0	0	.039	.318	.352
15.0	0	.054	.229	.283
20.0	0	.070	.191	.261

FIG. 5. Total absorption coefficient *vs* gamma-ray energy, (Air).

gamma-rays as the sum of the photoelectric absorption coefficient, the pair production absorption coefficient, and the Compton scattering coefficient. The total absorption coefficient gives the probability per cm of a removal of a gamma-ray from the primary beam, either by absorption or by scattering. Also, the total mean free path for all three processes is defined as the reciprocal of the total absorption coefficient and gives the probable distance traveled in which any one of the three processes may occur.

Let μ_{PE} be the photoelectric absorption coefficient, μ_{PP} the pair production absorption coefficient, μ_C the Compton scattering coefficient, and $\mu_t = \mu_{PE} + \mu_{PP} + \mu_C$ the total absorption coefficient. The values of these coefficients for lead, iron, concrete, water, and air are tabulated in Figs. 1 through 5 for various values of the photon energy, α. The photon energy, α, is expressed in units of mc^2, the rest energy of the electron, which is 0.51 Mev. In these figures, μ_t is also graphed as a function of α. In the photon energy range of present interest, μ_{PE} and μ_{PP} are very small compared with μ_C for sub-

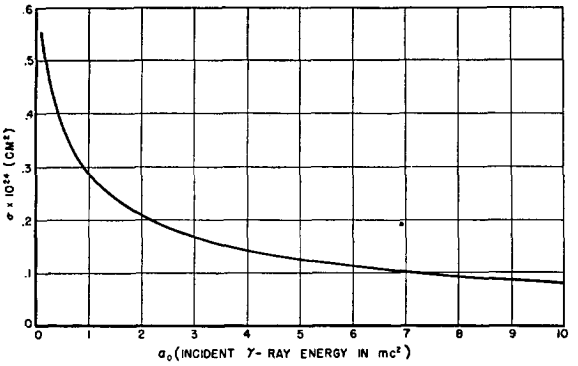


FIG. 6. Total Klein-Nishina cross section per electron *vs* incident gamma-ray energy.

stances such as concrete, water, and air. These substances contain only elements with low atomic numbers Z. Since μ_{PE} is essentially proportional to Z^5, and μ_{PP} is essentially proportional to Z^2, it is evident why these absorption effects become important in the elements of high atomic number.

SCATTERING OF GAMMA-RAYS

As has been pointed out, the scattering process for gamma-rays is Compton scattering. Compton scattering is an inelastic collision between a photon and an electron. If the incident photon of energy $\alpha_0 mc^2$ is deflected through the angle θ after collision with an electron initially at rest, the energy of the scattered photon, $\alpha_1 mc^2$, is determined from the laws of conservation of energy and momentum to be

$$\alpha_1=\frac{\alpha_0}{1+\alpha_0(1-\cos\theta)}. \tag{1}$$

The scattering cross section per electron, σ, for this process is given by the Klein-Nishina formula

$$\sigma=2\pi r_0^2\left\{\frac{1+\alpha_0}{\alpha_0^2}\left[\frac{2(1+\alpha_0)}{1+2\alpha_0}-\frac{1}{\alpha_0}\ln(1+2\alpha_0)\right]+\frac{1}{2\alpha_0}\ln(1+2\alpha_0)-\frac{1+3\alpha_0}{(1+2\alpha_0)^2}\right\}, \tag{2}$$

where r_0 is the classical radius of the electron ($r_0=e^2/mc^2=2.82\times10^{-13}$ cm), and α_0 is the incident photon energy in units mc^2. In Fig. 6, σ is plotted as a function of α_0. The differential cross section for scattering into the element of solid angle $d\Omega=2\pi\sin\theta d\theta$ at θ is

$$\frac{d\sigma}{d\Omega}=\frac{r_0^2}{2}\cdot\frac{1}{[1+\alpha_0(1-\cos\theta)]^2}\times\left\{1+\cos^2\theta+\frac{[\alpha_0(1-\cos\theta)]^2}{1+\alpha_0(1-\cos\theta)}\right\}. \tag{3}$$

The scattering cross section per unit solid angle, $d\sigma/d\Omega$, is shown in Fig. 7 as a function of the angle of scatter θ for various values of the incident energy α_0.

