



Edge states and interferometers in the Pfaffian and anti-Pfaffian states of the $\nu = \frac{5}{2}$ quantum Hall system

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We compute the tunneling current in a double point contact geometry of a quantum Hall system at filling fraction $\nu = \frac{5}{2}$, as a function of voltage and temperature, in the weak tunneling regime. We quantitatively compare two possible candidates for the state at $\nu = \frac{5}{2}$: the Moore-Read Pfaffian state, and its particle-hole conjugate, the anti-Pfaffian. We find that both possibilities exhibit the same qualitative behavior, and both have an even-odd effect that reflects their non-Abelian nature, but differ quantitatively in their voltage and temperature dependence.

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I. INTRODUCTION

Quantum Hall (QH) devices at certain filling fractions are the only systems known to be in topological phases. The $\nu = \frac{1}{3}$ Laughlin state is in an Abelian topological phase. The excitations of such a phase carry a fraction of an electron charge and have fractional statistics which are intermediate between bosonic and fermionic statistics. The fractional charge has been confirmed experimentally,¹⁻³ and experiments showing indications of fractional statistics have been recently performed.⁴

The observed⁵⁻⁷ quantum Hall state at filling fraction $\nu = \frac{5}{2}$ is the primary candidate for a system in a non-Abelian topological phase, and is believed to be described by the Moore-Read Pfaffian state^{8,9} as a result of numerical evidence.^{15,16} The excitations of the Pfaffian carry fractional charge $e/4$ and have non-Abelian braiding statistics: for given quasiparticle positions, there are several linearly independent quantum states of the system, and braiding the quasiparticles (qp's) causes a rotation in this space.¹⁰⁻¹⁴ In addition to their novelty, these properties could be useful for topological quantum computation.¹⁷

In the absence of Landau level mixing, the Hamiltonian of a half-filled Landau level is particle-hole symmetric. The Pfaffian state, if it is the ground state of such a Hamiltonian, spontaneously breaks particle-hole symmetry. The particle-hole conjugate of the Pfaffian, dubbed the anti-Pfaffian,^{18,19} has exactly the same energy as the Pfaffian in the absence of Landau level mixing. Hence, it is a serious candidate for the $\nu = \frac{5}{2}$ state observed in experiments, where Landau level mixing, which is not small, will favor one of the two states. Therefore, it is important to find experimental probes which can distinguish between these two states. Although the two states are related by a particle-hole transformation and are both non-Abelian, they differ in important ways: their quasiparticle statistics differ by Abelian phases, and the anti-Pfaffian has three counterpropagating neutral edge modes while the Pfaffian edge is completely chiral. In this paper we

consider edge tunneling experiments for both the Pfaffian and the anti-Pfaffian states, and we find quantitative differences between the two resulting from these distinctions.

The double point contact geometry has been proposed as a probe for non-Abelian statistics.^{12,20-25} In this setup, a QH bar is gated so that two constrictions are created, as shown in Fig. 1, and quasiparticles can tunnel from one edge to the other at either constriction. The dashed line in Fig. 1 serves as a reminder that the two edges are actually different sections of a single edge which is the boundary of the system; consequently, interedge tunneling satisfies topological conservation laws which are important in the non-Abelian case. An edge quasiparticle entering the sample from the left can tunnel to the lower edge through either point contact, and the measured tunneling current is sensitive to the interference between these two possible trajectories. The phase difference between the quantum amplitudes of these two trajectories depends on the applied voltage between the top and bottom edges, the magnetic flux enclosed between the two trajectories, and the number of quasiparticles localized in the bulk between the two trajectories. If the quasiparticles have non-Abelian statistics, the quantum state of the system can change when the edge quasiparticle encircles the localized

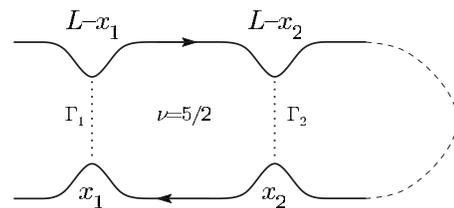


FIG. 1. The double point contact geometry. Edge quasiparticles can tunnel between the top and bottom edges at the point contacts $j=1,2$, with tunneling amplitude Γ_j . The dashed line serves as a reminder that both top and bottom edges are two sections of the same edge. The two ends of point contact j are two points on the same edge separated by a distance $L-2x_j$, where L is the length of the edge.

bulk ones, and the effect on the interference term is more than merely a phase shift. The Pfaffian and anti-Pfaffian states exemplify the most extreme case: if there is an even number of localized quasiparticles enclosed between the tunneling trajectories, there will be interference that depends on the magnetic flux and applied voltage, while in the presence of an odd number of bulk quasiparticles in the bulk, the interference pattern will be completely lost. We will recover these striking results using an explicit edge theory calculation.

The visibility of the interference pattern in the even quasiparticle case will be obscured by thermal smearing as well as the difference between the charged and neutral mode velocities. Naively, the latter is particularly acute in the anti-Pfaffian case, where the velocities have opposite sign. However, as we will see quantitatively from the edge state calculation below, the difference between the even and odd quasiparticle cases will be visible for sufficiently low temperature in both the Pfaffian and anti-Pfaffian states. The required temperature vanishes as the distance between the contacts or the difference in velocities is increased.

