Supporting Information for

**Metastable Olivine Wedge beneath the Japan Sea imaged by seismic interferometry**

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**Introduction**

This supplementary material includes Texts S1 to S4, Figures S1 to S13 and Tables S1 to S3. The texts S1 to S4 describe the inter-source interferometry theory, earthquake relocation, thermal modeling for the Japan slab and slab velocity profile setup respectively. The references cited here are included in the reference list of the main paper.
**Inter-source interferometry theory**

Given two sources that located at $x_1$ and $x_2$ being surrounded by a boundary $S$ of receivers $x'$, Curtis et al. (2009) derived the theory of inter-source interferometry based on the representation theorem and source-receiver reciprocity. For actual earthquake recordings in elastic media, the inter-source interferometric response between $x_1$ and $x_2$ in the frequency domain can be approximated as (Eq. 16 in Curtis et al. (2009) Supplementary Material):

\[
M_{ip}^2 M_{mq}^1 \partial_p \partial_q G_{im}^h(x_2|x_1) = iK \omega \oint_S u_n(x'|x_2)u_n^*(x'|x_1) \, dx' \quad (1)
\]

Where $M_{mq}^1$ represents the moment tensor component of source $x_1$ with a pair of opposite force couples acting in $m$th direction and separated in $q$th direction. $M^2$ corresponds to the moment tensor of source $x_2$, $\partial_q$ and $\partial_p$ are the partial derivatives with respect to the $q$th and $p$th direction at location $x_1$ and $x_2$ respectively. $G_{im}^h(x_2|x_1)$ is defined as

\[
G_{im}^h(x_2|x_1) = G_{im}(x_2|x_1) - G_{im}^*(x_2|x_1),
\]

where $G_{im}(x_2|x_1)$ represents the causal Green's function from $x_1$ to $x_2$ and $G_{im}^*(x_2|x_1)$ represents the acausal Green's function due to the wavefield propagating from receivers on $S$ to $x_2$ before refocusing at $x_1$. Subscripts $i$ and $m$ are the $i$th component of the displacement at $x_2$ and $m$th direction for a point source at $x_1$ respectively. $K$ is a constant and $\omega$ is the frequency. $u_n(x'|x_2)$ is the displacement recorded at location $x'$ from source $x_2$ and $u_n^*(x'|x_1)$ is the conjugate form of $u_n(x'|x_1)$.

Essentially, the integral at the right side of eq. 1 is the summation of cross-correlations of displacements recorded at receiver $x'$ from earthquake $x_1$ and $x_2$. To better understand the quantity that is estimated by the cross correlations, we rewrite the left side of eq. 1 as

\[
M_{ip}^2 M_{mq}^1 \partial_p \partial_q G_{im}^h(x_2|x_1) = M_{ip}^2 \partial_p \left[ M_{mq}^1 \partial_q G_{im}^h(x_2|x_1) \right] \quad (2)
\]

In the frequency domain, the term $M_{mq}^1 \partial_q G_{im}^h(x_2|x_1)$ is, by definition, the $i$th component displacement $u_i$ from source $x_1$ to ‘virtual receiver’ $x_2$:

\[
u_i(x_2|x_1) = M_{mq}^1 \partial_q G_{im}^h(x_2|x_1) \quad (3)
\]

Thus, eq. 1 can be written as:

\[
M_{ip}^2 \partial_p u_i(x_2|x_1) = iK \omega \oint_S u_n(x'|x_2)u_n^*(x'|x_1) \, dx' \quad (4)
\]
For the left side, $\partial_t u_i(x_2|x_1)$ is the dynamic strain at ‘virtual receiver’ $x_2$ triggered by passing waves from earthquake $x_1$. The moment tensor $M_{ip}$ of the earthquake occurred at $x_2$ determines the combination of dynamic strains measured in the inter-source interferometry.

This theory has been successfully applied to surface waves (Curtis et al., 2009), but in general, the above conclusion also works for other types of seismic wave, such as the body wave. Here we consider the compressional P wave displacement $u_i^P(x_2|x_1)$, substituting into eq. 4:

$$M_{ip} \partial_t u_i^P(x_2|x_1) = iK_\omega \oint u_n^P(x'|x_2) u_n^P(x'|x_1) \, dx'$$  \hspace{1cm} (5)$$

For a vertical strike slip earthquake, the moment tensor is given by $M_{xy} = M_{yx} = 1$ with other components being 0. Therefore, the left-hand side of Eq. 5 becomes:

$$\partial_y u_x^P(x_2|x_1) + \partial_x u_y^P(x_2|x_1) = e_{xy}^P(x_2|x_1) + e_{yx}^P(x_2|x_1)$$  \hspace{1cm} (6)$$

Where $e_{xy}^P$ and $e_{yx}^P$ are the horizontal transient strains at virtual receiver $x_2$. The strain combination for other types of earthquake mechanisms is summarized in Table S2.

