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A two-reservoir recycling model for mantle–crust evolution

(mantle depletion/crustal growth/trace elements/neodymium isotopes/strontium isotopes)

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ABSTRACT The exact solutions for the isotopic compositions and the concentrations of the two-reservoir model for mantle–crust evolution are given for arbitrary rates of crustal growth and of back flow to the mantle. The critical parameters are the chemical fractionation factors for crustal growth and refluxing and the integrated fractional mass-removal rates from the crust and the mantle. For the case where refluxing is proportional to crustal growth, all the solutions reduce to simple analytic expressions. The expression for the mean age of the mass of the crust with refluxing is given. If refluxing is significant, the model shows that highly incompatible elements have short residence times in the mantle. With plausible concentration values, material balance implies that the continents were derived from only a small fraction of the mantle.

There has been a renewed interest in models of crustal growth because of the remarkable regularities observed in the initial isotopic composition of Nd. This has led to estimates of the bulk Sm/Nd and Rb/Sr ratios for the earth and provides a basis for considering quantitative models of crustal evolution (1, 2). The formation of continental crust that is enriched in incompatible elements leaves behind a depleted mantle with the matter transport constrained to yield the bulk composition. In a previous work we have presented exact solutions for unidirectional transport from the mantle to the crust for arbitrary rates of crustal growth (3) and have shown the relationships between transport and isotopic abundances with radioactive decay. Other workers have presented analytical and numerical models for transport and isotopic evolution that includes backflow of continental crust to the mantle, using specific forms for the rates of crustal growth or species transport (4–10). Numerical solutions are extremely useful but, in our opinion, do not readily permit physical insight into the basic parameters governing such models. In this report we present some results on formal models of crustal evolution that explicitly include transport of material from the crust back to the mantle for *arbitrary* transfer rates and that may be of value in calculating particular models and in estimating the basic physicochemical parameters.

Notation and general equations for two-reservoir models

We will follow the notation of Jacobsen and Wasserburg (3). The crust grows from an initially undifferentiated mantle, which becomes depleted (reservoir 2) as a result of continuing continental crustal growth. The depleted mantle may make up all or part of the mantle. A portion of the mantle with bulk-earth concentrations of all elements may remain inactive through geologic history (reservoir 1). The depleted mantle (2) and the continental crust (3) are the only active reservoirs in this model. The total mass of 2 and 3 is conserved. This is model II of Jacobsen and Wasserburg (3), but it allows for backflow of crust to the mantle. Let the number of atoms of species i in reservoir j be N_{ij} and let the total mass of reservoir j be M_j. The concentration of species i in j is $C_{ij} = N_{ij}/M_j$. The species under consideration are: s, a stable nuclide with no radioactive parent; r, a radioactive nuclide with decay constant λ_r; and d a stable nuclide (of the same chemical species as s) which is the decay product of r.

For any two-reservoir model in which the continental crust has initially zero mass, conservation of mass requires $M_2(0) = M_2(\tau) + M_3(\tau)$. Initial concentrations in reservoir 2 are assumed to be bulk earth values such that $C_{i2}(0) = N_{i2}(0)/M_2(0) = N_{i1}(0)/M_1(0)$. Conservation of species requires

$$N_{i2}(0)\exp[-\lambda_i\tau] = N_{i2}(\tau) + N_{i3}(\tau) \qquad (i = s,r;\ i \neq d;\ \lambda_s = 0) \quad [1]$$

$$N_{d2}(0) + N_{r2}(0)[1 - \exp(-\lambda_r\tau)] = N_{d2}(\tau) + N_{d3}(\tau). \quad [2]$$

The time τ runs forward from the initial state at the formation of the earth. The time measured backward from the present will be called T, such that today $T = T_o - \tau$ where T_o is the age of the earth today. Fractional deviations of the isotopic ratios N_{dj}/N_{sj} in reservoirs $j = 2,3$ from the bulk earth values are given by $\epsilon^*_{dj}(\tau) \equiv [(N_{dj}/N_{sj})/(N_{d1}/N_{s1}) - 1] \times 10^4$. The enrichment factors are given by $f^{r/s}_j \equiv (N_{rj}/N_{sj})/(N_{r1}/N_{s1}) - 1$. The general relationships between ϵ^*_{dj} and $f^{r/s}_j$ for the two reservoirs is given in terms of the depletion of species s in reservoir 2 by the fundamental relationships:

$$\frac{\epsilon^*_{d2}(\tau)}{\epsilon^*_{d3}(\tau)} = \frac{f^{r/s}_2(\tau)}{f^{r/s}_3(\tau)} = 1 - \left[\frac{N_{s2}(0)}{N_{s2}(\tau)}\right] \quad [3a]$$

$$M_3(\tau)/M_2(0) = f^{r/s}_2(\tau)C_{s1}(\tau)/[f^{r/s}_2(\tau) - f^{r/s}_3(\tau)]C_{s3}(\tau). \quad [3b]$$

This is equivalent to equation 78 given in ref. 3.

