Supplemental Material
Floquet higher-order topological insulators and superconductors with space-time symmetries

Yang Peng[1, 2, 3, *]

[1] *Institute of Quantum Information and Matter and Department of Physics, California Institute of Technology, Pasadena, CA 91125, USA*
[2] *Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125, USA*
[3] *Department of Physics and Astronomy, California State University, Northridge, California 91330, USA*

K GROUPS FOR UNITARY LOOPS WITH AN ORDER-TWO SPACE-TIME SYMMETRY/ANTISYMMETRY

In this supplement, the explicit form of the K groups for unitary loops with an order-two space-time symmetry/antisymmetry, in different dimensions are listed.

$\delta_{\parallel} = 0$ *family*

In this family, the additional symmetry includes nonspatial symmetry, such as spin rotations with and without a simultaneous half-period time translation. We summarize the classification table for $\delta_{\parallel} = 0 \mod 2$ in complex symmetry classes with an order-two unitary symmetry in Table I. In Table II and III, we give the classification for $\delta_{\parallel} = 0 \mod 4$ in complex symmetry classes with an order-two antiunitary symmetry, and in real symmetry classes with an order-two unitary symmetry, respectively.

$\delta_{\parallel} = 1$ *family*

This family includes Floquet topological phases protected by reflection symmetry and time-glide symmetry, where only one direction of the momenta is flipped. We summarize the classification table for $\delta_{\parallel} = 1 \mod 2$ in complex symmetry classes with an order-two unitary symmetry in Table IV. In Table V and VI, we give the classification for $\delta_{\parallel} = 0 \mod 4$ in complex symmetry classes with an order-two antiunitary symmetry, and in real symmetry classes with an order-two unitary symmetry, respectively.

$\delta_{\parallel} = 2$ *family*

The additional symmetry includes twofold spatial rotation and twofold time-screw rotation, in which the momenta along two directions are flipped. For $\delta_{\parallel} = 2$, whose classification is the same as $\delta_{\parallel} = 0 \mod 2$ in complex symmetry classes with an order-two unitary symmetry, as shown in Table I. We summarize the classification for $\delta_{\parallel} = 2 \mod 4$ in complex symmetry classes with an order-two antiunitary symmetry, and in real symmetry classes with an order-two unitary symmetry in Table VII and VIII, respectively.

$\delta_{\parallel} = 3$ *family*

The order-two symmetry in this family includes inversion with and without a simultaneous half-period time translation. For $\delta_{\parallel} = 3$, whose classification is the same as $\delta_{\parallel} = 1 \mod 2$ in complex symmetry classes with an order-two unitary symmetry, as shown in Table IV. We summarize the classification for $\delta_{\parallel} = 3 \mod 4$ in complex symmetry classes with an order-two antiunitary symmetry, and in real symmetry classes with an order-two unitary symmetry in Table IX and X, respectively.

Table I. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 0 \mod 2$. Here, $\delta = d - D$.

Symmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_0^+, \hat{\mathcal{U}}_{T/2}^+$	A	$\mathcal{C}_{-\delta} \times \mathcal{C}_{-\delta}$	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AIII	$\mathcal{C}_{1-\delta} \times \mathcal{C}_{1-\delta}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$
$\overline{\mathcal{U}}_s^+$	A	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AIII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0

Table II. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time antiunitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 0 \mod 4$. Here, $\delta = d - D$.

Symmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{A}}_s^+$	A	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{A}}_{0,+}^+, \hat{\mathcal{A}}_{T/2,-}^+$	AIII	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\overline{\mathcal{A}}_0^+, \overline{\mathcal{A}}_{T/2}^-$	A	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,-}^-, \hat{\mathcal{A}}_{T/2,+}^-$	AIII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{A}}_s^-$	A	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{A}}_{0,+}^-, \hat{\mathcal{A}}_{T/2,-}^-$	AIII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\overline{\mathcal{A}}_0^-, \overline{\mathcal{A}}_{T/2}^+$	A	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,-}^+, \hat{\mathcal{A}}_{T/2,+}^+$	AIII	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Table III. Classification table for Floquet topological phases in real symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 0 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_{0,+}^{+}, \hat{\mathcal{U}}_{T/2,+}^{+}$	AI	$\mathcal{R}_{-\delta}\times\mathcal{R}_{-\delta}$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,++}^{+}, \hat{\mathcal{U}}_{T/2,+-}^{+}$	BDI	$\mathcal{R}_{1-\delta}\times\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,+}^{+}, \hat{\mathcal{U}}_{T/2,-}^{+}$	D	$\mathcal{R}_{2-\delta}\times\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^{+}, \hat{\mathcal{U}}_{T/2,+-}^{+}$	DIII	$\mathcal{R}_{3-\delta}\times\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+}^{+}, \hat{\mathcal{U}}_{T/2,+}^{+}$	AII	$\mathcal{R}_{4-\delta}\times\mathcal{R}_{4-\delta}$	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,++}^{+}, \hat{\mathcal{U}}_{T/2,+-}^{+}$	CII	$\mathcal{R}_{5-\delta}\times\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,+}^{+}, \hat{\mathcal{U}}_{T/2,-}^{+}$	C	$\mathcal{R}_{6-\delta}\times\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^{+}, \hat{\mathcal{U}}_{T/2,+-}^{+}$	CI	$\mathcal{R}_{7-\delta}\times\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}\oplus2\mathbb{Z}$	0	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}_2\oplus\mathbb{Z}_2$	$\mathbb{Z}\oplus\mathbb{Z}$
$\overline{\mathcal{U}}_{0,-}^{+}, \overline{\mathcal{U}}_{T/2,+}^{+}$	AI	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+-}^{+}, \hat{\mathcal{U}}_{T/2,++}^{+}$	BDI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\overline{\mathcal{U}}_{s,+}^{+}$	D	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-+}^{+}, \hat{\mathcal{U}}_{T/2,--}^{+}$	DIII	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\overline{\mathcal{U}}_{0,-}^{+}, \overline{\mathcal{U}}_{T/2,+}^{+}$	AII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+-}^{+}, \hat{\mathcal{U}}_{T/2,++}^{+}$	CII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\overline{\mathcal{U}}_{s,+}^{+}$	C	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,-+}^{+}, \hat{\mathcal{U}}_{T/2,--}^{+}$	CI	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-}^{+}, \hat{\mathcal{U}}_{T/2,-}^{+}$	AI	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^{+}, \hat{\mathcal{U}}_{T/2,-+}^{+}$	BDI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^{+}, \hat{\mathcal{U}}_{T/2,+}^{+}$	D	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^{+}, \hat{\mathcal{U}}_{T/2,-+}^{+}$	DIII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^{+}, \hat{\mathcal{U}}_{T/2,-}^{+}$	AII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^{+}, \hat{\mathcal{U}}_{T/2,-+}^{+}$	CII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^{+}, \hat{\mathcal{U}}_{T/2,+}^{+}$	C	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^{+}, \hat{\mathcal{U}}_{T/2,-+}^{+}$	CI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{0,+}^{+}, \overline{\mathcal{U}}_{T/2,-}^{+}$	AI	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-+}^{+}, \hat{\mathcal{U}}_{T/2,--}^{+}$	BDI	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\overline{\mathcal{U}}_{s,-}^{+}$	D	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+-}^{+}, \hat{\mathcal{U}}_{T/2,++}^{+}$	DIII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\overline{\mathcal{U}}_{0,+}^{+}, \overline{\mathcal{U}}_{T/2,-}^{+}$	AII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,-+}^{+}, \hat{\mathcal{U}}_{T/2,--}^{+}$	CII	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\overline{\mathcal{U}}_{s,-}^{+}$	C	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+-}^{+}, \hat{\mathcal{U}}_{T/2,++}^{+}$	CI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$