**Earthquake relocation**

We refined the centroid depth of D1 and S1 using ScS and sScS waveforms. Since the ScS and sScS phase propagate near-vertically through the earth interior at short distances, time differences between ScS and sScS are sensitive to the earthquake focal depth. We downloaded three component seismograms of regional stations (0-30 degrees) from F-net (Okada et al., 2004) and GSN (station MDJ), removed the instrumental response, rotated into tangential components, and filtered with a two-pole Butterworth band-pass filter of 0.02-0.05 Hz. We used a frequency-wavenumber method to synthesize ScS and sScS waveforms. In the calculation, the velocity model was constructed by combining the Crust1.0 (Laske et al., 2013) and IASP91 (Kennett & Engdahl, 1991). We cross-correlated the observed tangential seismograms with synthetic waveforms computed for different focal depths in a time window from 50 s before to 350 s after the predicted ScS (Figure S1). The highest correlation coefficient case corresponds to the optimal focal depth at each
station. We then averaged values over all the stations to estimate the centroid depths of S1 and D1 to be 359 km and 580 km, respectively. For the horizontal locations of D1 and S1, we adopted the results from ISC-EHB catalog based on which the Slab2.0 model was constructed (Hayes et al., 2018).

Given S1 as a reference earthquake, we relatively relocated the other two events S2 and S3 using the traveltime data documented by Japan Meteorological Agency (JMA). We performed a grid-search method to invert for the longitude, latitude and depth that minimizes the L2-norm misfit between predicted and reported traveltime differences. The horizontal locations and depths of all earthquakes are listed in Table S3, and the arrival time difference comparison between observed and predicted is shown in Figure S2. In fact, we found that the our relocated S2 and S3 locations are consistent with ISC-EHB results. So, for relocating deep earthquake D2, we directly applied the relative location of D2 and D1 from the ISC-EHB catalog.

**Thermal modeling for the Japan slab**

For modeling the two-dimensional slab thermal profile, we used a finite element code UNDERWORLD2 (Moresi et al., 2007) to solve the convection-diffusion equation.

\[
\frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) - \nabla \cdot (v T) \quad (7)
\]

where \(T, t, \kappa\) and \(v\) are the temperature, time, thermal diffusivity and mantle flow velocity. The thermal diffusivity is set to be \(10^{-6} \text{m}^2/\text{s}\). The initial thermal structure of subducting slab is constructed from a plate cooling model of a 95 km thick lithosphere with the age (~130 Ma) of Pacific plate at the trench position (Sdrolias & Müller, 2006). The mantle flow velocity field is given by the analytical solution for a corner flow model (Turcotte & Schubert, 2014), assuming a convergence velocity of 8 cm/yr (Sdrolias & Müller, 2006) and a 30° dip slab that is close to the slab surface delineated in Slab2.0 model (Hayes et al., 2018).

\[
\left\{ \begin{array}{l}
    v_x = \frac{3\pi}{\pi^2 - 9} - \frac{18 - 3\sqrt{3}\pi}{\pi^2 - 9} \arctan \frac{y}{x} + \left( \frac{3\pi}{\pi^2 - 9} x + \frac{18 - 3\sqrt{3}\pi}{\pi^2 - 9} y \right) \left( -\frac{x}{x^2 + y^2} \right), \\
    v_y = \frac{3\pi}{\pi^2 - 9} \arctan \frac{x}{y} + \left( \frac{3\pi}{\pi^2 - 9} x + \frac{18 - 3\sqrt{3}\pi}{\pi^2 - 9} y \right) \left( -\frac{y}{x^2 + y^2} \right)
\end{array} \right. 
\] 

