Crew, F. A. E., "Waved, an Autosomal Recessive Coat-Form Character in the Mouse," *Jour. Genetics*, **27**, (1933).

Dunn, L. C., "Linkage in Rats and Mice," *Genetics*, **5** (1920).

Ibsen, H. L., "Linkage in Rats," *Am. Nat.*, **54** (1916).

King, H. D., "Mutations in a Strain of Captive Gray Rats," *Proc. Sixth Inter. Cong. Genetics*, **2**, 50 (1932).

King, H. D., and Castle, W. E., "Linkage Studies of the Rat," *Proc. Nat. Acad. Sci.*, 390–399 (1935).

TITANOTHERE REMAINS FROM THE SESPE OF CALIFORNIA

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Introduction.—Until very recently, all occurrences of American titanotheres have been known from either Eocene or lower Oligocene horizons of the Rocky Mountains and western Great Plains. Absence of any trace of these mammals in Tertiary deposits located in the region to the west of the Cordilleran Province lends considerable interest to the presence of titanotheres in one of the Sespe horizons of southern California.

Remains of titanotheres at Locality 150 have been recognized for several years, since some of this material appeared early in the collection obtained during the quarrying operations at that locality. Brief mention of the occurrence of these mammals was made in preceding papers,[1] but a more extended statement was reserved in the hope that further collecting might reveal complete skull remains. Unfortunately, this hope has not been realized. The desire to refer to the group in a broad survey of the Sespe faunas and the recent discovery of titanothere material elsewhere in the far west, make it urgent at this time to attempt an identification of the type or types from the Sespe.

The rather numerous teeth and several jaws and jaw fragments, as well as foot material, of titanotheres occurring at Locality 150 contrasts markedly with the absence of such material in the collections obtained at the upper Eocene localities situated lower in the Sespe section. It substantiates a field observation that the Brontotheriidae make their first appearance or at least undergo their first noteworthy development in Sespe time during the stage represented at Locality 150.

It is a matter of some interest, particularly from the standpoint of establishing the relationship of the Sespe uppermost Eocene stage to the Duchesne River horizon of Utah, that the titanotheres occurring in the former are similar to those found in the latter. Indeed, so close is the