In the determination of the intensities of gamma-radiation as a function of the distance from the source, it is convenient to consider distances in two ranges.

1. Distances small compared with the mean free path.
2. Distances of the order of a mean free path and greater.

For distances small compared with the mean free path, the intensity is accurately given by the transmitted unscattered photons and the singly scattered photons. For distances of the order of a mean free path and greater, the contributions from the photons scattered twice, three times, etc., become increasingly important.

GAMMA-RAY INTENSITIES AT DISTANCES FROM THE SOURCE SMALL COMPARED WITH THE MEAN FREE PATH

The basic scattering problem is the calculation of the intensity at a point a given distance a from a monochromatic (or monoenergetic) point source emitting gamma-rays isotropically. Let N_0 be the number of photons emitted from the source with energy α_0. The number of photons per cm², which arrive at a receiver at distance a from the source, without suffering any collisions, is†

$$I^0=\frac{N_0}{4\pi a^2}\exp[-\mu_t(\alpha_0)a]. \tag{4}$$

The energy received per cm² in units of mc^2 is

$$E^0=\alpha_0 I^0=\frac{\alpha_0 N_0}{4\pi a^2}\exp[-\mu_t(\alpha_0)a]. \tag{5}$$

For distances $a\ll\lambda_t$, the only other contribution of significance comes from singly scattered photons. This intensity is readily computed as follows. The probable

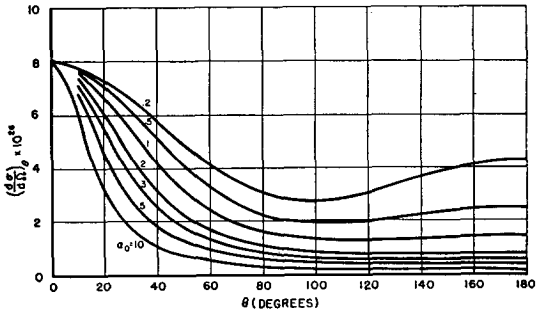


FIG. 7. Klein-Nishina angular distribution scattering cross section per unit solid angle *vs* angle of scatter.

† The exponential factor is easily understood as follows. In terms of the mean free path $\lambda_t=1/\mu_t$, the probability of a collision in the distance dx is dx/λ_t. If $f(x)$ is the probability that the particle will go a distance x without a collision, the probability that the particle will go a distance $x+dx$ without a collision is $f(x+dx)$, and

$$f(x+dx)=f(x)+\frac{df}{dx}dx=f(x)\left[1-\frac{dx}{\lambda_t}\right],$$

or

$$df/dx=-f(x)/\lambda_t=-\mu_t f(x).$$

Thus,

$$f(x)=e^{-\mu_t x}.$$

number of gamma-rays per cm² which reach the element of volume at P (see Fig. 8), is

$$\frac{N_0}{4\pi r_1^2}\exp[-\mu_t(\alpha_0)r_1];$$

the probability of scattering in the element of volume at P into the element of solid angle $d\Omega$ about θ is

$$n\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} dv_p d\Omega,$$

where n is the number of electrons per cm³, and $dv_p = 2\pi r_2^2 dr_2 \sin\theta d\theta$ is the element of volume at P; and $d\Omega = \Delta S/r_2^2$, where ΔS is the element of area at R as seen from P. The probability that a scattered photon goes from P to R without a collision is $\exp[-\mu_t(\alpha_1)r_2]$, where α_1 is given in terms of α_0 by Eq. (1).

Thus, the number of photons per cm² singly scattered from S to R is

$$d^2I^{(1)} = \frac{N_0}{4\pi r_1^2}\{\exp[-\mu_t(\alpha_0)r_1]\}n\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \cdot\frac{2\pi r_2^2 \sin\psi d\psi dr_2}{r_2^2}\exp[-\mu_t(\alpha_1)r_2];$$

$$d^2I^{(1)} = \frac{N_0 n}{2r_1^2}\{\exp[-\mu_t(\alpha_0)r_1 - \mu_t(\alpha_0)r_2]\}\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \times\sin\psi d\psi dr_2. \quad (6)$$

