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THE INCOHERENT SCATTERING OF ELECTROMAGNETIC WAVES BY FREE ELECTRONS

by

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ABSTRACT

In this paper the incoherent scattering of an electromagnetic wave by free electrons is examined theoretically. Under the assumption that the electrons have a Maxwellian velocity distribution, the scattered power and its frequency spectrum are calculated. The applicability of these results to ionospheric and laboratory plasmas is discussed.

Introduction

It is known that when the frequency ω of a monochromatic electromagnetic wave is considerably higher than the characteristic frequency ω_p of a plasma through which it is traveling $(\omega \gg \omega_p)$ the conventional macroscopic theory predicts that the wave will not be affected by the plasma. Yet, from a microscopic viewpoint it is clear that this cannot be so, because the incident wave undergoes Thomson scattering at each of the randomly moving plasma electrons* and consequently is scattered. Although this type of scattering is incoherent and hence relatively weak, it is detectable and may be used as a means of measuring electronic densities and temperatures. Attention was called to this possibility by Gordon[1] for the case of radio waves in the ionosphere and by Avakiants[2] for the case of light in a laboratory plasma. The existence of incoherent backscatter from the electrons of the ionosphere was confirmed by Bowles[3] using a vertically beamed radar.

To use this phenomenon as a quantitative diagnostic tool, it is necessary to derive mathematical formulas by which the electronic density and temperature can be deduced accurately from observed values of the scattered power and its spectrum. It is possible to handle certain aspects of this problem by assuming that the scattering is caused by statistical fluctuations of electron density.** However, we shall avoid this macroscopic approach by considering the problem from the more basic viewpoint of Thomson scattering.

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*Scattering by the plasma ions is several orders of magnitude smaller and hence negligible from the start.

**See, for example, Fejer's[4] recent paper in which he discussed the scattering of radio waves by density fluctuations of an ionized gas in thermal equilibrium.

Thomson Scattering by a Free Electron with Nonzero Initial Velocity

The point of departure in the present theory of incoherent scattering is the Thomson scattering of a plane monochromatic wave by a free electron having a nonzero initial velocity. Here the conventional formula for Thomson scattering is not adequate since it applies only to the case of zero initial velocity, i.e., to the case where the mean position of the electron is at rest. Therefore, for our present purpose it is necessary to derive a more general formula that takes into account the initial velocity of the electron.

To obtain such a generalization we recall that the electromagnetic field scattered by an electron initially at rest is given (in Gaussian units) by the well-known relations

$$\underline{E} = \frac{r_o}{r^3}\,\underline{r} \times (\underline{r} \times \underline{E}_o) \quad , \qquad \underline{H} = -\frac{r_o}{r^2}\,\underline{r} \times \underline{E}_o \tag{1}$$

in which $\underline{E}_o$ denotes the incident monochromatic electric field, r_o the classical radius of the electron, and $\underline{r}$ the vector distance from the electron to the observation point. These equations are based on the tacit assumption that the equation of motion of the electron is $m\ddot{\underline{d}} = -e\underline{E}_o$ where e,m denote the electronic charge and mass, and $\underline{d}$ the displacement of the electron from its fixed mean position, or in other words, that the velocity $\dot{\underline{d}}$ is small compared to the light velocity c .

Now we shall consider the case of an electron that has a nonzero initial velocity $\underline{v}$. Since this velocity is assumed to be moderate, only terms which are linear in $\underline{\beta}(= \underline{v}/c)$ are retained in the following calculations. We choose the frame of reference K' which moves with the velocity $\underline{v}$ with respect to the laboratory frame K . Since in K' the mean position of the electron is at rest, it follows by analogy with Eqs. (1) that the scattered field in K' is given by

$$\underline{E}' = \frac{r_o}{r'^3}\,\underline{r}' \times (\underline{r} \times \underline{E}_o') \quad , \qquad \underline{H}' = -\frac{r_o}{r'^2}\,\underline{r}' \times \underline{E}_o' \tag{2}$$

where $\underline{E}_o'$ is the incident electric field in K' and is related to the incident electromagnetic field $\underline{E}_o, \underline{H}_o$ in K by the Lorentz transformation $\underline{E}_o' = \underline{E}_o + \underline{\beta} \times \underline{H}_o$.