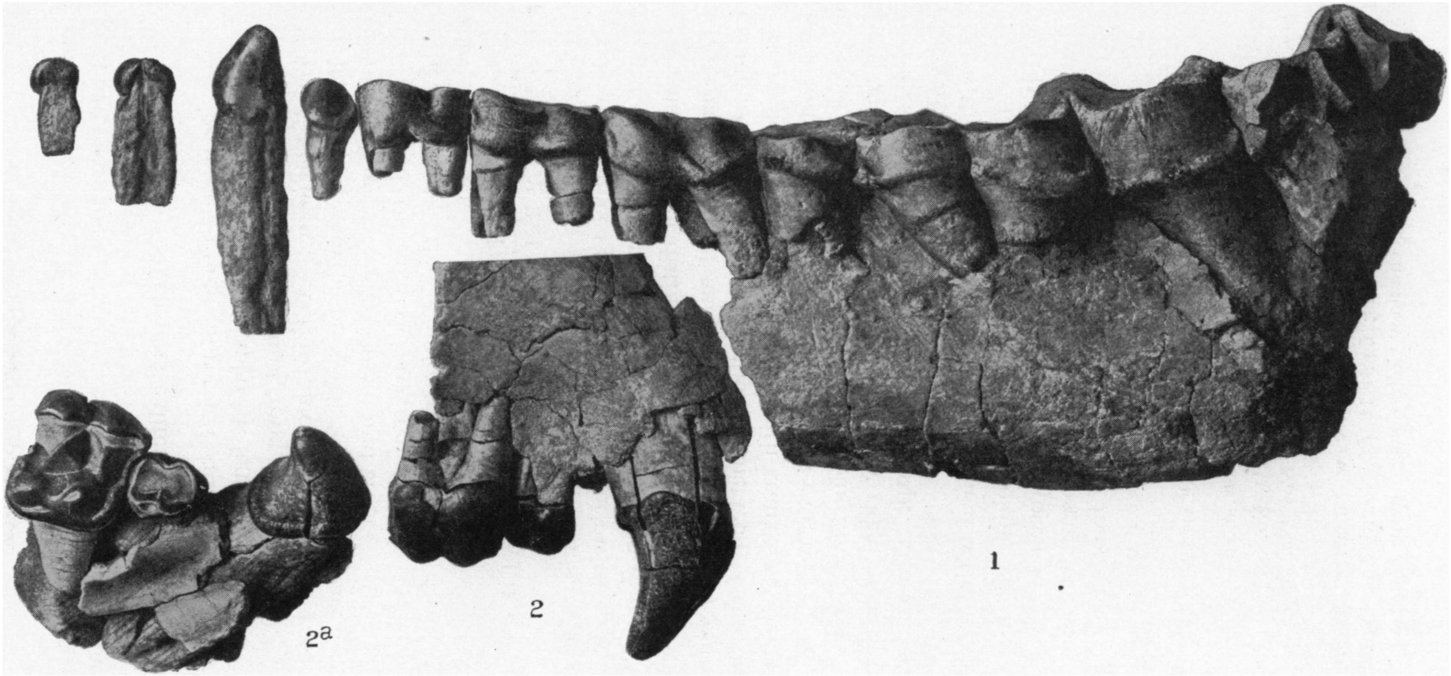

PLATE 1

Teleodus californicus, n. sp.

Figure 1, type specimen, left ramus and lower dentition, No. 1398, lateral view; $\times \frac{3}{4}$.

Figures 2, 2*a*, maxillary fragment with *C*, $P\underline{1}$ and $P\underline{2}$, No. 1834, lateral and occlusal views; $\times \frac{3}{4}$.

California Institute of Technology Collections. Sespe Uppermost Eocene, California.

resemblance between the two that there appear at present relatively few and apparently minor characters separating them specifically.

Teleodus californicus, n.sp.

Type Specimen.—Fragmentary left ramus with lower dentition, No. 1398, C.I.T. Vert. Pale. Coll., plate 1, figure 1.

Paratype.—Anterior portion of lower jaw, No. 1120.

Referred Specimens.—A third incomplete lower jaw, No. 1126, and miscellaneous lower and upper teeth, for the most part detached specimens, some of which are shown in plates 1 and 2.

Locality.—All the material referred to this titanothere comes from Locality 150 in the Brea Canyon section of the Sespe, north of the Simi Valley, California.

Characters.—$\frac{?}{2\text{–}3}, \frac{1}{1}, \frac{4}{4}, \frac{3}{3}$. Incisors with rounded, non-cingulate crowns. Third lower incisor absent in some individuals. Canines with crowns considerably reduced in size. Upper canines with crowns not noticeably enlarged at base. Very short diastema between C and $P\underline{1}$.

$P\underline{1}$ small; crown distinctly longer than wide with two external cusps, an internal cingulum, and with a cusp-like enlargement of the inner cingulum opposite forward portion of metacone (tritocone).

$P\underline{2}$ rectangular in cross-section with transverse diameter greater than anteroposterior. External surface convex on outer side of paracone and metacone. Hypocone (tetartocone) joined with protocone (deuterocone), but constriction of intervening low ridge in at least one specimen gives greater distinctness to the two cusps. Internal cingulum present or absent.

$P\underline{3}$ with outer face of paracone convex and with outer face of metacone flatter. Deep pit present adjacent to ectoloph. Hypocone (tetartocone) distinctly smaller than protocone (deuterocone) and well defined from latter cusp. Internal cingulum always absent for short distance at base of protocone; sometimes absent at base of hypocone.

$M\underline{2}$ and $M\underline{3}$ with anteroposterior diameter usually greater than trans-

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DESCRIPTION OF PLATE 2

Teleodus californicus, n. sp.

