Comments on "Plasma current drive by injection of photons with helicity" [Comm. Plasma Phys. Controlled Fusion 12, 165 (1989)]

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(Received 14 February 1989; accepted 3 August 1989)

Ohkawa has proposed a tokamak current-drive scheme [Comm. Plasma Phys. Controlled Fusion **12**, 165 (1989)], which relies on the injection of circularly polarized magnetohydrodynamic waves. It is shown here that the favorable current-drive efficiency predicted by Ohkawa is not attained because excessive power is dissipated by the fluctuating fields. The ratio of power absorbed by the plasma to the dc Ohmic power required to drive the same current is $P_{\text{absorbed}}/P_{\text{Ohmic}} \approx 4(B_a/b)^2$, where B_a is the static toroidal field strength and b is the strength of the fluctuating field.

Ohkawa[1] has proposed injecting waves with circular polarization that propagate and decay in the toroidal direction. It is argued that circularly polarized waves carry a uniform helicity flux that can, in principle, be used to replenish the helicity dissipated in a tokamak.

Consider a straight plasma column that exists for $z \geqslant 0$. Here we will use lower case letters to denote first-order quantities and upper case letters to denote equilibrium quantities. The vector potential of the wave is given by

$$\mathbf{a} = \tilde{a}(1, iS, 0)e^{i(kz-\omega t)}, \tag{1}$$

where $\tilde{a}$ and ω are real, $k = k_0 + i\gamma$ ($|\gamma| \ll |k_0|$), $S = +1$ (right-hand polarization), and $S = -1$ (left-hand polarization). The curl operation is equivalent to multiplication by Sk. Hence

$$\mathbf{b} = Sk\mathbf{a}, \quad \mathbf{j} = k^2\mathbf{a}, \quad \nabla\times\mathbf{j} = Sk^3\mathbf{a}, \quad \mathbf{e} = i\omega\mathbf{a}. \tag{2}$$

The helicity flux and the energy flux are, respectively, given by

$$\text{Re }\mathbf{e}\times\text{Re }\mathbf{a} = S\omega\tilde{a}^2e^{-2\gamma z}\hat{z}, \quad \text{Re }\mathbf{e}\times\text{Re }\mathbf{b} = \omega k\tilde{a}^2e^{-2\gamma z}\hat{z}. \tag{3}$$

In dimensionless units, with B_a denoting the strength of the axial field, the linearized magnetohydrodynamic equations read

$$\rho\frac{\partial\mathbf{u}}{\partial t} = \mathbf{j}\times B_a\hat{z}, \qquad \frac{\partial\mathbf{b}}{\partial t} = \nabla\times(\mathbf{u}\times B_z\hat{z}) + \eta\nabla^2\mathbf{b}, \tag{4}$$

which are Fourier analyzed to yield

$$-i\omega\rho\mathbf{u} = ikB_a\mathbf{b}, \tag{5}$$

$$-i\omega\mathbf{b} = ikB_a\mathbf{u} - \eta k^2\mathbf{b}. \tag{6}$$

Combining[1] Eqs. (5) and (6) yields the dispersion

$$k = (\omega/v_a)(1 - i\eta\omega/v_a^2)^{-1/2}, \tag{7}$$

where v_a is the Alfvén speed. Thus[1] for $\eta\omega/v_a^2 \ll 1$, $k \approx k_0 + i\gamma$, $k_0 = \omega/v_a$, and $\gamma = (\frac{1}{2})k_0(\eta\omega/v_a^2)$. The induction equation [Eq. (6)] becomes

$$\mathbf{u} \approx (-v_a/B_a)\mathbf{b} - (\eta k_0/2B_a)(i\mathbf{b}).$$

The induced voltage, or electromotive force (emf), is given by $\langle\text{Re }\mathbf{u}\times\text{Re }\mathbf{b}\rangle = \frac{1}{2}\text{Re }\mathbf{u}\times\mathbf{b}^*$. Using the above expression for $\mathbf{u}$ yields

$$\text{emf} = -S(\eta k_0/2B_a)k_0^2\tilde{a}^2e^{-2\gamma z}\hat{z}, \tag{8}$$

in agreement with Ref. 1 for the case where $S = +1$. Using $\text{emf} = \eta\mathbf{J}$ we retrieve Ohkawa's result for the driven dc current.