Let the fluxes of mass and species i from reservoir j to k be $\dot{M}_{jk}$ and J_{ijk}, respectively. The transport equations are:

$$\frac{dM_2}{d\tau} = -\frac{dM_3}{d\tau} = \dot{M}_{32} - \dot{M}_{23} \quad [4]$$

$$\frac{dN_{i2}}{d\tau} = J_{i32} - J_{i23} - \lambda_i N_{i2} \quad (i = s,r;\ i \neq d;\ \lambda_s = 0) \quad [5]$$

$$\frac{dN_{d2}}{d\tau} = J_{d32} - J_{d23} + \lambda_r N_{r2}. \quad [6]$$

The solutions for one reservoir are known from the other by the species and mass conservation equations.

Fluxes between the reservoirs

The preceding equations are valid for any two-reservoir model. We now make a specific assumption about the fluxes. Assume that a differential mass is removed from reservoir 2 as a partial melt and is added to the crust. The bulk mass flux into the crust is $\dot{M}_{23}$, and the concentration of a species i in the mass of partial melt ($\dot{M}_{23}\delta\tau$) is $c_{i3}(\tau)$. Let d_i be the fractionation factor of el-

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ement i in a differential segment of mantle melt that is to be added to the crust ($d_i = c_{i3}/C_{i2}$; concentration in melt/average concentration in mantle). The flux of species i from 2 to 3 is then given by

$$J_{i23}(\tau) = c_{i3}(\tau)\dot{M}_{23} = d_i C_{i2}(\tau)\dot{M}_{23} = d_i N_{i2}(\tau)\psi(\tau), \quad [7]$$

in which $\psi(\tau) \equiv \dot{M}_{23}/M_2$ is the fractional rate of mass removal from the depleted mantle. Similarly assume that the material being subducted back to the mantle is average crust with concentration C_{i3} chemically fractionated by a factor g_i (concentration of i in subducted material/concentration of i in average crust at that time). The flux of species i from 3 to 2 is then given by

$$J_{i32}(\tau) = g_i C_{i3}(\tau) M_{32} = g_i N_{i3}(\tau)\phi(\tau), \quad [8]$$

in which $\phi(\tau) \equiv \dot{M}_{32}/M_3$ is the fractional rate of mass removal from the continental crust. If the backflow is proportional to the area of the continents, we get $\phi(\tau) = E(\tau)/\rho h$ where $E(\tau)$ is the mass "erosion rate" per unit area and ρh is the average effective mass of a crustal column of unit area. The reciprocals of ψ and ϕ are the instantaneous residence times for the bulk mass of reservoirs 2 and 3, respectively.

Solutions of the transport equations

For the bulk mass flow we get by substituting the definitions for ψ and ϕ into Eq. **6**

$$-\frac{dM_2}{d\tau} = \frac{dM_3}{d\tau} = \psi(\tau)M_2(\tau) - \phi(\tau)M_3(\tau), \quad [9a]$$

and it follows that the mass of the crust as a function of time is given by

$$M_3(\tau) = M_2(0)\exp[-\Psi(\tau) - \Phi(\tau)] \times \int_o^\tau \psi(\xi)\exp[\Psi(\xi) + \Phi(\xi)]d\xi \quad [9b]$$

$$= \int_o^\tau M_2(\xi)\psi(\xi)\exp[-\Phi(\tau) + \Phi(\xi)]d\xi$$

where $\Psi(\tau) \equiv \int_o^\tau \psi(\xi)d\xi$ and $\Phi(\tau) \equiv \int_o^\tau \phi(\xi)d\xi$. The sum $\Psi(\tau) + \Phi(\tau)$ is the time-integrated fractional mass circulation and is the key time parameter of these equations. Define $a_i(\tau) \equiv d_i\psi(\tau) + g_i\phi(\tau)$ and then $A_i(\tau) \equiv \int_o^\tau a_i(\xi)d\xi = d_i\Psi(\tau) + g_i\Phi(\tau)$ is the time-integrated fractional species circulation. The transport equations (Eqs. **5** and **6**) for reservoir 2 for the isotopes may then be written:

$$\frac{dN_{i2}}{d\tau} + [\lambda_i + a_i(\tau)]N_{i2}(\tau) = g_i\phi(\tau)N_{i2}(0)\exp[-\lambda_i\tau] \quad (i = s,r; i \neq d; \lambda_s = 0) \quad [10]$$

$$\frac{dN_{d2}}{d\tau} + a_s(\tau)N_{d2}(\tau) = g_s\phi(\tau)\{N_{d2}(0) + N_{r2}(0) \times [1 - \exp(-\lambda_r\tau)]\} + \lambda_r N_{r2}(\tau). \quad [11]$$