By the Lorentz transformations

$$\underline{E} = \underline{E}' - \underline{\beta} \times \underline{H}' \quad , \qquad \underline{H} = \underline{H}' + \underline{\beta} \times \underline{E}' \tag{3}$$

the scattered field $\underline{E}, \underline{H}$ in K can be obtained from $\underline{E}', \underline{H}'$. Thus by substituting expressions (2) into Eqs. (3) we find that in the laboratory frame K the scattered field is given by*

$$\begin{aligned} \underline{E} &= \frac{r_o}{r'^3}\,\underline{r}' \times \left[\underline{r}' \times (\underline{E}_o + \underline{\beta} \times \underline{H}_o)\right] + \frac{r_o}{r'^2}\,\underline{\beta} \times (\underline{r}' \times \underline{E}_o) \\ \underline{H} &= -\frac{r_o}{r'^2}\,\underline{r}' \times (\underline{E}_o + \underline{\beta} \times \underline{H}_o) + \frac{r_o}{r'^3}\,\underline{\beta} \times \left[\underline{r}' \times (\underline{r}' \times \underline{E}_o)\right] \end{aligned} \tag{4}$$

where $\underline{r}' = \underline{r} - \underline{\beta} r$ and $r' = r - \underline{\beta} \cdot \underline{r}$.

The power scattered into an element of solid angle $d\Omega$ by definition is

$$W d\Omega = \frac{c}{4\pi}\,\underline{n} \cdot (\underline{E} \times \underline{H})\, r^2 d\Omega \tag{5}$$

where $\underline{n}$ is a unit vector in the scattering direction and $\underline{E}, \underline{H}$ are given

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*Avakiants obtained these expressions but with $\underline{r}'$ replaced by $\underline{r}$, and to this extent our Eqs. (4) disagree with his Eqs. (8).

by expressions (4). To compute this quantity the reference frame K is taken to be the Cartesian system shown in Fig. 1. The z-axis lies in the direction of propagation $\underline{k}$ of the incident wave. The angle that $\underline{E}_o$ makes with the yz-plane, i.e., the plane containing the unit vectors $\underline{k}$ and $\underline{n}$, is ϕ. The angle θ between $\underline{k}$ and $\underline{n}$ is the scattering angle. Substituting

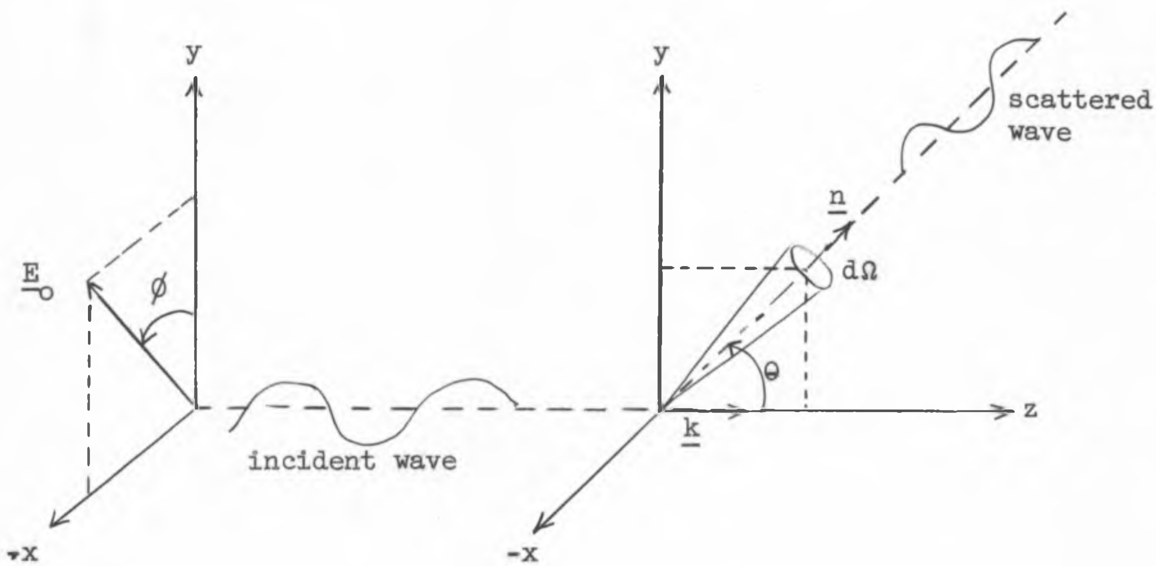

Fig. 1. Coordinate System

expressions (4) into definition (5) we obtain the following formula which gives the power scattered into a solid angle $d\Omega$ by a single free electron:

$$\begin{aligned} W d\Omega = \frac{c}{4\pi} E_o^2 r_o^2 (1 - \sin^2\theta \cos^2\phi)\, d\Omega + \frac{c}{4\pi} E_o^2 r_o^2 \Big[&\beta_x \sin 2\phi \sin\theta(\cos\theta - 1) + \\ &+ 2\beta_y \sin\theta(3 - 3\sin^2\theta\cos^2\phi - \sin^2\phi - \cos\theta\cos^2\phi) + \\ &+ 2\beta_z \left\{(3\cos\theta - 1)(1 - \sin^2\theta\cos^2\phi) - \cos\theta\right\} \Big]\, d\Omega . \qquad (6) \end{aligned}$$

If the incident wave is unpolarized the power scattered into a solid angle $d\Omega$ is obtained from Eq. (6) by averaging over the angle ϕ. Thus for an unpolarized incident wave

$$W d\Omega = \frac{c}{8\pi} E_o^2 r_o^2 (1 + \cos^2\theta)\, d\Omega \; +$$

$$+ \frac{c}{4\pi} E_o^2 r_o^2 \Big[\beta_y \sin\theta (3\cos^2\theta - \cos\theta + 2)$$

$$+ \beta_z (3\cos^3\theta - \cos^2\theta + \cos\theta - 1) \Big] d\Omega \quad . \tag{7}$$

The first terms of Eqs. (6) and (7) give the conventional expressions for the power scattered into a solid angle $d\Omega$ by an electron initially at rest; the others are correction terms which take into account the moderate initial velocity of the electron.

The differential scattering cross-section $d\sigma_e$ is simply related to $W d\Omega$ by

$$d\sigma_e = \frac{W d\Omega}{c E_o^2 / 4\pi} \tag{8}$$

where $cE_o^2/4\pi$ is the power density of the incident wave; and the total scattering cross-section is given by

$$\sigma_e = \int_0^{4\pi} d\sigma_e \, d\Omega \quad . \tag{9}$$

With the aid of expressions (6) and (7) we find from definition (9) that the total scattering cross-section is

$$\sigma_e = \frac{8}{3} \pi r_o^2 - \frac{16}{3} \pi r_o^2 \beta_z \quad . \tag{10}$$

This expression is valid for the case of a linearly polarized incident wave as well as that of an unpolarized incident wave. The first term is the conventional Thomson scattering cross-section of an electron.

Scattering by a Collection of Electrons in Random Motion

Suppose the scattering is done by a collection of randomly moving electrons. Clearly then, the phases of the scattered fields will be mutually random and hence the scattered radiation will be incoherent. This means that the resulting scattered intensity is proportional to $\sum_i (\underline{E}_i \times \underline{H}_i)$, where $\underline{E}_i, \underline{H}_i$ are the fields scattered by the i^{th} electron. If the scattering were coherent, then the scattered intensity would be proportional to $(\sum_i \underline{E}_i) \times (\sum_i \underline{H}_i)$. Thus we see that the scattered intensity in the case of incoherence is proportional to N , the number of electrons per unit volume whereas in the case of coherence it is proportional to N^2 . From this it follows that incoherent scatter is weaker than coherent scatter by a factor N .