Table IV. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 1 \mod 2$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_{0}^{+}, \hat{\mathcal{U}}_{T/2}^{+}$	A	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+}^{+}, \hat{\mathcal{U}}_{T/2,-}^{+}$	AIII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\overline{\mathcal{U}}_{s}^{+}$	A	$\mathcal{C}_{-\delta}\times\mathcal{C}_{-\delta}$	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-}^{+}, \hat{\mathcal{U}}_{T/2,+}^{+}$	AIII	$\mathcal{C}_{1-\delta}\times\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$	0	$\mathbb{Z}\oplus\mathbb{Z}$

Table V. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time antiunitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 1 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{A}}_s^+$	A	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,+}^+, \hat{\mathcal{A}}_{T/2,-}^+$	AIII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\overline{\mathcal{A}}_0^+, \overline{\mathcal{A}}_{T/2}^-$	A	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{A}}_{0,-}^-, \hat{\mathcal{A}}_{T/2,+}^-$	AIII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{A}}_s^-$	A	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,+}^-, \hat{\mathcal{A}}_{T/2,-}^-$	AIII	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\overline{\mathcal{A}}_0^-, \overline{\mathcal{A}}_{T/2}^+$	A	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{A}}_{0,-}^+, \hat{\mathcal{A}}_{T/2,+}^+$	AIII	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$

Table VI. Classification table for Floquet topological phases in real symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 1 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	BDI	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	D	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	DIII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CII	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	C	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{R}_{-\delta} \times \mathcal{R}_{-\delta}$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	BDI	$\mathcal{R}_{1-\delta} \times \mathcal{R}_{1-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\overline{\mathcal{U}}_{s,+}^+$	D	$\mathcal{R}_{2-\delta} \times \mathcal{R}_{2-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	DIII	$\mathcal{R}_{3-\delta} \times \mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{R}_{4-\delta} \times \mathcal{R}_{4-\delta}$	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	CII	$\mathcal{R}_{5-\delta} \times \mathcal{R}_{5-\delta}$	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0
$\overline{\mathcal{U}}_{s,+}^+$	C	$\mathcal{R}_{6-\delta} \times \mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CI	$\mathcal{R}_{7-\delta} \times \mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	BDI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	D	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	DIII	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	C	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CI	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	BDI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{s,-}^+$	D	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	DIII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{s,-}^+$	C	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{s,+-}^+ \hat{\mathcal{U}}_{T/2,++}^+$	CI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

Table VII. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time antiunitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 2 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{A}}_s^+$	A	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{A}}_{0,+}^+, \hat{\mathcal{A}}_{T/2,-}^+$	AIII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\overline{\mathcal{A}}_0^+, \overline{\mathcal{A}}_{T/2}^-$	A	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,-}^-, \hat{\mathcal{A}}_{T/2,+}^-$	AIII	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{A}}_s^-$	A	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{A}}_{0,+}^-, \hat{\mathcal{A}}_{T/2,-}^-$	AIII	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\overline{\mathcal{A}}_0^-, \overline{\mathcal{A}}_{T/2}^+$	A	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,-}^+, \hat{\mathcal{A}}_{T/2,+}^+$	AIII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$