Define

$$B_i(\tau) \equiv 1 + g_i\int_o^\tau \phi(\xi)\exp[A_i(\xi)]d\xi, \quad [12]$$

then the exact solution of Eq. **10** is:

$$N_{i2}(\tau) = N_{i2}(0)B_i(\tau)\exp[-\lambda_i\tau - A_i(\tau)] \quad (i = s,r; i \neq d; \lambda_s = 0). \quad [13]$$

The solution for the daughter isotope becomes:

$$N_{d2}(\tau) = \frac{N_{d2}(0)B_s(\tau) + \lambda_r N_{r2}(0)\int_o^\tau \{B_r(\xi)\exp[-A_r(\xi) + A_s(\xi)] - B_s(\xi) + B_s(\tau)\}\exp[-\lambda_r\xi]d\xi}{\exp[A_s(\tau)]}. \quad [14]$$

The chemical enrichment factor $f_2^{r/s}$ and the isotopic effects expressed as ϵ values for the depleted mantle are then given by

$$f_2^{r/s}(\tau) + 1 = \frac{B_r(\tau)}{B_s(\tau)}\exp[-A_r(\tau) + A_s(\tau)]. \quad [15]$$

$$\epsilon^*_{d2}(\tau) = \frac{Q^*_d(\tau)}{B_s(\tau)}\int_o^\tau f_2^{r/s}(\xi)\,B_s(\xi)\exp[\lambda_r(\tau - \xi)]d\xi \quad [16]$$

Here $Q^*_d(\tau) \equiv 10^4\lambda_r N_{r1}(\tau)/N_{d1}(\tau)$. Eqs. **13**, **15**, **16**, and **3** are the basic equations for calculations with this model for arbitrary rates of crustal growth and of refluxing. The form of these equations is the same as for model II of Jacobsen and Wasserburg (3) with no refluxing ($\Phi \equiv 0$). In this case $B_i = 1$, $dM_2/d\tau = -\dot{M}_{23}$ and $A_i(\tau) = d_i \ln[M_2(\tau)/M_2(0)]$.

For $\lambda\tau \ll 1$, we have $Q^*_d \simeq$ constant and the deviations ϵ^*_{dj} are

$$\epsilon^*_{dj}(\tau) = Q^*_d f_j^{r/s}(\tau)\tilde{t}_{r/s}, \quad [17]$$

in which

$$\tilde{t}_{r/s} \equiv \frac{1}{B_s(\tau)f_2^{r/s}(\tau)}\int_o^\tau f_2^{r/s}(\xi)B_s(\xi)d\xi. \quad [18]$$

We now derive an expression for the mean age of the crust for this model as a generalization of the treatment without refluxing by Jacobsen and Wasserburg (3). We assume that new crust is added in small random parcels that have an equal *a priori* probability of being refluxed. Then the probability that a parcel of matter added at time ξ survives at time τ is given by $\exp[-\Phi(\tau) + \Phi(\xi)]$. Thus the mass added per unit time to the continents at time ξ and remaining at time τ is given by $\dot{M}_{23}(\xi)\exp[-\Phi(\tau) + \Phi(\xi)]$. It follows that the mean age of the mass of the crust at time τ is given by

$$\bar{t}_{M3} = \frac{1}{M_3(\tau)}\int_o^\tau (\tau - \xi)\dot{M}_{23}(\xi)\exp[-\Phi(\tau) + \Phi(\xi)]d\xi = \int_o^\tau \frac{M_3(\xi)\exp[\Phi(\xi)]d\xi}{M_3(\tau)\exp[\Phi(\tau)]}. \quad [19]$$

Inspection of Eq. **19** shows (*i*) a low probability of survival for old crust requires Φ(4.5 AE) to be $\gg 1$ and (*ii*) the survivability of the crust at 2.8 AE requires $\Phi(4.5 \text{ AE}) - \Phi(\tau)$ for $\tau >$ 1.7 AE (2.8 AE age) to be small in order to explain the observed frequency of ages assuming a roughly uniform rate of crustal growth. This clearly demands that $\Phi(\tau)$ reach a large value early and that it only change slowly after that time.

Refluxing proportional to crustal growth

If we now consider a restricted relationship in which the fractional rate of refluxing $\dot{M}_{32}/M_3$ is proportional to the rate of crustal addition, then $\phi(\tau) = \beta\psi(\tau)$ and $\Phi(\tau) = \beta\Psi(\tau)$ where β is a constant. Then $A_i(\tau) = (d_i + \beta g_i)\Psi(\tau)$ and

$$B_i(\tau) = [d_i + \beta g_i\exp[A_i(\tau)]]/(d_i + \beta g_i), \quad [20]$$

and the solutions for an arbitrary growth function reduce to the simple forms

$$M_3(\tau)/M_2(0) = \{1 - \exp[-(1 + \beta)\Psi(\tau)]\}/(1 + \beta) \quad [21]$$

$$\frac{N_{i2}(\tau)}{N_{i2}(0)} = \exp[-\lambda_i\tau]\frac{\{d_i\exp[-(d_i + \beta g_i)\Psi(\tau)] + \beta g_i\}}{(d_i + \beta g_i)} \quad (i = s,r; i \neq d; \lambda_s = 0). \quad [22]$$