Figure 1, $M\underline{2}$ and $M\underline{3}$, No. 1004; figure 2, $P4$, No. 1011; figure 3, $M\underline{3}$, No. 1833; figure 4, $M3$, No. 1095; figures 5 and 6, $M\overline{3}$, Nos. 1835, 1836; figure 7, $M\underline{1}$ and $M\underline{2}$, Nos. 1094, 1009; occlusal views; × 2/3.

Figure 8, lower canine, No. 1831, lateral view; figures 9, 9*a*, 10, 10*a*, incisors, Nos. 1838, 1837, side and inner views; figure 11, $P3$, No. 1091, occlusal view; × 2/3.

California Institute of Technology Collections. Sespe Uppermost Eocene, California.

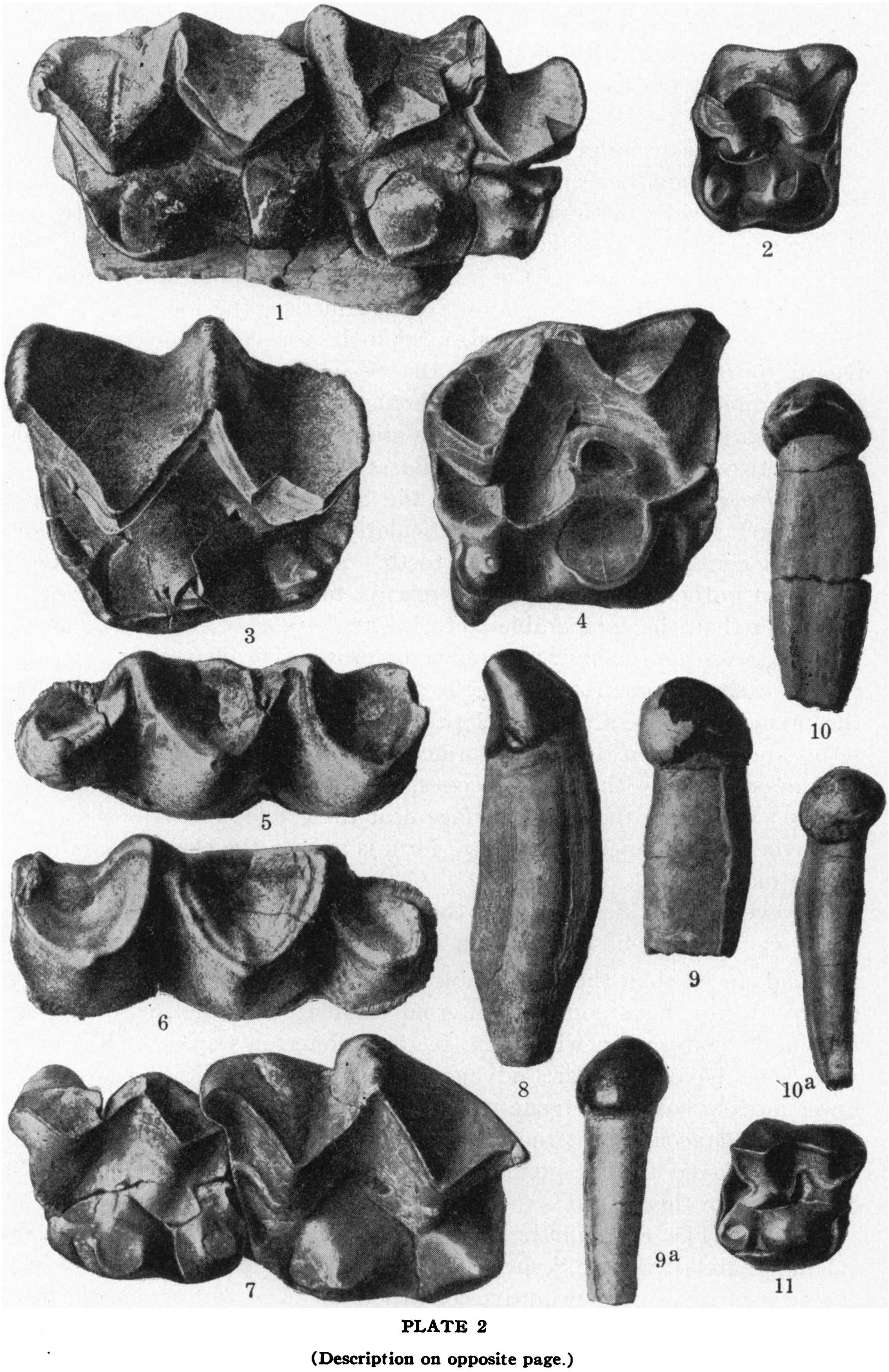

PLATE 2

(Description on opposite page.)