for arc corner (8)
\begin{equation}
\left\{ \begin{array}{l}
\dot{v}_x = \frac{(1-3\sqrt{3})\pi}{2\pi+3} - \frac{3\sqrt{3}-6}{5\pi+3} \arctan \frac{y}{x} + \left( \frac{-3}{5\pi+3} x + \frac{3\sqrt{3}-6}{5\pi+3} y \right) \left( \frac{-x}{x^2+y^2} \right) \\
\dot{v}_y = \frac{3\pi}{5\pi+3} - \frac{3}{5\pi+3} \arctan \frac{y}{x} + \left( \frac{-3}{5\pi+3} x + \frac{3\sqrt{3}-6}{5\pi+3} y \right) \left( \frac{-x}{x^2+y^2} \right)
\end{array} \right. \quad \text{for mantle corner (9)}
\end{equation}

where \( \dot{v}_x \) and \( \dot{v}_y \) are the horizontal and vertical components of the mantle flow respectively.

The simulated box dimension is 800 km \( \times \) 2400 km with a discretized element spacing of 1 km. In the simulation, we placed the Dirichlet boundary condition on the surface and Neumann boundary conditions at the other three sides for the temperature field. In the meantime, the velocity field is kept invariant. For solving the governing equation, variables in eq. 7 are non-dimensionalized with the following characteristic values:

\begin{equation}
\begin{array}{l}
x_i = d x_i' ; \quad \kappa = \kappa_0 \kappa' ; \quad v_i = \frac{\kappa_0}{d} d v_i' ; \quad t = \frac{d^2}{\kappa_0} t' ; \quad T = T_m \cdot T'
\end{array} \quad \text{(10)}
\end{equation}

where symbols with primes are dimensionless. \( \kappa_0 \) is the reference thermal diffusivity with a value of \( 10^{-6} \) m\(^2\)/s. \( d \) and \( T_m \) denote the mantle thickness and mantle temperature which are 800 km and 1450 °C for our case. The slab is diffused and advected over a duration of 30 Ma which is long enough for surface material descending to the depth with assumed convergence rate.

**Slab velocity profile setup**

Assuming an olivine-rich pyrolite assemblage, we mapped the thermal anomalies in the depth range of 200~800 km into velocity and density perturbations using a scaling from Cammarano et al., (2003). This scaling relation accounts for both anharmonic and anelastic effects with depth. For shallow velocity profile, we extended the seismic velocity and density perturbations inside slab at 200 km upward to the surface. The maximum P and S wave velocity perturbations within the slab are 4.5% and 6% respectively, which agree well with the inferred velocity anomalies from seismic waveform studies (Zhan et al., 2014). Our seismic velocity and density reference model is the IASP91 model (Kennett & Engdahl, 1991).
To simulate realistic coda waves, we added small scale heterogeneities in the lithosphere (above 200 km) described by a Von Kármán type autocorrelation function (Sato et al., 2012) given as:

$$P(k_x, k_z) = \frac{4\pi\kappa \varepsilon^2 a_x a_z}{(1 + a_x^2 k_x^2 + a_z^2 k_z^2)^{\kappa+1}}$$  (11)