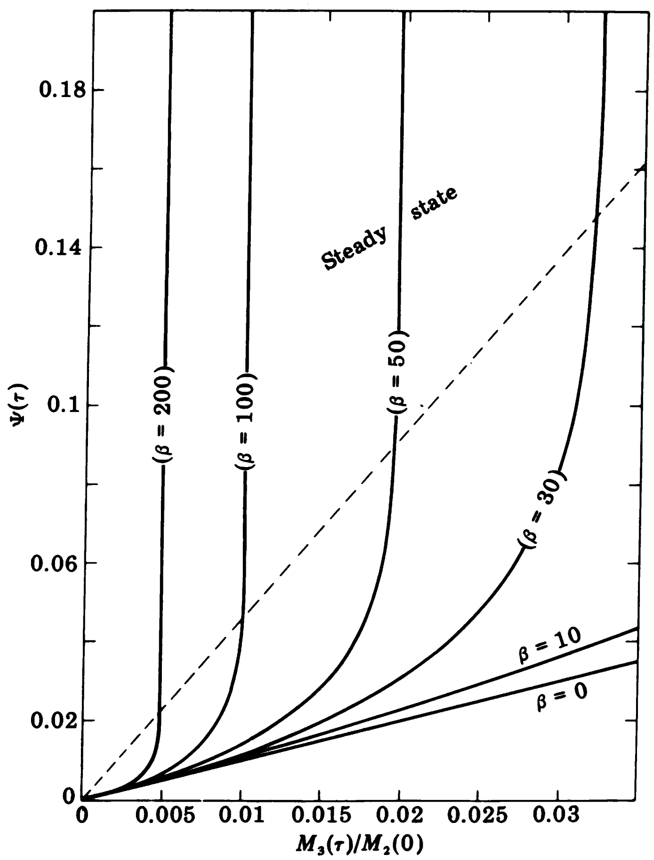

FIG. 1. Plot of $\Psi(\tau)$, the fractional mass circulation integral for the mantle vs. the degree of crust formation $M_3(\tau)/M_2(0)$ for the case where the fractional refluxing integral $\Phi(\tau) = \beta\Psi(\tau)$. At steady state $M_3^\infty/M_2(0) = 1/(1+\beta)$. The region above the dashed line shows where $M_3(\tau)/M_2(0)$ is within 1% of its steady state value.

The explicit relationship (Eq. 21) (see Fig. 1) allows a direct calculation of $\Psi(\tau)$ from a choice of $M_3(\tau)$. Hence, the solutions given here are explicit determinations of the state of the system at a time τ given a choice for the mass of the crust $M_3(\tau)$ and the parameters d_i, g_i, and β.

The expression for the enrichment factor f is given by

$$f_2^{r/s}(\tau)+1=\left[\frac{d_s+\beta g_s}{d_r+\beta g_r}\right]\times\left\{\frac{d_r\exp[-(d_r+\beta g_r)\Psi(\tau)]+\beta g_r}{d_s\exp[-(d_s+\beta g_s)\Psi(\tau)]+\beta g_s}\right\}, \qquad [23]$$

and the expression for ϵ_{d2}^* may be obtained by substituting Eqs. 20 and 23 into Eq. 16 or Eq. 17 if $\lambda\tau \ll 1$. The values of ϵ_{d3}^* and $f_3^{r/s}$ can then be calculated from Eqs. 3a, 22, and 23. If ϵ_{d2}^* is known and $\lambda\tau \ll 1$, then $\tilde{t}_{r/s}$ can be calculated from Eqs. 17 and 23.

The fraction x of the total amount of crust produced that has been refluxed $(0 \leq x \leq 1)$ is given by

$$x(\tau)=\frac{\int_o^\tau \dot{M}_{32}(\xi)d\xi}{\int_o^\tau \dot{M}_{23}(\xi)d\xi}=\frac{(\beta/u)\ln[1-u]+\beta}{(\beta/u)\ln[1-u]-1}. \qquad [24]$$

Here $u \equiv (1+\beta)M_3(\tau)/M_2(0)$. We also have the relationship

$$\Psi(\tau)=\frac{M_3(\tau)}{M_2(0)}\left[\frac{x(\tau)+\beta}{\beta[1-x(\tau)]}\right]. \qquad [25]$$