Since the differential scattering cross-section $d\sigma_e$ of a single electron is velocity-dependent, it is necessary at this point to make an assumption about the velocity distribution of the collection of electrons whose differential scattering cross-section $d\sigma_N$ we wish to find. We shall assume that their velocity distribution is Maxwellian. Since the scatter is incoherent and the velocity distribution is Maxwellian, it thus follows that the differential scattering cross-section of the collection of electrons is given by

$$d\sigma_N = Nc^3\left(\frac{m}{2\pi kT}\right)^{3/2} \iiint_{-\infty}^{\infty} d\sigma_e \exp\left\{-\frac{mc^2}{2kT}(\beta_x^2 + \beta_y^2 + \beta_z^2)\right\} d\beta_x d\beta_y d\beta_z \qquad (11)$$

where T is the kinetic temperature of the electrons and k is Boltzmann's constant. Dividing $Wd\Omega$ as given in Eqs. (6) and (7) by $cE_o^2/4\pi$, we obtain in accord with definition (8) the differential scattering cross-section $d\sigma_e$ for the two cases of polarization. Substituting the expressions of $d\sigma_e$ thus obtained into Eq. (11) we find that the differential scattering cross-section for a collection of randomly moving electrons is

$$d\sigma_N = N\, d\sigma_T \tag{12}$$

where $d\sigma_T$ is the Thomson scattering cross-section which is given by

$$d\sigma_T = r_o^2(1 - \sin^2\theta \cos^2\phi) \tag{13}$$

when the incident wave is linearly polarized, and by

$$d\sigma_T = \frac{1}{2} r_o^2(1 + \cos^2\theta) \tag{14}$$

when the incident wave is unpolarized. In the backward direction ($\theta = \pi$) $d\sigma_T$ has the value r_o^2 regardless of whether the incident wave is linearly polarized or unpolarized, and Eq. (12) reduces to $d\sigma_N = Nr_o^2$ where r_o is the classical radius of the electron.

An approximation is involved in the evaluation of the integral (11) when the integrand $d\sigma_e$ is given explicitly by expressions (6) and (7), arising from the fact that the limits of integration are infinite whereas expressions (6) and (7) are valid only for a limited range of velocities. Nevertheless, assuming that expressions (6) and (7) are valid for all values of velocity we expect the result of the integration to be accurate on the ground that the contribution to the integral of the high-velocity part of the range of integration is small. Clearly the quantity $mc^2/2kT$ in the exponent is enormously large for all normal temperatures and consequently the Maxwellian velocity distribution has a sharp taper.

Bandwidth of the Spectrum

According to the Doppler effect, when an incident wave of frequency ω_o falls on a moving electron, the frequency ω of the scattered field is given rigorously by

$$\omega = \omega_o \frac{1 - \underline{\beta} \cdot \underline{k}}{1 - \underline{\beta} \cdot \underline{n}} \tag{15}$$

or for moderate velocities by

$$\omega = \omega_o (1 - \underline{\beta} \cdot \underline{k})(1 + \underline{\beta} \cdot \underline{n}) \quad . \tag{16}$$

Consequently the frequency shift $\Delta\omega$, in the case of moderate velocities, is approximately

$$\Delta\omega = \omega - \omega_o = \omega_o (\underline{\beta} \cdot \underline{n} - \underline{\beta} \cdot \underline{k}) \quad . \tag{17}$$

Since $\underline{n}$ and $\underline{k}$ lie in the yz-plane, the x-component of $\underline{\beta}$ does not come into play. Therefore, we may assume without loss of generality that $\underline{\beta}$ lies in the yz-plane, as shown in Fig. 2. We let ψ be the angle between $\underline{\beta}$ and $\underline{n}$ and as before Θ is the angle between $\underline{n}$ and $\underline{k}$.

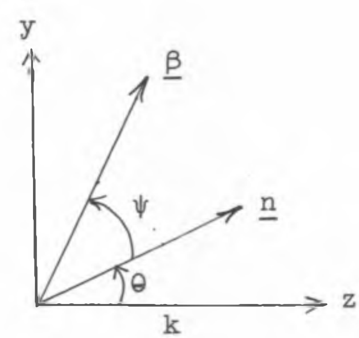

Fig. 2. Bandwidth is maximum when $\psi = (\pi - \Theta)/2$

Then Eq. (17) becomes

$$\Delta\omega = \omega_o \beta \left\{ \cos\psi - \cos(\psi + \Theta) \right\} \tag{18}$$

where β is the magnitude of $\underline{\beta}$. For Θ, β, ω_o fixed, $\Delta\omega$ is a maximum when $\psi = (\pi - \Theta)/2$. Hence the maximum value of $\Delta\omega$ is

$$(\Delta\omega)_{max} = 2\omega_o \beta \sin\frac{\theta}{2} \quad . \tag{19}$$

We define the bandwidth of the spectrum to be twice the average of $(\Delta\omega)_{max}$, that is,

$$\text{bandwidth} = 2\left\langle(\Delta\omega)_{max}\right\rangle = 4\omega_o \sin\frac{\theta}{2} \langle\beta\rangle \quad . \tag{20}$$