where $P$ is the power spectral density function (PSDF), $\kappa$ is the Hurst exponent which is assigned as 0.5 in our model. $k_i$ is the wavenumber in $i$th direction. $a_x$ and $a_z$ are the correlation distances in the x and z components respectively and $\varepsilon$ is the mean square fractional fluctuation. For the velocity and density structures above 200 km, we imposed isotropic scatters outside the slab with $a_x = a_z = 5$ km and $\varepsilon = 2\%$, and elongated scatters within the slab with $a_x = 0.5$ km, $a_z = 10$ km and $\varepsilon = 2.5\%$, which is suggested by long duration coda wave observation in Japan (Furumura & Kennett, 2005).
Figure S1. Focal depth determination of events S1 and D1 using ScS and sScS. (a). Map view of deep earthquakes D1 and S1 and broadband stations used for determining the focal depth. (b). left panel shows the ScS and sScS tangential waveform comparison between observations (black) and synthetics (red) for earthquake S1. Right panel indicates the optimal focal depth of individual station by searching for the highest cross-correlation coefficient. The averaged optimal focal depth is 359 km for S1. (c). similar to (b) but for earthquake D1, the averaged focal depth is 580 km.
**Figure S2.** Traveltime differences as a function of azimuth for different events. The circle and triangle symbols represent traveltime differences of S3-S1 and S2-S1 respectively. Observed and predicted traveltime differences are indicated in blue and red respectively.
**Figure S3.** 0.2–2 Hz inter-source interferometry benchmark results. (a). P wave velocity perturbation profile derived from the thermal modeling. Heterogeneities are imposed above 200 km. The slab and MOW geometries are delineated by black solid line and black dashed line respectively. A 5% velocity reduction is placed within MOW. Deep earthquakes D1 and S1 are used here. Waveforms of a linear array (blue inverted triangles) on the surface are calculated. (b). Inter-source interferometry benchmark result for a case of thermal slab without MOW (NO MOW case). The lower panel shows the cross-correlation record section of the linear array as a function of distance to D1. The upper panel is the waveform comparison between stacked (black) and predicted (red) strain response from D1 to S1. (c). similar to (b). but for the case of slab with MOW. Note that the waveform polarity flipped after introducing the MOW.
Figure S4. Snapshots of wavefield propagating from deep source D1 (purple star) to virtual receiver S1 (magenta triangle). The snapshots are shown for (a) 5.0 s, (b) 30.0 s and (c) 54.8 s after the origin time of D1. The slab and MOW geometries are delineated by black solid line and black dashed line respectively. Both P and S wavefields are shown in the snapshots. Positive and negative wavefield are indicated in red and blue respectively.
**Figure S5.** Inter-source interferometry results of all the six earthquake pairs (a–f) at the frequency band of 0.2–2 Hz. In each subplot, the lower panel is the record section of cross-correlations at individual Hi-net stations and the top panel represents the stacked cross-correlation waveform.
Figure S6. First motion analysis for all deep earthquakes (a~e) used in this study. The fault planes are from JMA solutions. Red dots represent Hi-net stations used in the inter-source interferometry. Black crosses and circles indicate negative and positive polarities identified on F-net seismograms respectively.
Figure S7. Oceanic crust fails to reproduce observed inter-source interferometry results. (a). Slab profile and source-receiver configuration. The grey and cyan area represent the slab and crust respectively. The crust is 8 km thick with 8% reduction in velocity and density. Synthetic waveforms at three virtual receivers (magenta triangles) are calculated for cases of deep earthquakes (purple stars) inside and outside the crust. (b). 0.2~2 Hz waveform comparison between observations (black) and synthetics (red). The red solid lines and red dashed lines indicate synthetics for cases of deep earthquake inside and outside the crust respectively.
Figure S8. Frequency dependence of the inter-source interferometry. (a). Slab profile and source-receiver configuration. The gray and cyan region represent the slab and MOW respectively. The P-wave velocity within MOW decreases 5%. A virtual linear array (magenta triangle) is placed from the slab upper interface toward the slab center at a depth of 359 km. Deep earthquake D1 (purple star) is within MOW with a depth of 580 km. (b). Record section of 0.2~2 Hz waveforms for the virtual array. (c). Similar to (b). but for 0.2~5 Hz waveforms. Note the rapid change of 0.2~5 Hz waveforms (shown in red) at the distance range of 50~70 km, while 0.2~2 Hz waveforms are kept the same.
**Figure S9.** 0.2–5 Hz inter-source interferometry benchmark results for event pairs: (a) D1-S1, (b) D1-S2 and (c) D1-S3. The lower panel shows the coda wave cross-correlation record section of a linear array on the surface (Figure S3a). The upper panel is the waveform comparison between stacked (black) and synthetic (red) strain response.
Figure S10. Inter-source interferometry results at 0.2~5 Hz. Similar to Figure S5 but for the frequency band of 0.2~5 Hz.
Figure S11. Bootstrapping tests of the 0.2~5 Hz inter-source interferometry results from all six deep earthquakes pairs (a~f). The red waveforms are the stacked cross-correlations and grey shadow zones indicate the 95% confidence intervals (2σ). The number of stacked cross-correlations are noted in the top right corner. Note that the splitting waveforms cannot be alternatively interchanged with a single phase within two standard deviations.
Figure S12. 5 Hz waveform comparison of observations (black) and synthetics (red) with different P-wave velocity anomalies within MOW for (a) D1-S1 pair, (b) D1-S2 pair and (c) D1-S3 pair. The velocity perturbation is indicated in the left of each trace. Note that synthetics with a P-wave perturbation of -4% to -5% generally fit observations.
**Figure S13.** Trade-off between thermal modeling parameterizations and cut-off temperature ($T_{mow}$). (a). Slab profile with different MOW geometries. The cyan region represents our proposed model (8 cm/yr, 30 Ma; $T_{mow}=664^\circ$C) in Figure 3a. The red solid line and red dashed line indicate suggested MOW geometries for a slab with a subducting velocity of 8 cm/yr and thermal evolution time of 25 Ma, and a slab with a subducting velocity of 9 cm/yr and thermal evolution time of 30 Ma respectively. The slab subducting velocity, thermal evolution time and kinetic cut-off temperature ($T_{mow}$) for each scenario are shown in the upper left corner. Note that although $T_{mow}$ differs tens of degrees, the overall dimensions are nearly invariant. (b). 0.2~5 Hz waveform comparison of observations (black) and synthetics (red) with different MOW models shown in (a).
<table>
<thead>
<tr>
<th>Method</th>
<th>Study area</th>
<th>Thickness at 410-km [km]</th>
<th>Depth extend [km]</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traveltime inversion</td>
<td>SW. Japan</td>
<td>N/A</td>
<td>~500†</td>
<td>Lidaka &amp; Suetsugu, 1992</td>
</tr>
<tr>
<td>Traveltime inversion</td>
<td>Japan Sea</td>
<td>50</td>
<td>560</td>
<td>Jiang et al., 2008</td>
</tr>
<tr>
<td>Traveltime inversion</td>
<td>Japan Sea</td>
<td>50</td>
<td>570</td>
<td>Jiang &amp; Zhao, 2011</td>
</tr>
<tr>
<td>Traveltime inversion</td>
<td>N. Japan Sea</td>
<td>N/A</td>
<td>580</td>
<td>Jiang et al., 2015</td>
</tr>
<tr>
<td>Receiver function</td>
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<td>~450†</td>
<td>Kawakatsu &amp; Yoshioka, 2011</td>
</tr>
<tr>
<td>Coda wave duration</td>
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<td>600</td>
<td>Furumura et al., 2016</td>
</tr>
<tr>
<td>Deep double seismic zone</td>
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<td>&lt;500†</td>
<td>Likada &amp; Furukawa, 1994</td>
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<tr>
<td>Deep double seismic zone</td>
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<td>460†</td>
<td>Wiens et al., 1993</td>
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<td>Teleseismic waveform</td>
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<td>Kanashima et al., 2007</td>
</tr>
</tbody>
</table>