The principle conceptual difficulty in analyzing interedge tunneling stems from the non-Abelian nature of the bulk state, which causes ambiguities in edge correlation functions (or, more properly, *conformal blocks*). We show how these are resolved, following Refs. 26 and 27 and further refinements introduced in Refs. 22 and 23.

The paper is organized as follows. In Sec. II we set up the perturbative calculation to lowest order, explain the ambiguity that arises in evaluating correlation functions due to the non-Abelian nature of the edge, and show how to resolve this ambiguity, following Refs. 26 and 27. In Sec. III, we find the expected tunneling current behavior as a function of bias voltage and temperature in the Pfaffian state, taking into consideration the different velocities of charged and neutral modes on the edge. We show that for sufficiently low temperature, interference will be visible in the even quasiparticle case. In Sec. IV, we repeat the calculation for the anti-Pfaffian state and show the quantitative differences with the Pfaffian case.

II. TUNNELING OPERATORS AND CONFORMAL BLOCKS

We now set up the calculation of the tunneling current to lowest order and discuss the basic issues which arise. The Pfaffian and anti-Pfaffian cases are conceptually similar, so we focus on the Pfaffian for the sake of concreteness. The edge theory of the Pfaffian state has a chiral bosonic charge mode and a chiral neutral Majorana mode^{26–29}

$$\mathcal{L}_{\text{Pf}}(\psi, \phi) = \frac{2}{4\pi} \partial_x \phi (\partial_t + v_c \partial_x) \phi + i\psi (\partial_t + v_n \partial_x) \psi. \quad (1)$$

Both modes propagate in the same direction, but will have different velocities in general. One expects the charge velocity v_c to be larger than the neutral velocity v_n . The electron operator is a charge 1 fermionic operator as follows:

$$\Phi_{el} = \psi e^{i\sqrt{2}\phi}, \quad (2)$$

and the $e/4$ quasiparticle operator is

$$\Phi_{1/4} = \sigma e^{i\phi/2\sqrt{2}}, \quad (3)$$

where σ is the Ising spin field of the Majorana fermion theory.²⁷ When interedge tunneling is weak, we expect the amplitude Γ for charge- $e/4$ to be transferred from one edge to the other to be larger than for higher charges $ne/4$, which should be $\sim \Gamma^n$. It is also the most relevant tunneling operator in the renormalization group sense,^{26,27} so we will focus on it. Since it is relevant, its effective value grows as the temperature is decreased, eventually leaving the weak tunneling regime. We assume that the temperature is high enough that the system is still in the weak tunneling regime and a perturbative calculation will be valid, but still much lower than the bulk energy gap.

Following Ref. 30, we write the tunneling Hamiltonian in the form

$$H_i(t) = \Gamma_1 e^{-i\omega_j t} V_1(t) + \Gamma_2 e^{i\Phi/4\Phi_0} e^{-i\omega_j t} V_2(t) + \text{H.c.} \quad (4)$$

The frequency $\omega_j = \frac{eV}{4}$ is the Josephson frequency for a charge $e/4$ quasiparticle with voltage V applied between the top and bottom edges. The difference in the magnetic fluxes enclosed by the two trajectories around the interferometer is Φ . We have chosen a gauge in which the vector potential is concentrated at the second point contact so that Φ enters only through the second term above. Both edges are part of the boundary of the same Hall droplet, so we can denote the point on the upper edge which is on the other side of point contact j from x_j by $L - x_j$, where $j=1, 2$ and L is large. The operator $V_j(t)$ tunnels a quasiparticle between x_j and $L - x_j$ as follows:

$$V_j(t) = \sigma(x_j, t) \sigma(L - x_j, t) e^{(i/\sqrt{8})\phi(x_j, t)} e^{-(i/\sqrt{8})\phi(L - x_j, t)}. \quad (5)$$

The current operator can be easily found from the commutator of the tunneling Hamiltonian and the charge on one edge as follows:

$$I(t) = \frac{ie}{4} [\Gamma_1 e^{-i\omega_j t} V_1(t) - \text{H.c.}] + \frac{ie}{4} [\Gamma_2 e^{i\Phi/4\Phi_0} e^{-i\omega_j t} V_2(t) - \text{H.c.}]. \quad (6)$$

To lowest order in perturbation theory, the tunneling current is found to be

$$\langle I(t) \rangle = -i \int_{-\infty}^t dt' \langle 0 | [I(t), H_i(t')] | 0 \rangle. \quad (7)$$

In order to compute the current, we substitute Eqs. (4) and (6) into Eq. (7). We obtain

$$I(t) = \frac{e}{4} \sum_{j,k} \Gamma_j \Gamma_k^* e^{i(j-k)\Phi/4\Phi_0} \times \int_{-\infty}^t dt' e^{i\omega_j(t'-t)} (\langle V_j(t) V_k^\dagger(t') \rangle - \langle V_k^\dagger(t') V_j(t) \rangle). \quad (8)$$