Since

$$\langle\beta\rangle = \sqrt{\frac{8kT}{\pi mc^2}} \quad ,$$

it follows from Eq. (20) that

$$\text{bandwidth} = 4\omega_o \sin\frac{\theta}{2} \sqrt{\frac{8kT}{\pi mc^2}} \quad . \tag{21}$$

Thus we see that the bandwidth is proportional to $T^{1/2}$ and varies with the scattering angle θ as $\sin(\theta/2)$. When $\theta = 0$ the bandwidth is zero; it monotonically increases as θ goes from 0 to π, and has its maximum in the backward direction $\theta = \pi$.

Spectrum of the Scattered Radiation

To compute the spectrum of the scattered radiation we recall from Eqs. (8) and (11) that the power scattered into a solid angle $d\Omega$ by a collection of electrons is

$$(Wd\Omega)_N = Nc^3\left(\frac{m}{2\pi kT}\right)^{3/2} \iiint_{-\infty}^{\infty} Wd\Omega \, e^{-\frac{mc^2}{2kT}(\beta_x^2+\beta_y^2+\beta_z^2)} d\beta_x d\beta_y d\beta_z \quad . \tag{22}$$

In the integrand $Wd\Omega$ is the power scattered into a solid angle $d\Omega$ by a single electron. Actually $\beta_x, \beta_y, \beta_z$ cannot exceed unity and hence the infinite limits which are required by the statistics of the problem are incompatible with the actual limits dictated by the physics of the problem. However, the factor $mc^2/2kT$ is so large that the exponential function drops off very sharply from its maximum value, which occurs at $\beta_x = \beta_y = \beta_z = 0$, and consequently the value of the integral is rather insensitive to the replacement of the infinite limits by arbitrary finite ones as long as they include a reasonable interval about the origin in β-space. Hence we can write Eq. (22) as

$$(Wd\Omega)_N = Nc^3\left(\frac{m}{2\pi kT}\right)^{3/2} \int\int\int_{-b}^{b} Wd\Omega\, e^{-\frac{mc^2}{2kT}(\beta_x^2+\beta_y^2+\beta_z^2)}\, d\beta_x d\beta_y d\beta_z \quad , \qquad (23)$$

where the limits are $-b$ to b, b being somewhat less than unity. Since

$$\beta_z = \frac{\omega_o - \omega}{\omega_o(1 - \cos\Theta)} + \frac{\beta_y \sin\Theta}{1 - \cos\Theta} \quad , \qquad (24)$$

which follows from Eq. (16), we may transform the β_z-integration to an ω-integration. Noting that the Jacobian of the transformation is

$$\frac{\partial\beta_z}{\partial\omega} = -\frac{1}{\omega_o(1 - \cos\Theta)} \qquad (25)$$

we see that expression (23) becomes

$$(Wd\Omega)_N = \int d\omega \left\{ Nc^3\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{d\Omega}{\omega_o(1-\cos\Theta)} \int_{-b}^{b}\int_{-b}^{b} W\, e^{-\frac{mc^2}{2kT}(\beta_x^2+\beta_y^2+\beta_z^2)}\, d\beta_x d\beta_y \right\}$$

(26)

where β_z in the exponential and in W of the integrand is given by Eq. (24). The quantity in curly brackets is the frequency spectrum of the scattered radiation. That is, the spectrum of the power scattered into a solid angle $d\Omega$ in the frequency band $d\omega$ is given by

$$(W\, d\Omega\, d\omega)_N = \frac{d\Omega\, d\omega}{\omega_o(1-\cos\Theta)} I \tag{27}$$

where

$$I = Nc^3 \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-b}^{b}\int_{-b}^{b} W\, e^{-\frac{mc^2}{2kT}(\beta_x^2+\beta_y^2+\beta_z^2)}\, d\beta_x\, d\beta_y \quad . \tag{28}$$