†: values are not given in the literature, inferred from the figures or contexts.

**Table S1.** Comparison of metastable olivine wedge geometry proposed from previous seismic studies. The light green rows are proposed MOW geometries at other subduction zones.
<table>
<thead>
<tr>
<th>Source Mechanism</th>
<th>Strain Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 degree dip slip</td>
<td>$e_{zz} - e_{yy}$</td>
</tr>
<tr>
<td>Vertical dip slip</td>
<td>$-e_{yz} - e_{zy}$</td>
</tr>
<tr>
<td>Vertical strike slip</td>
<td>$-e_{xy} - e_{yx}$</td>
</tr>
<tr>
<td>Isotropic explosion</td>
<td>$e_{xx} + e_{yy} + e_{zz}$</td>
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</tbody>
</table>

**Table S2.** Combinations of strain components measured for different source mechanisms. Here we take coordinates (x,y,z) at the source as (North, East, Down).
<table>
<thead>
<tr>
<th>Event ID</th>
<th>YYYY/MM/DD</th>
<th>Magnitude [mb]</th>
<th>Depth [km]</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>2009/08/10</td>
<td>4.8</td>
<td>580</td>
<td>130.58</td>
<td>43.52</td>
</tr>
<tr>
<td>D2</td>
<td>2011/01/07</td>
<td>4.8</td>
<td>561</td>
<td>131.04</td>
<td>43.02</td>
</tr>
<tr>
<td>S1</td>
<td>2010/02/05</td>
<td>5.0</td>
<td>359</td>
<td>135.78</td>
<td>40.64</td>
</tr>
<tr>
<td>S2</td>
<td>2012/07/17</td>
<td>4.4</td>
<td>353</td>
<td>135.56</td>
<td>41.18</td>
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<tr>
<td>S3</td>
<td>2017/11/25</td>
<td>N/A</td>
<td>380</td>
<td>135.28</td>
<td>41.70</td>
</tr>
</tbody>
</table>

**Table S3.** Earthquake origin date, magnitude and relocation results for earthquakes used in our inter-source interferometry study. The earthquake date and magnitude information are from ISC-EHB catalog.