Therefore, we must compute the correlation function

$$\begin{aligned} \langle V_j(t) V_k^\dagger(t') \rangle &= \langle \sigma(x_j, t) \sigma(L - x_j, t) \sigma(L - x_k, t') \sigma(x_k, t') \rangle \\ &\times \langle e^{(i/\sqrt{8})\phi(x_j, t)} e^{-(i/\sqrt{8})\phi(L - x_j, t)} e^{(i/\sqrt{8})\phi(L - x_k, t')} \\ &\times e^{-(i/\sqrt{8})\phi(L - x_k, t')} \rangle. \end{aligned} \quad (9)$$

This correlation function is at the heart of our calculation. The correlations involving the bosonic fields are straightforward to calculate and, in the limit of a long sample $L \rightarrow \infty$, the bosonic correlation function breaks into a product of two-point correlation functions of fields on the same edge.

$$\begin{aligned} \langle e^{(i/\sqrt{8})\phi(x_j, t)} e^{-(i/\sqrt{8})\phi(x_k, t')} \rangle &\langle e^{-(i/\sqrt{8})\phi(L - x_j, t)} e^{(i/\sqrt{8})\phi(L - x_k, t')} \rangle \\ &= \prod_{r=\pm} [\delta + i(v_c(t - t') + r(x_j - x_k))]^{-1/8}. \end{aligned} \quad (10)$$

However, the four σ correlation function is actually ill defined without further information, namely, the fusion channels of the four σ operators. (Technically, the correlation function is what is called a *conformal block*.) These are determined by the physical situation, as we elaborate on this below.

In the Ising conformal field theory, the σ operators have nontrivial fusion rules.

$$\sigma \times \sigma = I + \psi. \quad (11)$$

A correlation function of $2n$ σ particles is nonvanishing only if all of the operators fuse together to the identity, but there are a number of ways in which the fields can do that. In the four σ operators case, the correlation $\langle \sigma(z_1) \sigma(z_2) \sigma(z_3) \sigma(z_4) \rangle$ has two different conformal blocks corresponding to the two possible fusions. In the standard notation explained, for instance, in this context in Ref. 27, these two conformal blocks or fusion channels are

$$\mathcal{F}_c \equiv \frac{1}{2} \begin{array}{c} 1 \quad 2 \quad 3 \quad 4 \\ \hline I \quad | \quad | \quad c \quad | \quad | \quad I \end{array},$$

where $c=1$ or ψ is the fusion product of the fields at the space-time points z_1 and z_2 . Their explicit forms are

$$\begin{aligned} \mathcal{F}_I &= \left(\frac{1}{z_{12}z_{34}(1-x)} \right)^{1/8} (1 + \sqrt{1-x})^{1/2}, \\ \mathcal{F}_\psi &= \left(\frac{1}{z_{12}z_{34}(1-x)} \right)^{1/8} (1 - \sqrt{1-x})^{1/2}, \end{aligned} \quad (12)$$

where $z_{ij} = z_i - z_j$ and $x = z_{12}z_{34} / z_{13}z_{24}$.

Now for an obvious question: which conformal block enters the perturbative calculation? As explained in Refs. 26 and 27, when there are no quasiparticles in the bulk, the correct choice is the conformal block in which the σ operators in the tunneling operator $V_j(t)$, i.e., $\sigma(x_j, t)$ and $\sigma(L - x_j, t)$, fuse to the identity. Since all this operator does is transfer a quasiparticle from one side of the Hall sample to the other, it should not change the topological charge on the edge, which would involve the creation of a fermion. In the bottom half of Fig. 2(a), we show two successive tunneling events. Each can be envisioned as the creation out of the

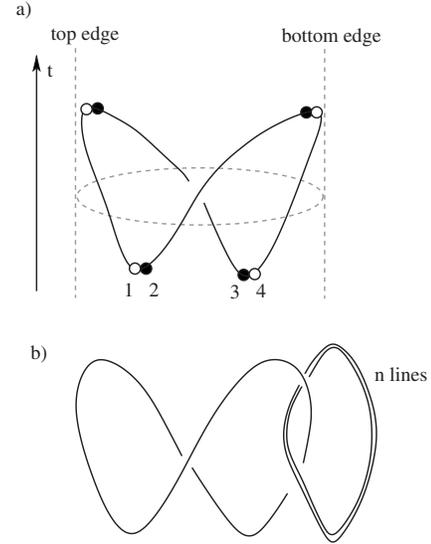


FIG. 2. (a) The knot corresponding to the matrix element between the state in which two quasiparticle-quasihole pairs, 1,2 and 3,4 are created out of the vacuum (top half) and go to opposite edges and a state in which the two quasiparticles on each edge (e.g., 1,3 on the top edge) fuse to I . Equivalently, it is one element of the F matrix, which transforms between the basis of conformal blocks in which 1,2 has a fixed fusion channel (and therefore, 3,4 does as well) and the basis in which 1,3 has a fixed fusion channel. (b) The same matrix element with n quasiparticles in the bulk. The n quasiparticles are assumed to have been created in pairs in the distant past, with one member of each pair taken inside the interferometer and the other member left outside. The two tunneling events are assumed to occur at different point contacts. The figure then gives the matrix element between the states in which 1,2 and 3,4 are created out of the vacuum (top half), go to opposite edges, and encircle the bulk quasiparticles; and a state in which 1,3 fuse to I .