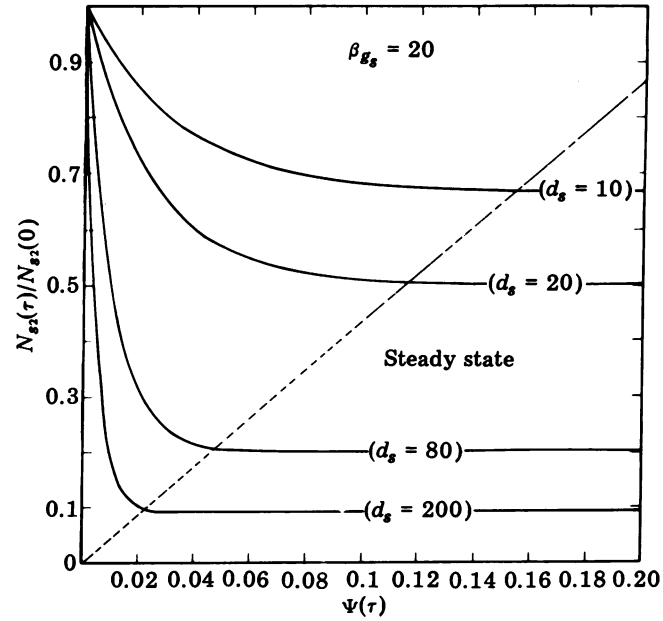

FIG. 2. Plot of the degree of depletion of an incompatible stable species s in the mantle $N_{s2}(\tau)/N_{s2}(0)$ vs. the fractional mass circulation integral $\Psi(\tau)$ for a refluxing of $\beta g_s = 20$. The region to the right of the dashed line shows where $N_{s2}(\tau) - N_{s2}(0)$ is within 1% of its steady state value $[N_{s2}^\infty/N_{s2}(0) = \beta g_s/(d_s + \beta g_s)$ at steady state]. Note that the stronger the depletion in the mantle is, the faster the system reaches steady state.

Most of the equations can be immediately evaluated given the value of the matter circulation integral $\Psi(\tau) + \Phi(\tau) = (1+\beta)\,\Psi(\tau)$ for any choice of parameters. Graphs of the depletion of a stable species in the mantle as a function of $\Psi(\tau)$ are shown in Fig. 2.

The condition for a quasi steady state is given for $\Psi(\tau) \rightarrow \infty$ (with $\lambda_r\tau$ bounded) and reduces to

$$M_3^\infty = M_2(0)/(1+\beta) \qquad [26]$$

$$N_{i2}^\infty = N_{i2}(0)\exp[-\lambda_i\tau]\beta g_i/(d_i+\beta g_i) \qquad (i = s,r;\ i \neq d;\ \lambda_s = 0) \qquad [27]$$

$$f_{2\infty}^{r/s}+1=\frac{1+(d_s/\beta g_s)}{1+(d_r/\beta g_r)} \qquad [28]$$

$$\tilde{t}_{r/s}^\infty=\frac{1}{(d_s+\beta g_s)\psi_\infty},\ \tilde{t}_{M3}^\infty=\frac{1}{\beta\psi_\infty} \qquad [29]$$

$$\epsilon_{d2}^{*\infty}=-Q_d^*(\tau)\left\{\frac{[(d_r/\beta g_r)-(d_s/\beta g_s)]}{[1+(d_r/\beta g_r)][(d_s+\beta g_s)\psi_\infty-\lambda_r]}\right\}. \qquad [30]$$

If $d_i/\beta g_i = 1$, this results in a depletion of $N_{s2}^\infty/N_{s2}(0)$ by a factor of 2. For large depletion, $d_i/\beta g_i \gg 1$. In particular, if we were to require that the depletion in the mantle be a factor of 11, this gives $d_i/\beta g_i = 10$. We also note that for any reasonable choices of ψ_∞ and β that $\tilde{t}_{M3}^\infty$ is much larger than 4.5 AE.† From Eq. 22 we have $d_s\{[N_{s2}(0)/N_{s3}(\tau)] - 1\} \geq \beta g_s$. This inequality may be used to put rather strict limits on the amount of refluxing for elements with $d_s \gg 1$, if an independent estimate can be made of d_s and $N_{s2}(0)/N_{s3}(\tau)$.

Changing reflux rates

The general treatment with Ψ and Φ, each arbitrary and independent, permits the treatment of changing reflux rates relative to growth rates. For simplicity it is convenient to con-

† "AE" is equivalent to 10^9 years.