verse diameter. Pits form adjacent to ectoloph and hypocone is small. Hypocone in some specimens representing $M\underline{3}$ with one or more subsidiary cuspules on adjacent cingulum.

$P\overline{1}$, small.

$M\overline{3}$ may have well-developed posterior cingulum on posterior lobe.

The Sespe titanotheres are as small as or smaller than *Teleodus uintensis*. *T. californicus* is distinguished principally by a precocious reduction of the lower incisors from three to two in some individuals.

Comparisons.—Reviewing the numerous types of titanotheres described from the Eocene and Oligocene of North America, the following genera, because of their geologic position, need to be especially considered with regard to possible relationship to the Sespe species, namely (1) *Protitanotherium,* (2) *Diplacodon,* (3) *Eotitanotherium,* (4) *Ateleodon* and (5) *Teleodus.* Of these, the first three are definitely Eocene in age, while the last two are from the Lower and Basal Oligocene.

(1) *Protitanotherium* differs from the Sespe form in decidedly larger size, larger size of canines, and particularly in the larger size and more primitive construction of the incisor teeth. A longer post-canine diastema is present in the skull. $P\overline{1}$ and $P\overline{2}$, relative to size of posterior premolars, are larger than the comparable teeth in the Sespe species.

(2) *Diplacodon* exhibits a number of progressive characters. In this respect it shows some resemblance to the Sespe type. Thus, for example, the premolars possess well developed tetartocones and rudimentary fossettes are present on the crown surface; the premolar tritocones are large and are subequal with the protocones, and the molars have very distinct fossettes or pits on the crown surface near the ectoloph.

Further resemblance to the Sespe form is seen in the elongate character of the molars.

However, in the Sespe species there is a shorter diastema behind the upper canine; $P\underline{2}$ has acquired a rectangular cross-section and is thus more advanced than the comparable tooth in *Diplacodon;* the premolars usually do not have complete internal cingula; and the incisors (if we may make comparison with these teeth in Peterson's species *Diplacodon superbum*) have the more advanced rounded crowns. Moreover, the lower incisors have undergone more reduction.

(3) *Eotitanotherium* is more progressive than *Diplacodon* in the characters displayed by the premolars and makes a closer approach to the Sespe form in this regard.

However, like *Protitanotherium, Eotitanotherium* is a large type, considerably larger than the Sespe species. Moreover, as in *Protitanotherium,* the incisors are of more primitive construction. The post-canine diastema in the skull is distinctly longer and the molars do not show the antero-posterior elongation seen in *Diplacodon* and in the Sespe form.

The three preceding types are from Eocene horizons definitely earlier in time than that represented by Locality 150.

(4) *Ateleodon*, according to Schlaikjer,[2] shows great reduction or even loss of the lower canines and incisors. In this regard the Yoder genus exhibits an advance beyond the stage represented by the Sespe specimens. In the latter the incisors and canines are present, although these teeth were on occasion small in size. The Sespe form resembles *Ateleodon* in size. It is unfortunate that the incompleteness of the material from the Yoder horizon limits severely the characterization of *Ateleodon*.

(5) *Teleodus*, or more particularly the species, *T. uintensis*, described by Peterson[3] from the Duchesne River, makes a close approach in its characters to the Sespe form.

Some specimens from the Sespe are smaller, some slightly larger than comparable materials from the Duchesne River, but on the whole there is considerable similarity in character of size between the forms from California and Utah. This difference among individuals from Locality 150 evidently expresses variation within a specific group and doubtless is due also to a difference in sex. Thus far no structural differences appear to furnish a basis for recognizing more than one specific type at the California locality.