Further, the singly-scattered energy received per cm² (in units mc^2) is just

$$d^2E^{(1)} = \alpha_1 d^2I^{(1)}. \quad (7)$$

One may express r_1 and r_2 in terms of θ and ψ, so that Eqs. (6) and (7) become

$$I^{(1)}(a) = \frac{N_0 n}{2a}\int_0^\pi d\psi\int_\psi^\pi d\theta\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \cdot\exp\left\{-\left[\mu_t(\alpha_0)\frac{a\sin\psi}{\sin\theta} + \mu_t(\alpha_1)\frac{a\sin(\theta-\psi)}{\sin\theta}\right]\right\}; \quad (8)$$

$$E^{(1)}(a) = \frac{N_0 n}{2a}\int_0^\pi d\psi\int_\psi^\pi d\theta\alpha_1\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \cdot\exp\left\{-\left[\mu_t(\alpha_0)\frac{a\sin\psi}{\sin\theta} + \mu_t(\alpha_1)\frac{a\sin(\theta-\psi)}{\sin\theta}\right]\right\}. \quad (9)$$

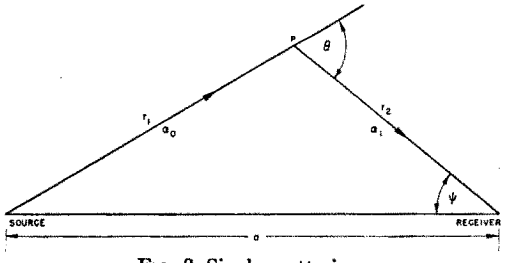


FIG. 8. Single scattering.

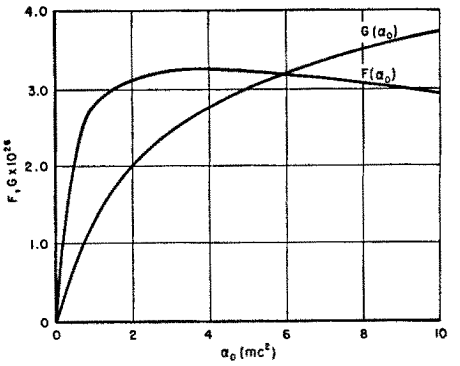


FIG. 9. $F(\alpha_0)$, $G(\alpha_0)$ *vs* α_0.

The integrals in Eqs. (8) and (9) may be greatly simplified by an approximation which is quite good for energetic gamma-rays, say of energies 2 mc^2 or greater. It is evident (see Fig. 7) that large angle scattering is much less probable at high energies than small angle scattering; further, the exponential factors most heavily weight small values of r_1 and r_2. These conditions mean also that α_1 is near α_0 in the important range of integration. Thus, one makes only a slight overestimate of the intensity if one assumes that

$$\exp[-\mu_t(\alpha_0)r_1 - \mu_t(\alpha_1)r_2] = \exp\left\{-\left[\mu_t(\alpha_0)\frac{a\sin\psi}{\sin\theta} + \mu_t(\alpha_1)\frac{a\sin(\theta-\psi)}{\sin\theta}\right]\right\} \approx \exp[-\mu_t(\alpha_0)a]. \quad (10)$$

With this approximation

$$I^{(1)}(a) = \frac{N_0 n}{2a}\{\exp[-\mu_t(\alpha_0)a]\}\int_0^\pi \theta\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} d\theta; \quad (11)$$

$$E^{(1)}(a) = \frac{N_0 n}{2a}\{\exp[-\mu_t(\alpha_0)a]\}\int_0^\pi a_1\theta\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} d\theta. \quad (12)$$

The integrals occurring in Eqs. (11) and (12) are functions only of α_0, so that

$$I^{(1)}(a) = \frac{N_0 n}{2a}\{\exp[-\mu_t(\alpha_0)a]\}F(\alpha_0); \quad (13)$$

$$E^{(1)}(a) = \frac{N_0 n}{2a}\{\exp[-\mu_t(\alpha_0)a]\}G(\alpha_0). \quad (14)$$

It should be noted that, to a good approximation, $I^{(1)}(a)$ and $E^{(1)}(a)$ vary with a, as $\exp[-\mu_t(\alpha_0)a]/a$, while $I^{(0)}(a)$ and $E^{(0)}(a)$ vary as $\exp[-\mu_t(\alpha_0)a]/a^2$. $F(\alpha_0)$ and $G(\alpha_0)$ are given as functions of α_0 in Fig. 9.