vacuum of a quasiparticle-quasihole pair in the bulk. Saying that they are created “out of the vacuum” is equivalent to saying that they fuse to I . The quasiparticle then goes to one edge and the quasihole goes to the other. A second tunneling event (either at the same or a different point contact) occurs in the same way. Let us, for the sake of concreteness call the quasiparticle and quasihole which are created in the first tunneling process 1 and 2; in the second tunneling process, 3 and 4 are created. (For these purposes, there is no need to distinguish between quasiparticles and quasiholes.) Let us assume that quasiparticles 1 and 3 go to the top edge while 2 and 4 go to the bottom edge. If the two edges are independent (as occurs in the $L \rightarrow \infty$ limit), this process has a non-zero amplitude only if 1,3 fuse to I and 2,4 fuse to I , as depicted in the top half of Fig. 2(a). [I is depicted by the absence of a line. If a fermion were the result of fusing the two quasiparticles, there would be a wavy line emanating upward from each of the two fusion points at the top of Fig. 2(a).] This picture can be interpreted as the matrix element between the state in which quasiparticle-quasihole pairs 1,2 and 3,4 are created in the bulk and go to the edges (bottom) and the state in which quasiparticles 1,3 fuse to I and 2,4 fuse to I (top).

Hence the correlation function in Eq. (9) is actually the conformal block as follows:

$$\mathcal{F}_I = \frac{(x_j, t)(L - x_j, t)(L - x_k, t')(x_k, t')}{I \quad I \quad I \quad I} .$$

On the other hand, in the $L \rightarrow \infty$ limit, we expect the σ correlation in Eq. (9) to break into a product of correlators of fields on the same side of the sample.

$$\langle \sigma(x_j, t) \sigma(x_k, t') \rangle \langle \sigma(L - x_j, t) \sigma(L - x_k, t') \rangle. \quad (13)$$

As noted above, this correlation function is nonvanishing only if the fields on the same side of the sample fuse to the identity. This conformal block is given pictorially by

$$\mathcal{G}_I = \frac{(x_j, t)(x_k, t')(L - x_j, t)(L - x_k, t')}{I \quad I \quad I \quad I} .$$

In the \mathcal{G}_c 's, we specify the fusions of fields on the same side of the edge rather than opposite sides of a point contact. In the $L \rightarrow \infty$ limit, \mathcal{G}_ψ vanishes. The conformal blocks \mathcal{G}_c are linear combinations of the \mathcal{F}_c 's; both form bases for the two-dimensional vector space of conformal blocks. The basis change between the two is called the F matrix, which is part of the basic data characterizing a topological phase. We can write

$$\mathcal{F}_I = a_I \mathcal{G}_I + a_\psi \mathcal{G}_\psi, \quad (14)$$

where the coefficients a_I and a_ψ are two of the entries in the F matrix. They can be calculated by computing the Kauffman bracket for a braid that corresponds to this change of basis, as was done in Refs. 26 and 27.

$$a_I = \frac{1}{\sqrt{2}} \quad (\text{no qp's in bulk}). \quad (15)$$

For the purposes of our calculation, we only need the long sample limit of the correlation function \mathcal{F}_I . As explained above, we find that it is proportional to \mathcal{G}_I , which can be easily evaluated since it is simply the product of two two-point correlation functions [it can also be obtained by taking the large- L limit of the expression for G_I as in Eq. (12)].

$$\mathcal{F}_I|_{L \rightarrow \infty} = a_I \mathcal{G}_I|_{L \rightarrow \infty} = a_I \prod_{\epsilon=\pm} [\delta + i(v_n(t - t') + \epsilon(x_j - x_k))]^{-1/8}. \quad (16)$$

We now generalize this to the case in which there are n quasiparticles in the bulk between the two point contacts. Correlation functions in which all of the fields are at the same point contact are unchanged. However, as pointed out in Refs. 22 and 23, when two tunneling processes occur at different point contacts, the two quasiparticle-quasihole pairs are created out of the vacuum as before, but quasiparticle 1 must encircle the bulk quasiparticles before it can be fused with quasiparticle 3. This difference modifies the matrix element with the state in which 1, 3 fuse to I and 2, 4 fuse to I , as depicted in Fig. 2(b). Let us consider the simplest case, in which there is a single quasiparticle in the bulk. We can imagine that a quasiparticle-quasihole pair was created in the distant past and one member of the pair was brought into the

interferometer while the other member was left outside. Then we create the quasiparticle-quasihole pairs 1, 2 and 3, 4 and take 4 around the bulk quasiparticle. This process is depicted in the bottom half of Fig. 2(b). We can compute the resulting a_I by computing the matrix element between the resulting state and the state in which 1, 3 fuse to I (as do 2, 4). This matrix element can be computed from the Kauffman bracket of the link in Fig. 2(b) or, equivalently, by using the F and R matrices of the theory. By either method, we find $a_I = 0$. The reason is that, after 4 is taken around the bulk quasiparticle, either 1, 3 or 2, 4 (but not both) must fuse to ψ rather than I . Therefore, there is no amplitude for 1, 3 and 2, 4 to fuse to I . For the same reason, $a_\psi = 0$, so even for L finite, there is no contribution from such a process. In fact, the same result is obtained for any odd n since an odd number of quasiparticles must fuse to σ . Therefore, their effect is the same as if there were a single quasiparticle in the bulk.

$$a_I = 0 \quad (\text{odd number } n \text{ of qp's in bulk}). \quad (17)$$

For n even, the n bulk quasiparticles can fuse to either I or ψ . The former case is the same as in the absence of quasiparticles; in the latter case, there is an additional minus sign which is acquired when a σ goes around a ψ .

$$a_I = \pm \frac{1}{\sqrt{2}} \quad (\text{even number } n \text{ of qp's in bulk}). \quad (18)$$

With the correct conformal block in hand, as specified by the corresponding value of a_I , we can now give a meaning to expressions such as (9) and can use Eq. (7) to compute the current through our interferometer.