sider time regimes in which the refluxing rates are markedly different and in which a simple scaling law exists, such as $\phi(\tau)/\psi(\tau) = \beta(\tau)$. The simplest of these is $\phi = \beta_1\psi$ for $[0 \leqslant \tau \leqslant \tau_1]$ and $\phi = \beta_2\psi$ for $[\tau_1 < \tau]$ where β_1 and β_2 are constants. This gives $\Phi(\tau) = \beta_1\Psi(\tau)$ for $(\tau < \tau_1)$ and $\Phi(\tau) = \beta_1\Psi(\tau_1) + \beta_2[\Psi(\tau) - \Psi(\tau_1)]$ for $(\tau_1 < \tau)$. The exact solution may then be found by substitution in Eqs. **12 to 16,** which yields a generalization of the results as given for a single regime in the previous section. If we require that there be very little crust remaining at time τ_1 for a given growth law $[\Psi(\tau)]$, then this demands that $(\beta + 1)\Psi(\tau_1) \gg 1$ and $\beta + 1 \gg 1$. For example, if the amount of crust created between the origin of the earth and 3.6 AE is about one-fourth of the current mass of the crust $(M_3(T_o)/M_2(0) = 0.02)$, and if it is 75% destroyed by 3.6 AE (i.e., $x \sim 0.75$ in Eq. **24**) then we obtain $\beta \approx 800$.

RESULTS

The exact solutions to the two-box model of crustal evolution with refluxing are given in terms of the basic parameters d_i and g_i and the fractional crustal growth and refluxing functions $\Psi(\tau)$ and $\Phi(\tau)$. The solutions reduce to the same basic form as derived earlier for models without refluxing (3). In particular it is shown that isotopic deviations from bulk earth values are given by $\epsilon^*_{d2}(\tau) = Q^*_d f_2^{r/s}(\tau)\, \tilde{t}_{r/s}(\tau)$ for $\lambda\tau \ll 1$ (Eq. **17**), which is identical in form to that obtained without refluxing. Thus, if ϵ^*_{d2} and $f_2^{r/s}$ are fixed, the numerical value of the time parameter $\tilde{t}_{r/s}$ is equal to the value for the analogous time parameter $\tilde{t}_{r/s}$ without refluxing (given in ref. 3); however, the interpretation of the "time" $\tilde{t}_{r/s}$ in terms of crustal evolution is much more complicated (cf. Eq. **18**). The exact solutions for arbitrary rates of crustal growth and refluxing may be readily calculated by quadrature. For the class of solutions where $\Phi(\tau)/\Psi(\tau) = \beta$ (constant), the time only appears indirectly through $\Psi(\tau)$ and β or $M_3(\tau)/M_2(0)$. These models may easily be evaluated with a standard hand calculator.

Whereas the detailed evolution and interpretation of isotopic patterns may be somewhat complex because of the variety of parameters, the problem is basically controlled by material balance between two reservoirs. It follows that there are very simple relationships between intensive quantities, such as ϵ^*_{d2} and ϵ^*_{d3} or $f_2^{r/s}$ and $f_3^{r/s}$, and the degree of depletion in the mantle (cf. Eq. 3). These imply strong constraints on all such models through the mass ratio $M_3(\tau)/M_2(0)$ and the concentrations in the crust and mantle. Insofar as the chemical abundances and mass ratio are known, this fixes a relationship between the reflux parameters. For example, if we choose the concentrations and $M_3(\tau)/M_2(0)$ as given in ref. 3, which are in italics in Table 1, then all of the other parameters given in Table 1 except $\tilde{t}_{r/s}$ result from using Eq. 3 and the definitions of f and ϵ values. The time parameters $\tilde{t}_{Sm/Nd}$ and $\tilde{t}_{Rb/Sr}$ may then be calculated from the values of f and ϵ (Eq. **17**). If for simplicity we assume that

Table 2. Estimates for characteristic times (τ_s) for circulation of stable isotopes of some elements between mantle and crust for various mass fractions of refluxed crust x

	$\beta = 0, x = 0$	$\beta = 9.52, x = 0.1$	
	d_s	d_s	$\tau_s(AE)$
I. 28% mantle $M_3(T_o)/M_2(0) = 0.02$			
Rb ($g_{Rb} = 3$)	134.3	402.3	0.47
Rb ($g_{Rb} = 1$)	134.3	175.5	1.1
Sr ($g_{Sr} = 1$)	20.30	20.61	6.6
Sm ($g_{Sm} = 1$)	14.08	14.23	8.4
Nd ($g_{Nd} = 1$)	26.34	26.91	5.5
Bulk mass	≡1	≡1	19.0
II. Whole mantle $M_3(T_o)/M_2(0) = 0.005592$			
Rb ($g_{Rb} = 3$)	428.2	1047	0.63
Rb ($g_{Rb} = 1$)	428.2	523.7	1.3
Sr ($g_{Sr} = 1$)	17.61	17.67	13.9
Sm ($g_{Sm} = 1$)	12.79	12.83	15.3
Nd ($g_{Nd} = 1$)	21.86	21.96	12.8
Bulk mass	≡1	≡1	20.4