It is customary to express the radiological effects of gamma-rays in terms of the ionization they produce. The conventional unit of this ionization is the roentgen. The roentgen is here taken to be an ionization density

of one electrostatic unit of charge (1 esu) per cm³ of standard air as measured in an ionization chamber with negligible wall thickness. The conversion factor relating gamma-ray intensity with the ionization produced in air is readily found by calculation of the energy transfer to the electrons in air. The energy transfer per cm³ of air is the product of the gamma-ray energy intensity (the energy in units of mc^2 per cm²) by an appropriate absorption coefficient μ_A (cm⁻¹). At the point where it is desired to determine the ionization, let α be the energy of the gamma-rays, and let α' be the energy after a scatter. The energy transfer coefficient, μ_A, is found by expressing the differential cross section [Eq. (3)] in terms of α and α' instead of the initial energy α and the angle of scatter θ. When the differential cross section is so expressed, it will be denoted by $d\sigma/d\alpha'$. Then,

$$\mu_A(\alpha)=\frac{n}{\alpha}\int_{\alpha/(1+2\alpha)}^{\alpha}(\alpha-\alpha')\frac{d\sigma}{d\alpha'}d\alpha', \tag{15}$$

where n is the number of electrons per cm³ of standard air. μ_A is shown as a function of α in Fig. 10. The average energy required to produce an ion pair in air will be taken as 66×10^{-6} mc^2, and the ion charge will be taken as 4.80×10^{-10} esu, so that the ion charge produced in air per cm³ by the energy transfer per cm³, $E\mu_A$, is

$$r\left(\frac{\text{esu}}{\text{cm}^3}\right)=\frac{4.80\times10^{-10}}{66\times10^{-6}}E\mu_A$$
$$=0.727\times10^{-5}\,E\mu_A \text{ roentgens}, \tag{16}$$

where E is the gamma-ray energy intensity (the energy in units of mc^2 per cm²).

The ionization in roentgens produced by the unscattered gamma-rays, from a point source emitting N_0 photons of energy α_0, at a point receiver distant a from the source, is from Eq. (5)

$$r^0(a)=0.727\times10^{-5}\mu_A(\alpha_0)\frac{\alpha_0N_0}{4\pi a^2}\exp[-\mu_t(\alpha_0)a]. \tag{17}$$

The ionization in roentgens produced by singly scattered gamma-rays, from a point source emitting N_0 photons of energy α_0, at a point receiver distant a from the source, is approximately from Eq. (12)

$$r^{(1)}(a)=0.727\times10^{-5}\frac{N_0n}{2a}\{\exp[-\mu_t(\alpha_0)a]\}$$
$$\times\int_0^{\pi}\mu_A(\alpha_1)\alpha_1\theta\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta}d\theta. \tag{18}$$

Equation (18) may be further approximated by replacing the energy spectrum of the scattered gamma-rays by a mean energy $\bar{\alpha}_1$, where

$$\bar{\alpha}_1=\frac{E^{(1)}}{I^{(1)}}=\frac{G(\alpha_0)}{F(\alpha_0)}. \tag{19}$$

Then, an approximate value for the ionization in roentgens of the singly scattered radiation is [Eq. (14)],

$$r^{(1)}(a)\approx0.727\times10^{-5}E^{(1)}(a)\mu_A(\bar{\alpha}_1)$$
$$=0.727\times10^{-5}\mu_A(\bar{\alpha}_1)\frac{N_0n}{2a}G(\alpha_0)$$
$$\times\exp[-\mu_t(\alpha_0)a]. \tag{20}$$

Equations (17) and (20) determine the total ionization in roentgens from a point source when the distance a is much less than the mean free path. The function $[r^{(0)}(a)+r^{(1)}(a)]/N_0$, that is, the roentgens received per emitted gamma-ray, is plotted in Fig. 11 for various values of the gamma-ray energy α_0 when the medium is air.