In the preceding discussion, we have focused on the neutral sector of the theory, where the interesting non-Abelian effects occur. However, there is also a charged sector of the theory. The full conformal theory describing the edge includes both parts. As a result, there are additional phases which result from the change of basis when there are quasiparticles in the bulk. Furthermore, we must exercise a little more care in distinguishing quasiparticles from quasiholes since they have different Abelian phases. By recalculating Fig. 2(b) with the Abelian part of the theory included, we find that a_I acquires an additional phase $n\pi/4$ when there are n quasiparticles in the bulk and 1 and 4 are quasiparticles while 2 and 3 are quasiholes. The opposite phase results when 2 and 3 are quasiparticles while 1 and 4 are quasiholes.

III. TEMPERATURE AND VOLTAGE DEPENDENCE OF THE CURRENT THROUGH AN INTERFEROMETER

To lowest order in Γ_1, Γ_2 , the current naturally breaks into the sum of three terms as follows:

$$I = I_1 + I_2 + I_{12}, \quad (19)$$

where

$$I_j = \frac{e}{4} |\Gamma_j|^2 \int_{-\infty}^0 dt e^{i\omega t} (\langle V_j(0) V_j^\dagger(t) \rangle - \langle V_j^\dagger(t) V_j(0) \rangle), \quad (20)$$

and

$$I_{12} = \frac{e}{4} \Gamma_1 \Gamma_2^* \int_{-\infty}^0 dt e^{i\omega t} (\langle V_1(0) V_2^\dagger(t) \rangle - \langle V_2^\dagger(t) V_1(0) \rangle) + \text{c.c.} \quad (21)$$

I_j , $j=1,2$ would be the backscattered current if only point contact j were present. I_{12} is due to interference between the process in which a quasiparticle tunnels between the two edges at x_1 and the process in which it continues to x_2 and tunnels there. As a result, I_{12} depends on the magnetic flux and the number of bulk quasiparticles between the two point contacts; it reflects the non-Abelian statistics of quasiparticles, namely, the difference between even and odd numbers of bulk quasiparticles. Meanwhile, I_1, I_2 , and I_{12} all depend on the bias voltage and temperature. In this section we quantitatively analyze the dependence of I on all of these parameters.

We first consider the zero-temperature case. The single-point contact current term $I_1 + I_2$ is identical to the backscattering current due a single impurity in a Luttinger liquid. The current is a power law in voltage as follows:

$$I_1 + I_2 = \frac{1}{\sqrt{2}} \frac{e}{4} \frac{\pi}{\Gamma\left(\frac{1}{2}\right)} (|\Gamma_1|^2 + |\Gamma_2|^2) v_n^{-1/4} v_c^{-2/8} \times \text{sgn}(V) \left(\frac{e|V|}{4}\right)^{-1/2}. \quad (22)$$

The factor of $\frac{1}{\sqrt{2}}$ is a_l discussed in the previous section. We now consider I_{12} . For an odd number of quasiparticles in the bulk,

$$I_{12} = 0 \quad (\text{odd number } n \text{ of qp's in bulk}). \quad (23)$$

For an even number n of quasiparticles in the bulk, I_{12} can be evaluated analytically in the special case in which the two velocities are equal.

$$I_{12} = \pm \frac{1}{\sqrt{2}} \frac{e}{4} \frac{\pi^{3/2} 2^{9/4}}{\Gamma\left(\frac{1}{4}\right)} |\Gamma_1| |\Gamma_2| \cos\left(\frac{\Phi}{4\Phi_0} + n\frac{\pi}{4} + \alpha\right) \times \text{sgn}(V) |V|^{-1/2} \left(\frac{e|x_1 - x_2|}{4v}\right)^{1/4} J_{-1/4}\left(\frac{e|x_1 - x_2|}{4v}|V|\right). \quad (24)$$

In this expression, the \pm sign is obtained if the quasiparticles in the bulk fuse to total non-Abelian charge 1 or ψ , respectively; $J_{-1/4}$ is the Bessel function; Φ is the flux enclosed in the interference loop; and n is the (even) number of bulk quasiparticles inside the loop. The phase $n\pi/4$ is a statistical phase due to the Abelian part of the theory. The phase α is $\arg(\Gamma_1 \Gamma_2^*)$. When the charge and neutral velocities are not equal, the current and differential conductance will oscillate at two different frequencies as seen in Fig. 3, and both charge and neutral velocities can be extracted from the two different periods. The smaller period corresponding to the fast oscillations is roughly $\frac{16\pi}{e|x_1 - x_2|} (1/v_n + 1/v_c)^{-1}$, and the larger period corresponding to the oscillations of the envelope is roughly $\frac{16\pi}{e|x_1 - x_2|} (1/v_n - 1/v_c)^{-1}$.