Let us now consider the effect of the ac current $\mathbf{j}$. First consider the helicity "dissipation" resulting from the ac fields. This is $\eta\langle\text{Re }\mathbf{j}\cdot\text{Re }\mathbf{b}\rangle = \eta\frac{1}{2}\text{Re }\mathbf{j}\cdot\mathbf{b}^* = \eta Sk_0^3\tilde{a}^2e^{-2\gamma z}$. Hence

$$\mathbf{J}\cdot B_a\hat{z} = -\tfrac{1}{2}\langle\text{Re }\mathbf{j}\cdot\text{Re }\mathbf{b}\rangle. \tag{9}$$

It follows that the ac magnetic helicity dissipation is twice as large as the dc dissipation and is of the opposite sign. (Since $\langle\mathbf{j}\cdot\mathbf{b}\rangle$ is not positive definite, it is not strictly correct to call this a "dissipation"; however, following the conventional nomenclature, we will continue to do so.) Ohkawa neglects the ac dissipation term entirely. The role of ac magnetic helicity dissipation is further explored in the work of Bellan.[2]

Since $\mathbf{j}$ and $\mathbf{b}$ are parallel, Eq. (9) implies $|J_z| \sim |\mathbf{j}|\,|\mathbf{b}|/2B_a$. Thus the driven dc current is a small fraction of the fluctuating current, since we clearly want $|\mathbf{b}| \ll B_a$. The power absorbed by the plasma is given by $P_{\text{absorbed}} \sim \eta J^2 + \eta\langle\text{Re }\mathbf{j}\cdot\text{Re }\mathbf{j}\rangle$. Hence

$$P_{\text{absorbed}} \sim \eta J^2[1 + 4(B_a/b)^2]. \tag{10}$$

Since $b \ll B_a$, it follows that $P_{\text{absorbed}}/P_{\text{Ohmic}} \approx 4(B_a/b)^2 \gg 1$, where P_{Ohmic} is the power required to drive the same current by conventional Ohmic heating.

It is interesting to note that both the helicity flux and the emf flip sign if S flips sign, whereas the energy flux is always in the $+\hat{z}$ direction. Thus if **J** is parallel to $B_a \hat{z}$, then we have an example of current drive by ac helicity *extraction*.

In summary, the attractive current-drive efficiency quoted by Ohkawa is not attained since excessively large power must be dissipated by the fluctuating fields. It should be noted, finally, that the magnetohydrodynamic equations break down at frequencies approaching the ion-cyclotron frequency. At such frequencies the $\langle \mathbf{j} \times \mathbf{b} \rangle$ Hall emf plays an important role (the Rotamak and related schemes[3] rely on this mechanism) and the current-drive efficiency improves.

[1]T. Ohkawa, Comm. Plasma Phys. Controlled Fusion **12**, 165 (1989).
[2]P. M. Bellan, Phys. Rev. Lett. **57**, 2383 (1986).
[3]I. R. Jones, Comm. Plasma Phys. Controlled Fusion **10**, 115 (1986).

Response to Schalit and Bellan

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(Received 12 June 1989; accepted 3 August 1989)

The ac helicity effect discussed by Schalit and Bellan does not impact the current drive efficiency obtained by Ohkawa.

Schalit and Bellan correctly point out that Ohkawa[1] did not include the helicity term due to the fluctuating current and the wave magnetic field. They then incorrectly conclude that the current drive efficiency predicted by Ohkawa is unattainable when this "ac helicity" effect is included. In fact, they predict the same efficiency as Ohkawa; they have simply chosen another form to present the results. They conclude from their Eq. (10) that $P_{\rm abs}/P_{\rm Ohmic} \gg 1$, because $b \ll B_a$. However, if the electromotive force (emf) of Eq. (8) is set equal to ηJ, it is readily seen that for a tokamak-sized current density $\sim B_a k_0$, a wave magnetic field $b \sim B_a$ is required and thus Ohmic-like efficiencies are obtained. Of course, $b \sim B_a$ is impractical. The way this point was made in Ref. 1 was by noting that resistive absorption is far too weak to drive practical currents. That is why enhanced absorption with minority ion cyclotron resonance damping was suggested as an alternative.

One further comment on the current drive efficiency may be useful. Usually current drive efficiency is measured by some ratio like J/P, the current driven divided by the power absorbed. It can be seen from Eq. (8) that $J \propto b^2$ and then from Eq. (10) that $P_{\rm abs} \propto b^2$ and thus the current drive efficiency J/P is independent of the wave amplitude b. The point is that in this collisional model J/P is large and independent of b but J and P are both very small for $b \ll B_a$.

Finally, we have recently considered a warm multifluid model[2] and found that cyclotron damping by a minority ion species can drive appreciable current with an efficiency estimated as $IR/P = 0.27$ A m/W at $T_e = 40$ keV, where I is the total plasma current, R is the major radius, and P is the absorbed power. Unlike the collisionally damped case, the power here can be totally absorbed. Further, we find in this case that rf helicity injection [the divergence of the helicity flux of Eq. (3)] does drive dc current in the plasma while the ac helicity dissipation is negligible.

[1]T. Ohkawa, Comments Plasma Phys. Controlled Fusion **12**, 165 (1989).
[2]V. S. Chan, R. L. Miller, and T. Ohkawa, submitted to Phys. Fluids B.