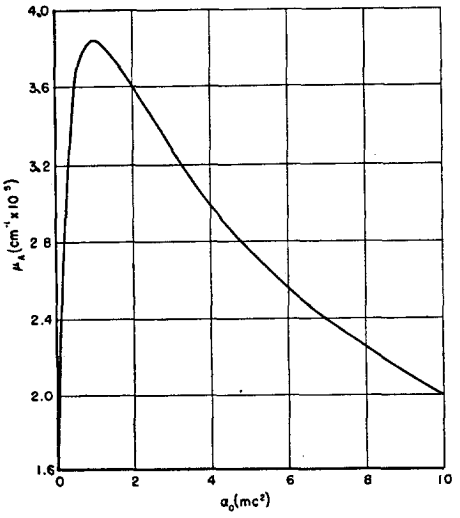


FIG. 10. Absorption coefficient for energy transfer by gamma-rays to air (STP).

GAMMA-RAY INTENSITIES AT DISTANCES FROM THE SOURCE COMPARABLE WITH THE MEAN FREE PATH AND GREATER

For distances of the order of a mean free path or greater, the contributions of the gamma-rays which have been scattered twice, three times, etc., become of increasing importance. The determination of these contributions is a problem of numerical complexity which increases rapidly with the order of scatter. Approximate calculations of the transmission of gamma-rays with multiple scatterings are available;‡ an important feature of these approximations is an essential simplification of the statistics of the sequence of scatterings; e.g., Hirschfelder's approximation consists in replacing the angular scattering distribution by some fixed mean angle of scatter. This approximation overestimates the trans-

‡ Hirschfelder, Magee, and Hull, Phys. Rev. 78, 852 (1948).

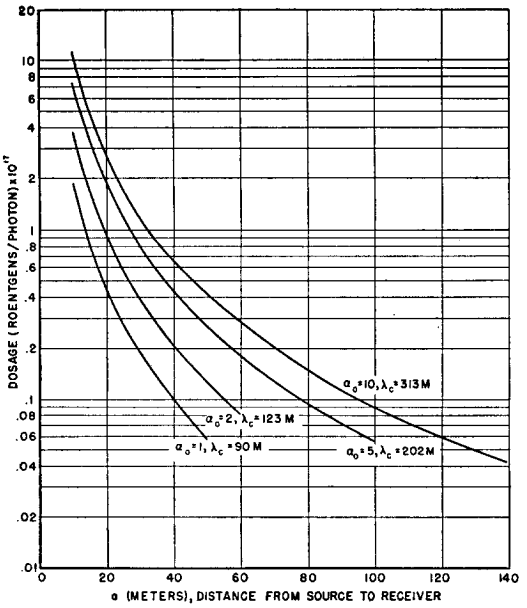


FIG. 11. Dosage in air (STP) per emitted photon from a point source to a point receiver.

mission of the radiation. A more accurate analysis is being made by several investigators, including Mr. H. Kahn and Dr. G. Peebles at RAND Corporation. These studies have been carried far enough so that the total transmitted gamma-ray intensity may be expressed in terms of the incident intensity by means of semi-empirical factors.

The qualitative behavior of the radiation, in the energy range 1 to 10 mc^2 considered here, which has traveled through a thickness of the medium corresponding to many mean free paths may be readily understood. The contribution to the energy intensity, at large distances, of photons which have been scattered through one or more large angles will be small: first, because any large angular deflection requires a considerable reduction in the photon energy; and secondly, because not only does the total absorption coefficient increase, but the probability of another large angle scatter is also increased. One, therefore, expects that the transmitted energy spectrum at large distances will have a mean energy not much less than the initial energy. An important contribution to the transmitted energy intensity will be from photons scattered through relatively small angles.