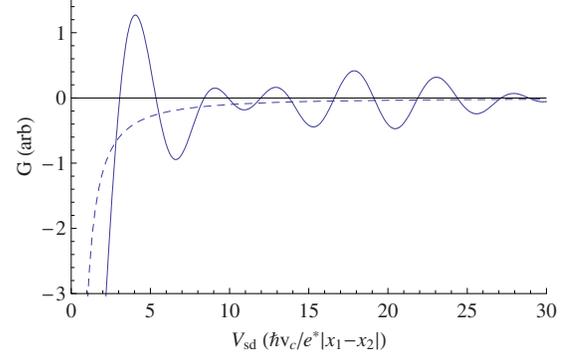


FIG. 3. (Color online) The differential conductance as a function of applied voltage at low temperature, for $v_n = 0.75v_c$. The dashed line is the conductance with an odd number of quasiparticles in the interference loop, and the solid line is for an even number. The charge and neutral velocities can be extracted from the two oscillation periods.

Finite-temperature correlation functions can be obtained from the zero-temperature correlation functions by a conformal transformation from the plane to the cylinder, which amounts to the following substitution:

$$\frac{1}{(\delta + i(t \pm x/v))^{1/8}} \rightarrow \left(\frac{\pi T}{\sin[\pi T(\delta + i(t \pm x/v))]} \right)^{1/8}. \quad (25)$$

We find that the general form of the current is

$$I_1 + I_2 = (|\Gamma_1|^2 + |\Gamma_2|^2) |V|^{-1/2} A\left(\frac{eV/4}{k_B T}\right), \quad (26)$$

$$I_{12} = |\Gamma_1| |\Gamma_2| \cos\left(\frac{\Phi}{4\Phi_0} + n\frac{\pi}{4} + \alpha\right) \text{sgn}(V) |V|^{-1/2} \times B_n\left(\frac{e|x_1 - x_2|}{4v_c} |V|, \frac{e|x_1 - x_2|}{4v_n} |V|, \frac{eV/4}{k_B T}\right), \quad (27)$$

where $B_{2n+1}(x, y, z) = 0$, and $A(x)$ and $B_{2n}(x, y, z)$ are scaling functions which reduce to Eqs. (22) and (24) in the $T=0$ limit: $A(\infty) = \text{const}$, $B_{2n}(x, x, 0) \propto x^{1/4} J_{-1/4}(x)$. In the opposite limit, $k_B T > eV$, $A(x) \sim x^{3/2}$ as $x \rightarrow 0$, so that the conductance due to a single point contact is $\sim T^{-3/2}$. The explicit form of $A(x)$ is

$$A(x) = \frac{1}{\sqrt{2}} \frac{e}{4} \frac{\pi \sqrt{x}}{\Gamma\left(\frac{1}{2}\right)} \left| \Gamma\left(\frac{1}{4} + i\frac{x}{2\pi}\right) \right|^2 \sinh(x/2).$$

$B_{2n}(x, y, z)$ is more complicated, but it simplifies in the limit that $(x+y)/z$ is large, where $B_{2n}(x, y, z) \sim e^{-(x+y)/z}$. Consequently, there is an effective dephasing length³¹

$$L_\phi = \frac{\beta}{2\pi} \left(\frac{1/8}{v_c} + \frac{1/8}{v_n} \right)^{-1}, \quad (28)$$

such that

$$I_{12} \propto e^{-|x_1-x_2|/L_\phi} \cos\left(\frac{\Phi}{4\Phi_0} + n\frac{\pi}{4} + \alpha\right). \quad (29)$$

Interference is only visible if the interferometer is smaller than L_ϕ . Equivalently, there is a characteristic temperature scale²³ T^* .

$$k_B T^* = \frac{1}{2\pi|x_1-x_2|} \left(\frac{1/8}{v_c} + \frac{1/8}{v_n}\right)^{-1}. \quad (30)$$

Interference is only visible for $T < T^*$ since Eq. (29) can be rewritten as

$$I_{12} \propto e^{-T/T^*} \cos\left(\frac{\Phi}{4\Phi_0} + n\frac{\pi}{4} + \alpha\right). \quad (31)$$

For fixed v_c , decreasing v_n causes T^* and L_ϕ to decrease. If v_n becomes very small, interference will only be visible at extremely low temperatures or for extremely small interferometers (which, of course, suffer from other problems). In the extreme limit, $v_n=0$, interference will not be visible at all. Numerical studies³² indicate that the two velocities might be quite different, in which case, it will be important that interferometry experiments be done at sufficiently low temperatures. Using commonly accepted values of edge velocities (see, for instance, Ref. 33) of $v_c \approx 5 \times 10^4$ m/s and $v_n = 0.1v_c$, we estimate the dephasing length L_ϕ to be about $4 \mu\text{m}$ at a temperature of 10 mK. We will see below that the direction of the propagation of the neutral mode is irrelevant for these dc interference measurements. Even when the neutral modes propagate in opposition to the charge modes, as in the anti-Pfaffian state, interference can be observable, and the dephasing length is only a function of the magnitude of the velocities of the edge modes.