Table VIII. Classification table for Floquet topological phases in real symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 2 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	BDI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	D	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	DIII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	C	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	BDI	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\overline{\mathcal{U}}_{s,+}^+$	D	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	DIII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	CII	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\overline{\mathcal{U}}_{s,+}^+$	C	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{R}_{-\delta} \times \mathcal{R}_{-\delta}$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	BDI	$\mathcal{R}_{1-\delta} \times \mathcal{R}_{1-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	D	$\mathcal{R}_{2-\delta} \times \mathcal{R}_{2-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	DIII	$\mathcal{R}_{3-\delta} \times \mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{R}_{4-\delta} \times \mathcal{R}_{4-\delta}$	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CII	$\mathcal{R}_{5-\delta} \times \mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	C	$\mathcal{R}_{6-\delta} \times \mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CI	$\mathcal{R}_{7-\delta} \times \mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}\oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	BDI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\overline{\mathcal{U}}_{s,-}^+$	D	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	DIII	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\overline{\mathcal{U}}_{s,-}^+$	C	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	CI	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Table IX. Classification table for Floquet topological phases in complex symmetry classes supporting an additional order-two space-time antiunitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 3 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{A}}_s^+$	A	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,+}^+, \hat{\mathcal{A}}_{T/2,-}^+$	AIII	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\overline{\mathcal{A}}_0^+, \overline{\mathcal{A}}_{T/2}^-$	A	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{A}}_{0,-}^-, \hat{\mathcal{A}}_{T/2,+}^-$	AIII	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{A}}_s^-$	A	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{A}}_{0,+}^-, \hat{\mathcal{A}}_{T/2,-}^-$	AIII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\overline{\mathcal{A}}_0^-, \overline{\mathcal{A}}_{T/2}^+$	A	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{A}}_{0,-}^+, \hat{\mathcal{A}}_{T/2,+}^+$	AIII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0

Table X. Classification table for Floquet topological phases in real symmetry classes supporting an additional order-two space-time unitary symmetry with flipped parameters $\delta_\parallel = d_\parallel - D_\parallel = 3 \mod 4$. Here, $\delta = d - D$.

Symmetry or antisymmetry	Class	Classifying space	$\delta=0$	$\delta=1$	$\delta=2$	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$	$\delta=7$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	BDI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	D	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	DIII	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,+}^+, \hat{\mathcal{U}}_{T/2,-}^+$	C	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,++}^+, \hat{\mathcal{U}}_{T/2,+-}^+$	CI	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AI	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	BDI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{s,+}^+$	D	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	DIII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{0,-}^+, \overline{\mathcal{U}}_{T/2,+}^+$	AII	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	CII	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\overline{\mathcal{U}}_{s,+}^+$	C	$\mathcal{C}_{-\delta}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CI	$\mathcal{C}_{1-\delta}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{R}_{1-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	BDI	$\mathcal{R}_{2-\delta}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	D	$\mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	DIII	$\mathcal{R}_{4-\delta}$	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{R}_{5-\delta}$	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CII	$\mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,-}^+, \hat{\mathcal{U}}_{T/2,+}^+$	C	$\mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$\hat{\mathcal{U}}_{0,--}^+, \hat{\mathcal{U}}_{T/2,-+}^+$	CI	$\mathcal{R}_{-\delta}$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AI	$\mathcal{R}_{-\delta} \times \mathcal{R}_{-\delta}$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	BDI	$\mathcal{R}_{1-\delta} \times \mathcal{R}_{1-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$\overline{\mathcal{U}}_{s,-}^+$	D	$\mathcal{R}_{2-\delta} \times \mathcal{R}_{2-\delta}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	DIII	$\mathcal{R}_{3-\delta} \times \mathcal{R}_{3-\delta}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$
$\overline{\mathcal{U}}_{0,+}^+, \overline{\mathcal{U}}_{T/2,-}^+$	AII	$\mathcal{R}_{4-\delta} \times \mathcal{R}_{4-\delta}$	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0	0
$\hat{\mathcal{U}}_{0,-+}^+, \hat{\mathcal{U}}_{T/2,--}^+$	CII	$\mathcal{R}_{5-\delta} \times \mathcal{R}_{5-\delta}$	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0	0
$\overline{\mathcal{U}}_{s,-}^+$	C	$\mathcal{R}_{6-\delta} \times \mathcal{R}_{6-\delta}$	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$	0
$\hat{\mathcal{U}}_{0,+-}^+, \hat{\mathcal{U}}_{T/2,++}^+$	CI	$\mathcal{R}_{7-\delta} \times \mathcal{R}_{7-\delta}$	0	0	0	$2\mathbb{Z} \oplus 2\mathbb{Z}$	0	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$