If $E^{(0)}(a)$ is the unscattered transmitted energy intensity, the total transmitted energy intensity may be written

$$E^t(a)=B[\alpha_0, a\mu_t(\alpha_0)]E^{(0)}(a), \tag{21}$$

and for a point source emitting N_0 gamma-rays isotropically

$$E^t(a)=B[\alpha_0, a\mu_t(\alpha_0)]\frac{N_0\alpha_0}{4\pi a^2}\exp[-\mu_t(\alpha_0)a]. \tag{22}$$

Since the mean transmitted energy is near α_0, one has approximately for the ionization in roentgens

$$\begin{aligned} r^t(a)&\approx 0.727\times 10^{-5}\mu_A(\alpha_0)E^t(a)\\ &=0.727\times 10^{-5}\mu_A(\alpha_0)B[\alpha_0, a\mu_t(\alpha_0)]\frac{N_0\alpha_0}{4\pi a^2}\\ &\qquad\times\exp[-\mu_t(\alpha_0)a]. \end{aligned} \tag{23}$$

For light elements in which the absorption coefficient $\mu_t(\alpha_0)$ comes primarily from Compton scattering so that $\mu_t(\alpha_0)\approx\mu_C(\alpha_0)$, the factor $B[\alpha_0, a\mu_t(\alpha_0)]$ will vary only slightly from element to element. For heavy elements the factor $B[\alpha_0, a\mu_t(\alpha_0)]$ will also vary with atomic number Z. Transmission in heavy elements will be discussed under "Gamma-Ray Shielding."

Experimental observations have been made in air of the transmission of gamma-rays for which $\alpha_0=3$. These observations show that B becomes a slowly varying factor for large values of $a\mu_C(\alpha_0)$ for $\alpha_0=3$. This is evident from Fig. 12, in which the logarithm of the product of a^2 by the observed ionization is plotted against a. For larger values of α_0, one would expect the factor $B[\alpha_0, a\mu_C(\alpha_0)]$ to increase relatively slowly with increasing α_0.

In Fig. 12, the calculated values of $a^2r^0(a)$ have been drawn for reference; the calculated value of $a^2[r^0(a)+r^{(1)}(a)]$ is also shown.

TRANSMISSION THROUGH A VARIABLE DENSITY MEDIUM

A special problem of importance is the transmission of gamma-rays through an inhomogeneous medium. The

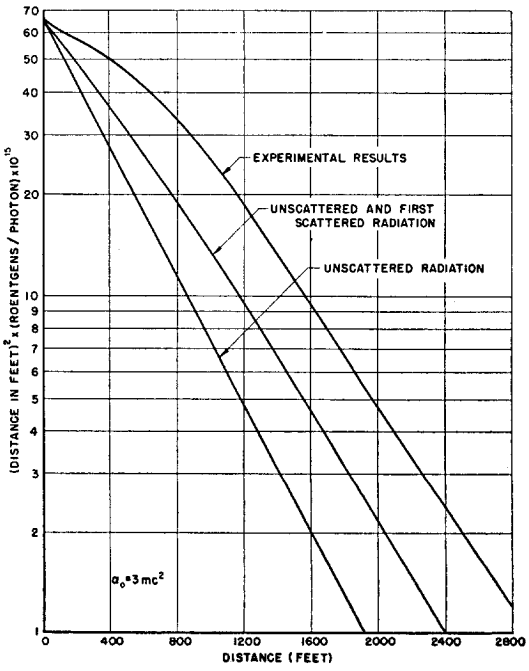


FIG. 12. Dosage in air per emitted photon from a point source to a point receiver.

effect of the inhomogeneity of the medium is taken into account by revising the exponential attenuation factor $\exp[-\mu_t(\alpha_0)r]$ to become $\exp[-\int_0^r \mu_t(\alpha_0, r)dr]$. Consider, as an example, the transmission of gamma-rays through a variable atmosphere. For air, where $\mu_t(\alpha_0)\approx\mu_C(\alpha_0)$, the effect of the variable atmosphere is taken into account by replacing $\mu_C(\alpha_0)a$ with

$$\int_0^a \mu_C(\alpha_0, z)dz.$$

Thus,

$$E^t(a)=B[\alpha_0, a\bar{\mu}_C(\alpha_0)]\frac{N_0\alpha_0}{4\pi a^2}\times\exp\left[-\int_0^a \mu_C(\alpha_0, z)dz\right]. \quad (24)$$

The factor B is a slowly varying function of a for the large values of $a\mu_C(\alpha_0)$ of present interest. One has

$$\mu_C(\alpha_0, z)=\frac{A}{M}Z\sigma(\alpha_0)\rho(z),$$

where A is Avogadro's number, M is the molecular weight of air, Z is the effective atomic number of air, and $\rho(z)$ is the atmospheric density. Thus,

$$\int_0^a \mu_C(\alpha_0, z)dz=\frac{A}{M}Z\sigma(\alpha_0)\int_0^a \rho(z)dz.$$

This density integral may be evaluated.