Figure 3 shows the differential conductance $\partial I/\partial V$ at a temperature much lower than T^* for both even and odd numbers of bulk quasiparticles. As may be seen from this figure, the difference between even and odd numbers of quasiparticles is still very dramatic, even for finite temperature and different charge and neutral velocities. The even quasiparticle differential conductance passes through zero twice at voltages which are small enough that the odd quasiparticle differential conductance is still appreciable (and, of course, due entirely to I_1+I_2).

IV. ANTI-PFAFFIAN EDGE

If one ignores Landau level mixing, then the Hamiltonian for the $\nu=\frac{5}{2}$ QH system is particle-hole symmetric when there is exactly half an electron per flux quantum (ignoring the filled Landau levels). The Pfaffian state, on the other hand, does not possess this symmetry. The particle-hole conjugate of the Pfaffian state, the anti-Pfaffian ($\overline{\text{Pf}}$),^{18,19} has the same energy in the absence of Landau level mixing as the Pfaffian, and should be considered a candidate for the observed $\nu=\frac{5}{2}$ state, even with finite Landau level mixing.

The edge theory of the anti-Pfaffian can be considered by considering a Pfaffian state of holes in a filled $\nu=1$ Landau level as follows:

$$\begin{aligned} \mathcal{L} = & \frac{1}{4\pi} \partial_x \phi_1 (-i\partial_t + v_1 \partial_x) \phi_1 + \mathcal{L}_{\text{Pf}}(\psi_1, \phi_2) \\ & + \frac{1}{4\pi} 2v_{12} \partial_x \phi_1 \partial_x \phi_2 + \xi(x) \psi_1 e^{i(\phi_1-2\phi_2)} + \text{H.c.} \end{aligned} \quad (32)$$

Here, $\mathcal{L}_{\text{Pf}}(\psi_1, \phi_2)$ is the Pfaffian edge action (1) but for counterpropagating edge modes. The coupling v_{12} is short-ranged Coulomb repulsion between the edge mode of the filled Landau level and the charged edge mode of the Pfaffian state of holes while $\xi(x)$ is random tunneling of electrons between the $\nu=1$ edge and the edge of the Pfaffian of holes. For large v_{12} and arbitrarily weak ξ or for small v_{12} and sufficiently large ξ , the theory flows in the infrared to a theory of a forward propagating bosonic charge mode and three *backward propagating* neutral Majorana modes as follows.^{18,19}

$$\mathcal{L}_{\overline{\text{Pf}}} = \frac{2}{4\pi} \partial_x \phi_\rho (\partial_t + v_c \partial_x) \phi_\rho + \sum_{a=1,2,3} i\psi_a (-\partial_t + v_n \partial_x) \psi_a. \quad (33)$$

We will discuss quasiparticle tunneling in this phase of the anti-Pfaffian edge. The three Majorana fermions form an $\text{SU}(2)_2$ triplet, which means that the non-Abelian statistics due to this part of the theory are associated with $\text{SU}(2)_2$ Chern-Simons theory.¹² The electron operator in this theory is $(\psi_2 - i\psi_3)e^{i2\phi_\rho}$. The charge $e/4$ quasiparticles are the primary fields $\phi_{1/2}^\pm e^{i\phi_\rho/2}$, where $\phi_{1/2}^\pm$ are the spin-1/2 fields of $\text{SU}(2)_2$, and can be written in terms of the Ising order and disorder fields σ_a and μ_a . The $\phi_{1/2}^\pm$ fields consist of linear combinations of products of three σ_a or μ_a operators, and therefore has dimension $3/16$. Consequently, the $e/4$ quasiparticle operator in the anti-Pfaffian state has dimension $1/4$, as opposed to dimension $1/8$ in the Pfaffian case. This difference in the scaling dimension causes the Pfaffian and anti-Pfaffian to have different temperature and voltage dependence for transport through point contacts which, in principle, allows one to experimentally distinguish between the two states. Another important difference is that in the anti-Pfaffian case, the charge $e/2$ quasiparticle operator has the same scaling dimension as the $e/4$ quasiparticle and its tunneling is just as relevant, but one expects the bare tunneling element for the $e/2$ quasiparticle to be smaller than the $e/4$ one ($|\Gamma_{e/2}| \sim |\Gamma_{e/4}|^2$).

The above discussion implies that $e/4$ quasiparticle tunneling is the dominant one also in the anti-Pfaffian case. The tunneling current calculation in the double quantum point setup proceeds in a very similar fashion to the Pfaffian case. To lowest order, we must compute four quasiparticle correlation functions, and the relevant conformal block is the one in which quasiparticle fields on both ends of a point contact should fuse the identity. In the long sample limit, we seek the projection of these correlation functions on the conformal block in which quasiparticles on the same edge fuse to the identity.