$\Phi = \beta\Psi$ and independently choose a value for β, then Ψ (Eq. **21**) and the fraction x of refluxed crust (Eq. **24**) can be calculated. A further independent choice of g_i then determines d_i (Eq. **22**). If we take $\beta = 9.52$ and $M_3(\tau)/M_2(0) = 0.02$, then $\Psi = 0.02246$ and 10% of the crust was recycled (Eq. **24**). With chosen g_i values, the corresponding values of d_i are determined and given in the second and third columns of Table 2. We assume $g_s = 1$ for Sr, Nd, and Sm. Rb is strongly enriched in the upper crust ($\sim$ a factor of 3), so if upper crust is refluxed, then $g_{Rb} \approx 3$. Thus, we have calculated d_{Rb} by assuming both $g_{Rb} = 1$ and $g_{Rb} = 3$. We note that the d values for Sr, Nd, and Sm in Table 2 are only 1% to 2% higher than those for no refluxing ($x = 0$). However, the value for d_{Rb} clearly changes drastically for refluxing, as the value without refluxing is $d_{Rb} = 134.3$. The lower half of Tables 1 and 2 were computed for the case in which the whole mantle was involved in crustal formation. The whole mantle model gives $\tilde{t}_{Sm/Nd} = 7.6$ AE. Because this parameter clearly cannot exceed 4.5 AE, this shows that such a model is inconsistent with the primary input values we selected. The whole mantle model also gives a very high Rb concentration for the bulk crust of 102 ppm. This results in extremely large values of d_{Rb}. If the d_{Rb} values in Table 2 for $x = 0.1$ are too large (175–1050), then for a given mass fraction of refluxing x, the value of d_{Rb} may be reduced by increasing $M_3(\tau)/M_2(0)$, thus making the size of the depleted mantle $M_2(0)$ smaller. Both of these calculations strongly indicate that most crustal growth involves only the upper mantle, in agreement with earlier calculations (3, 8). This conclusion is unaffected by refluxing and is a direct consequence of the material balance conditions and the concentrations used.

The characteristic time for circulation of a stable species s

Table 1. Self-consistent sets of present day ($\tau = T_o$) parameters for the two-reservoir model with backflow of crust to the mantle

Reservoir	Rb	Sr	$f^{Rb/Sr}$	ϵ_{Sr}	Sm	Nd	$f^{Sm/Nd}$	ϵ_{Nd}
Undepleted mantle	*0.6287*	*22*	≡0	≡0	*0.4097*	*1.26*	≡0	≡0
I. 28% mantle	$M_3(T_o)/M_2(0) =$ *0.02*; $\tilde{t}_{Rb/Sr} = 1.796$ AE; $\tilde{t}_{Sm/Nd} = 1.416$ AE							
Continental crust	29.35	*370*	1.776	53.27	5.072	*26*	*−0.4*	−14.23
Depleted mantle	0.04257	14.90	*−0.9*	*−27*	0.3145	0.7551	0.2811	*10*
II. Whole mantle	$M_3(T_o)/M_2(0) =$ *0.005592*; $\tilde{t}_{Rb/Sr} = 1.796$ AE; $\tilde{t}_{Sm/Nd} = 7.627$ AE							
Continental crust	102.2	*370*	8.670	260.1	5.072	*26*	*−0.4*	−76.66
Depleted mantle	0.05728	20.04	*−0.9*	*−27*	0.3835	1.121	0.05218	*10*

Concentrations are given in parts per million by weight. The parameters in italics are the primary input values from Jacobsen and Wasserburg (3). The other parameters are calculated from these values. The $\tilde{t}_{r/s}$ values were calculated using $Q^*_{Nd} = 25.13 \times 10^{-9}$ yr^{-1} and $Q^*_{Sr} = 16.70 \times 10^{-9}$ yr^{-1}. The table includes the new chondritic values for Sm/Nd by Jacobsen and Wasserburg (11).

is $\tau_s \equiv 1/(d_s + \beta g_s)\psi$. If ψ is constant, then using Eq. **25** we get

$$\tau_s = \frac{\tau\beta(1 - x)M_2}{(d_s + \beta g_s)(x + \beta)M_3}. \quad [31]$$

We reemphasize the importance of the Rb–Sr decay system in the depleted oceanic mantle because of the strong depletion in Rb (3). By using the d_s values in the first half of Table 2, the τ_s values may be calculated for the case $M_3(\tau)/M_2(0) = 0.02$ and $x = 0.1$ for a total time $\tau = 4.5$ AE. For $d_{\mathrm{Rb}} = 176$ and $g_{\mathrm{Rb}} = 1$, we get $\tau_{\mathrm{Rb}} = 1.1$ AE. If we assume that upper crust with $g_{\mathrm{Rb}} = 3$ is refluxed, then $d_{\mathrm{Rb}} = 402$ and $\tau_{\mathrm{Rb}} = 0.47$ AE. Smaller values of M_2/M_3 would decrease τ_{Rb}, whereas use of the whole mantle $M_3/M_2 = 0.005592$ will increase τ_{Rb} for the same mass fraction of refluxing as shown in the second half of Table 2. The short values for τ_{Rb} mainly reflect the short residence time of Rb in the mantle ($= 1/d_{\mathrm{Rb}}\psi$), whereas the residence time of Rb in the crust ($= 1/\beta g_{\mathrm{Rb}}\psi$) is relatively long. Estimates of τ_s for Sr, Sm, Nd, and bulk mass are also given in Table 2. The time it takes for $N_{s2}(\tau) - N_{s2}(0)$ to change to be within 5% of its steady state value is $3\tau_s$. So for $\tau = 4.5$ AE, we get $\tau_s = 1.5$ AE. It follows that Rb may have been close to a steady state distribution in the upper mantle over much of geologic time with any significant refluxing. The same is true for all other highly incompatible elements, including Ba, K, U, and Th. In contrast, elements with much smaller values of d_s as Sr, Nd, and Sm have $\tau_s > 4$ AE and are far away from steady state with refluxing.