For example, one may approximate the atmosphere in the troposphere by a dry adiabatic atmosphere in which

$$p=p_0e^{-(gz/RT)},$$

and

$$T=T_0-\beta z,$$

where g is the acceleration of gravity, R is the gas constant (in ergs per g per °K), T is the temperature in °K, p is pressure, and β is the dry adiabatic lapse rate which

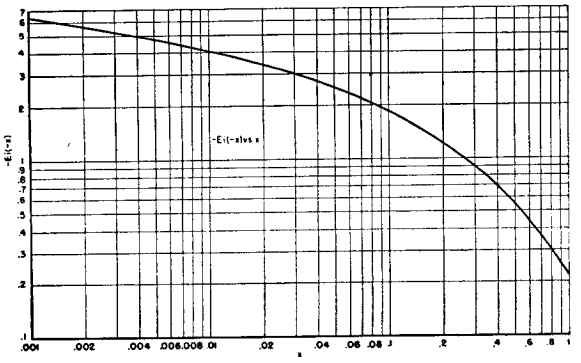


FIG. 13. Exponential integral function.

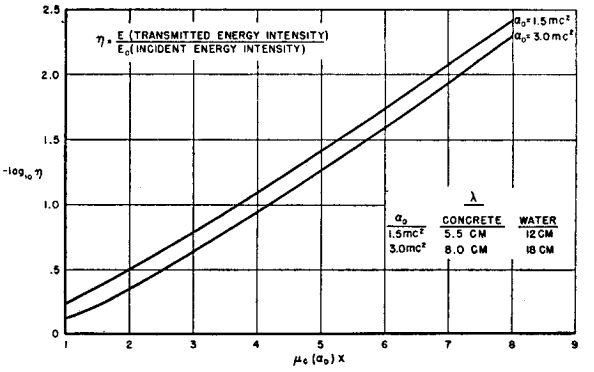


FIG. 14. Energy attenuation of gamma-rays in water and concrete.

is 9.8°K per 1000 meters. Then, one finds directly that

$$\int_0^a \rho(z)dz=e^{g/R\beta}\frac{\rho_0T_0}{\beta}\left[Ei\left(-\frac{g}{R\beta}\frac{T_0}{T_0-\beta a}\right)-Ei\left(-\frac{g}{R\beta}\right)\right],$$

and finally

$$\int_0^a \mu_C(\alpha_0, z)dz=\frac{A}{M}Z\sigma(\alpha_0)e^{g/R\beta}\frac{\rho_0T_0}{\beta}\times\left[Ei\left(-\frac{g}{R\beta}\frac{T_0}{T_0-\beta a}\right)-Ei\left(-\frac{g}{R\beta}\right)\right]. \quad (25)$$

A graph of the exponential integral function $-Ei(-x)$ is given in Fig. 13.

GAMMA-RAY SHIELDING

Shielding with Light Elements

In light elements, the total absorption coefficient (in the energy range of present practical interest), is given for the most part by the Compton scattering coefficient. The shielding problem is essentially characterized by considering the attenuation of a plane beam of gamma-rays incident upon a plane slab of the shielding material of thickness X. When the distance from the source is large, the incident gamma-radiation may be taken as a plane beam.

The attenuation of a plane beam of the radiation incident upon the shield may be readily calculated, for thicknesses small compared with the mean free path, by determination of the unscattered and singly-scattered transmitted intensities. If one has I_0 photons per cm² of energy α_0 normally incident upon a plane slab, the unscattered transmitted intensity is

$$I^0(X)=I_0\exp[-\mu_C(\alpha_0)X], \quad (26)$$

and the transmitted energy intensity is

$$E^0(X)=\alpha_0I_0\exp[-\mu_C(\alpha_0)X]. \quad (27)$$

The singly-scattered transmitted intensity $I^{(1)}(X)$ is given by

$$I^{(1)}(X)=2\pi nI_0\int_0^{\pi/2}\sin\theta d\theta \times\exp[-\mu_C(a_1)X/\cos\theta]\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta}\int_0^X dx \times\exp\left\{-x\left[\mu_C(\alpha_0)+\frac{\mu_1(\alpha_1)}{\cos\theta}\right]\right\},$$