$\text{SU}(2)_2$ non-Abelian statistics are similar to the Ising statistics that appear in the Pfaffian. In the $\text{SU}(2)_2$ theory there are only three particle types, 0, $1/2$, and 1, with the fusion rule

$$\frac{1}{2} \times \frac{1}{2} = 0 + 1, \quad (34)$$

which is analogous to the fusion rule in Eq. (11). Hence, the enumeration of conformal blocks in $SU(2)_2$ theory is the same as in the Ising theory if we identify the operators I , σ , and ψ with 0, 1/2, and 1 operators, respectively. Also, the matrix elements of the F matrix, which describes the change of basis between different fusion channels, turn out to be the same in both theories, up to a phase.³⁵ An equation analogous to Eq. (16) holds for the anti-Pfaffian case also, but with different power laws since the spin-1/2 operator has a different scaling dimension than the σ operator.

$$\mathcal{F}_I|_{L \rightarrow \infty} = \tilde{\alpha}_I \mathcal{G}_I|_{L \rightarrow \infty} = \tilde{\alpha}_I \prod_{\epsilon=\pm} [\delta + i(v_n(t-t') - \epsilon(x_j - x_k))]^{-3/8}. \quad (35)$$

The tunneling current behavior in the anti-Pfaffian case is qualitatively the same as but quantitatively different from the Pfaffian case. One might worry that no interference should take place at all since the $e/4$ quasiparticle operator is made up of a bosonic part moving in one direction and a fermionic part moving in the opposite direction, and in a semiclassical picture these two parts are moving away from each other. In fact, the sign of the neutral mode velocity makes no difference, as may be seen by comparing Eqs. (16) and (35). As a result of the product over $\epsilon=\pm$, the sign of the neutral mode velocity drops out of the problem. The point is that the quantum mechanical tunneling process involves creating a quasiparticle and a quasihole, and regardless of the chirality of the mode, one excitation will move to the left and one to the right. We note that this breakdown of semiclassical intuition represented by the insensitivity to the neutral mode direction is a feature of a dc measurement. A finite frequency measurement might be more sensitive to the difference between the charge and neutral velocities.

At zero temperature in the anti-Pfaffian state,

$$I_1 + I_2 = \frac{1}{\sqrt{2}} \frac{e}{4} \pi (|\Gamma_1|^2 + |\Gamma_2|^2) v_n^{-6/8} v_c^{-2/8} \text{sgn}(V). \quad (36)$$

The conductance will behave as V^{-1} ; the differential conductance will be sharply peaked at $V=0$ (with a peak width of order $k_B T$) and vanishing elsewhere. For $k_B T > eV$, the conductance varies as T^{-1} . In both cases, there are quantitative differences from the Pfaffian.

Again, for an odd number of quasiparticles in the interference loop,

$$I_{12} = 0. \quad (37)$$

For an even number of bulk quasiparticles, the tunneling current will oscillate with magnetic field and voltage, similar to the Pfaffian case. Again, for charge and neutral velocities which are equal in absolute value (although opposite in sign), I_{12} can be found analytically.

$$I_{12} = \pm \frac{1}{\sqrt{2}} \frac{e}{4} \frac{2\pi^{3/2}}{\Gamma\left(\frac{1}{2}\right)} |\Gamma_1| |\Gamma_2| \cos\left(\frac{\Phi}{4\Phi_0} + n\frac{\pi}{4} + \alpha\right) \times \text{sgn}(V) J_0\left(\frac{e|x_1 - x_2|}{4v_c} |V|\right). \quad (38)$$

Although the phase acquired in the anti-Pfaffian state by an $e/4$ quasiparticle in going around another $e/4$ quasiparticle is different (in either fusion channel) from in the Pfaffian state, the phase acquired by an $e/4$ quasiparticle in going around a charge $e/2$ is $\pm i$ in either state, with the minus sign corresponding to the presence of a neutral fermion.

A difference between the absolute values of the neutral and charge velocities will again be evident through a beating pattern in the differential conductance. I_{12} is exponentially decaying with temperature with characteristic scale as follows:

$$k_B T^* = \frac{1}{2\pi|x_1 - x_2|} \left(\frac{1/8}{v_c} + \frac{3/8}{v_n}\right)^{-1}, \quad (39)$$

and the corresponding dephasing length is

$$L_\phi = \frac{\beta}{2\pi} \left(\frac{1/8}{v_c} + \frac{3/8}{v_n}\right)^{-1}. \quad (40)$$

V. DISCUSSION

As we have seen from the preceding formulas, the Pfaffian and anti-Pfaffian state have qualitatively similar behavior in a two-point-contact interferometer. In particular, the reversal of the neutral modes in the latter state makes little difference. However, the temperature and voltage dependences of the backscattered current are quantitatively different. The difference is clear in the behavior of a single-point contact, where the associated power laws are different, $I \sim V^{-1/2}$ in the case of the Pfaffian and $I \sim V^0$ in the case of the anti-Pfaffian. However, there are also differences in the detailed temperature and voltage dependence of the interference contribution to the current, as may be seen from Eqs. (24) and (38).

The relative insensitivity of quantum interference effects to the difference between the charge and neutral mode velocities runs counter to semiclassical thinking (and shows its limitations): naively, one might think that when a quasiparticle decays into its charged and neutral parts, interferometry would be hopeless. Fortunately, this is not the case, as explicit calculation shows. This also argues well for the suitability of either one for quantum computation along the lines of Refs. 17 and 34. The downside is that the experimental difference between the Pfaffian and anti-Pfaffian states is muted. It can be extracted from the behavior in an interferometer, but it would still be useful to have a probe which is more sensitive to the direction of the neutral modes.

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