CONCLUSIONS

Formal expressions for the concentrations and isotopic composition in crust and mantle are given for models for the growth of the continents from a homogeneous mantle layer. These models yield very short residence times for highly incompatible elements in the mantle and require that the mass of the mantle from which the continents are derived is a small fraction of the total mass of the mantle. Although models controlled by refluxing can give relatively uniform concentrations of incompatible trace elements over much of geologic time, there are several difficulties. It is not possible to derive materials with $\epsilon^*_{\mathrm{Nd}}(\tau) = 0$ from the mantle over the past ~2–3 AE. This requires that magmas with $\epsilon^*_{\mathrm{Nd}}(\tau) = 0$ are "accidental" mixtures of crust and depleted mantle in the appropriate proportions. From these considerations we infer that a two-layer model (crust and depleted mantle), in which the mantle layer is continuously depleted (with or without refluxing) is not an adequate explanation of the observations. We conclude that a three- or four-layer model is required that includes (*i*) a lower undepleted mantle which continues to provide material to (*ii*) the depleted upper mantle and (*iii*) the crust. The upper part of the depleted upper mantle (~200 km?) behind subduction zones may be a buffer zone, which rapidly refluxes the material that is transported off the continents into the upper depleted (oceanic) mantle. This zone rapidly (0.2 AE) returns most of the refluxed material back to orogenic belts on the continents along with some of the upper-mantle material. The main transport is then considered to be the concurrent formation of continental crust with the creation of newly depleted mantle, which reduces the mass of undepleted mantle as in model I in ref. 3. The refluxing is then a mechanism that recreates crust from preexisting crust plus some depleted mantle and changes the age pattern on the crust but does not supply the main source of primary new crustal material. This type of model will require joining models I and II (3) with shallow refluxing. In all these considerations, it must be respected that the bulk-earth parameters are still not sufficiently well established and that changes in this basic reference state will have substantial quantitative and qualitative effects on our understanding. It still appears necessary to consider two distinct periods of crustal evolution. The early period (prior to 3.6 AE) in which refluxing was very high, transporting material to depths, and a subsequent period in which the refluxing was substantially smaller. These time scales are comparable to the decay of ^{235}U and undoubtedly reflect a major change in the vigor of crustal growth and destruction with the decreasing heat flow. The results obtained in this work show that the formal problem of mantle differentiation and crustal growth with refluxing can be reduced to a simple analytical form that is readily susceptible to numerical evaluation and to physical interpretation.

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1. DePaolo, D. J. & Wasserburg, G. J. (1976) *Geophys. Res. Lett.* **3**, 743–746.
2. O'Nions, R. K., Hamilton, P. J. & Evensen, N. M. (1977) *Earth Planet. Sci. Lett.* **34**, 13–22.
3. Jacobsen, S. B. & Wasserburg, G. J. (1979) *J. Geophys. Res.* **84**, 7411–7427.
4. Armstrong, R. L. (1968) *Rev. Geophys. Space Phys.* **6**, 175–199.
5. Russel, R. D. (1972) *Rev. Geophys. Space Phys.* **10**, 529–549.
6. Russel, R. D. & Birnie, D. J. (1974) *Phys. Earth Planet. Inter.* **8**, 158–166.
7. Doe, B. R. & Zartman, R. E. (1979) in *Geochemistry of Hydrothermal Ore Deposits*, ed. Barnes, H. L. (Wiley, New York), 2nd Ed., pp. 22–70.
8. O'Nions, R. K., Evensen, N. M. & Hamilton, P. J. (1979) *J. Geophys. Res.* **84**, 6091–6101.
9. Armstrong, R. L. & Hein, S. M. (1973) *Geochim. Cosmochim. Acta* **37**, 1–18.
10. Richter, F. M. & Ribe, N. M. (1979) *Earth Planet. Sci. Lett.* **43**, 212–222.
11. Jacobsen, S. B. & Wasserburg, G. J. (1980) *Earth Planet. Sci. Lett.* **50**, 139–155.