$$I^{(1)}(X)=I_0\exp[-\mu_C(\alpha_0)X]\int_0^{\pi/2}2\pi\sin\theta d\theta n\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \times\left[\frac{1-\exp\left\{-\frac{X}{\cos\theta}[\mu_C(\alpha_1)-\mu_C(\alpha_0)\cos\theta]\right\}}{\frac{\mu_C(\alpha_1)}{\cos\theta}-\mu_C(\alpha_0)}\right], \quad (28)$$

and the transmitted, singly-scattered energy intensity is

$$E^{(1)}(X)=E_0\exp[-\mu_C(\alpha_0)X] \times\int_0^{\pi/2}\frac{2\pi\sin\theta d\theta}{1+\alpha_0(1-\cos\theta)}n\left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0,\theta} \times\left[\frac{1-\exp\left\{-\frac{X}{\cos\theta}[\mu_C(\alpha_1)-\mu_C(\alpha_0)\cos\theta]\right\}}{\frac{\mu_C(\alpha_1)}{\cos\theta}-\mu_C(\alpha_0)}\right]. \quad (29)$$

The primary interest is in the transmitted energy intensity, since this is directly correlated with ionization produced at a receiver. The ionization produced in roentgens by the transmitted intensity is

$$r=0.727\times10^{-5}[\mu_A(\alpha_0)E^{(0)}(X)+\mu_A(\bar{\alpha}_1)E^{(1)}(X)], \quad (30)$$

where $\bar{\alpha}_1$ is very nearly $E^{(1)}(X)/I^{(1)}(X)$ and may even be replaced by α_0 with only a small loss in accuracy.

For thicknesses of several mean free paths, one has recourse to the semi-empirical factor as discussed under "Gamma-Ray Intensities at Distances from the Source Comparable with the Mean Free Path and Greater." The transmitted energy intensity is

$$E[\mu_C(\alpha_0)X]=E_0\exp[-\mu_C(\alpha_0)X]B[\alpha_0,\mu_C(\alpha_0)X]. \quad (31)$$

Attenuation coefficients for water and concrete are shown in Fig. 14 for $\alpha_0=1.5$ and 3.

TABLE I. Lead.

X (cm)	$B(X)$
0	1.0
2	1.2
4	1.4
6	1.6
8	1.9
10	2.2
12	2.5
14	2.8
16	3.1
18	3.5
20	4.0
22	4.5
24	5.0

Shielding with Heavy Elements

For heavy elements, μ_{PP} and μ_{PE} make important contributions to μ_t. In elements of high atomic number, such as lead, and in the energy range of present concern, the photons which are degraded in energy by Compton scattering have rapidly increasing probability of complete absorption. As a result, the calculation of the attenuation of gamma-rays in such elements is a problem involving the determination of a relatively small number of scatterings.

Numerical calculations have been made of the total energy intensity transmitted through various thicknesses of lead for several values of the incident energy α_0. These calculations indicate the persistence of the "hard core" in the transmitted energy; that is, the energy spectrum of the transmitted beam is heavily weighted toward the incident photon energy α_0. If one writes the total transmitted energy intensity $E(X)$ as

$$E(X)=E_0B(X)\exp[-\mu_t(\alpha_0)X], \quad (32)$$

where E_0 is the incident energy intensity. Then $B(\alpha_0, X)$ has been calculated to have the approximate values given in Table I for $\alpha_0=10$.

Further calculations for different values of the incident energy α_0 show that the ratio of the total transmitted energy intensity to the unscattered energy intensity varies only slightly with the incident photon energy in the range $2\leqslant\alpha_0\leqslant10$ and is, thus, approximately only a function of the thickness X. Thus, one may use Table I to obtain rough values of $B(X)$ for this range of incident gamma-ray energies.

The ionization in roentgens produced by the transmitted energy is, thus, approximately

$$r=0.727\times10^{-5}\mu_A(\alpha_0)E_0B(X)\exp[-\mu_t(\alpha_0)X]. \quad (33)$$