Compared with skull No. 11754 Carnegie Mus., the maxillary fragment with C, $P\underline{1}$ and $P\underline{2}$, No. 1834 C.I.T., plate 1, figures 2, 2*a*, is similar in size with $P\underline{1}$ slightly more reduced. Two lower incisors are present in the type, No. 1398, plate 1, figure 1. A distinct difference in size prevails between these teeth, for the lateral, presumably second, incisor is considerably larger than the medial one. In the larger jaw of the paratype, No. 1120, the third incisor is likewise absent. However, in a third jaw from the Sespe, No. 1126, remnants of the forward alveolar wall and external septum suggest the presence of a small, laterally compressed alveolus for $I\overline{3}$. Unfortunately, in No. 1126, the crown of the canine immediately adjacent has been pushed into the alveolus in such a way as to damage considerably that portion of the jaw lying between the canine and $I\overline{2}$. This has prohibited a very clear demonstration of the presence of a third incisor, but there nevertheless remains, on the basis of the evidence at hand, considerable probability that $I\overline{3}$ was present. In absence or reduced size of $I\overline{3}$, the Sespe species differs from *Teleodus uintensis*. The characters displayed by the incisors in the former are certainly more like those in Oligocene titanotheres than like those in Eocene types.

Plate 2 illustrates a number of teeth representing the upper and lower dentitions of titanotheres from Locality 150. Individually, these specimens resemble comparable teeth in *T. uintensis*.

[1] Stock, C., *Proc. Nat. Acad. Sci.*, **18**, 522 (1932); *Ibid.*, **19**, 762 (1933).

[2] Schlaikjer, E. M., *Mus. Comp. Zoöl. Bull.*, **76**, 83–84 (1935).
[3] Peterson, O. A., *Ann. Carnegie Mus.*, **20**, art. 14 (1931).

DIFFERENTIABLE MANIFOLDS IN EUCLIDEAN SPACE[1]

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We present here a summary of some theorems on the imbedding of abstract differentiable manifolds in Euclidean space E^n and on the approximation to such manifolds by analytic manifolds. As a corollary it is noted that any differentiable manifold may be given an analytic Riemannian metric.

I. *The Imbedding of a Differentiable Manifold in Euclidean Space.*—Let M be a topological space with neighborhoods $U_1, U_2, \ldots$. Let each U_i be homeomorphic with the interior of the unit m-sphere S^m. If U_i and U_j have common points U_{ij}, then the homeomorphisms of U_i and U_j with S^m induce a mapping of one part of S^m on another part. If all such maps are of class C^r (i.e., have continuous partial derivatives through the rth order), $r \geqq 1$, with non-vanishing Jacobian, we say M is *differentiable*, and *of class* C^r.

If M is in E^n and each point of M is in a neighborhood which may be defined by expressing $n - m$ of the coördinates in terms of the remaining m, the functions being of class C^r, then M is of class C^r in the above sense; we say M *is of class* C^r *in* E^n. Suppose M is of class C^r, and is mapped into E^n. The n coördinates at points of M are n functions defined over M. If these functions are of class C^s (with the obvious definition for $s \leqq r$), and are independent (so that m independent directions at any point of M go into m independent directions in E^n), we call the map of M in E^n a *regular* C^r*-map*. Such a map is locally one-one: a neighborhood of any point of M is mapped in a one-one manner in E^n.

THEOREM I. *Any m-manifold of class* C^r ($r \geqq 1$ *finite or infinite*) *may be imbedded by a regular* C^r*-map in* E^{2m}, *and by such a map in a one-one manner in* E^{2m+1}.

The proof runs as follows: If M is closed, a finite number of neighborhoods $U_1, \ldots, U_\nu$ cover M. Corresponding to these neighborhoods we define functions $f_1, \ldots, f_\mu$, $\mu = (m + 1)\nu$, of class C^r over M, which, used as coördinates, map M in a regular C^r-manner in E^μ. If $r = 1$, we next approximate to M by a manifold of class C^2. We now project M or the new manifold along straight lines into spaces of lower dimension, till we