Substituting expressions (6) and (7) into expression (28), we find after changing the limits from $-b,b$ to $-\infty,\infty$ that

$$(W\, d\Omega\, d\omega)_N = \frac{Nc^2\, r_o^2\, E_o^2}{8\pi\, \omega_o} \sqrt{\frac{m}{\pi kT}}\; \frac{1-\sin^2\Theta\cos^2\phi}{1-\cos\Theta} \left[1 - \frac{3(\omega_o-\omega)}{\omega_o}\right] \times$$

$$\times \exp\left(-\frac{mc^2(\omega_o-\omega)^2}{4\omega_o^2\, kT(1-\cos\Theta)}\right) d\Omega\, d\omega \tag{29}$$

for the case of linear polarization, and

$$(W\, d\Omega\, d\omega)_N = \frac{Nc^2\, r_o^2\, E_o^2}{16\pi\, \omega_o} \sqrt{\frac{m}{\pi kT}}\; \frac{1+\cos^2\Theta}{\sqrt{1-\cos\Theta}} \left[1 - \frac{3(\omega_o-\omega)}{\omega_o}\right]$$

$$\exp\left(-\frac{mc^2\,(\omega_o-\omega)^2}{4\omega_o^2\, kT(1-\cos\Theta)}\right) d\Omega\, d\omega \quad , \tag{30}$$

for the case of an unpolarized incident wave.

Limits of Applicability of the Theory

According to the conventional theory of electromagnetic wave propagation in a plasma the index of refraction is given by

$$\eta = \sqrt{1 - (\omega_p/\omega)^2} \quad . \tag{31}$$

From this we see that when the frequency ω of the incident wave is high compared to the plasma frequency ω_p, the incident wave travels undisturbed with the velocity of light. Therefore, from the viewpoint of the conventional theory, the condition that must be satisfied to permit the free passage of an electromagnetic wave is

$$\omega \gg \omega_p \quad . \tag{32}$$

However, this condition is not sufficient to guarantee that the electrons of the plasma be effectively free of collective effects. It appears that the stronger condition

$$\omega > \omega_D = \frac{2\pi c}{\lambda_D} \tag{33}$$

where λ_D is the Debye length and ω_D the Debye frequency, has to be satisfied for this to be true. Since

$$\omega_p = (4\pi Ne^2/m)^{1/2} \tag{34}$$

and

$$\lambda_D = (kT/4\pi Ne^2)^{1/2}$$

it follows that

$$\frac{\omega_D}{\omega_p} = \frac{2\pi c}{\lambda_D \omega_p} = \frac{2\pi c}{\sqrt{kT/m}} \gg 1 \quad . \tag{35}$$

The inequality results from the fact that since $\sqrt{kT/m}$ is the most probable speed of a collection of electrons having a Maxwellian distribution of velocities it is less than c. By virtue of this inequality, if condition (33) is satisfied, then condition (32) is satisfied as a consequence.

In ionospheric applications condition (33) is satisfied at heights of 1000 km or higher for a radar frequency of about 500 Mc/sec, and hence the phenomenon of incoherent scatter prevails there.

With the advent of the laser, which is now a working source of coherent light, it is possible to examine the incoherent scatter from the electrons of various types of laboratory plasmas. In such applications the above theory is valid when a photon suffers only a single collision with an electron and then leaves the plasma region. This means that the mean free path of the electron has to exceed the linear dimensions of the plasma region.

Summary

In this paper the problem of the incoherent scattering of electromagnetic waves by free electrons in thermal equilibrium is examined theoretically. The starting point is the scattering cross-section of a single free electron initially in motion. The formula for the cross-section is derived by calculating the scattered field $\underline{E}',\underline{H}'$ in the frame of reference K' in which the electron is at rest, and then obtaining from $\underline{E}',\underline{H}'$, by a Lorentz transformation, the scattered field $\underline{E},\underline{H}$ in the laboratory frame of reference. From a knowledge of $\underline{E},\underline{H}$ the differential scattering cross-section $d\sigma_e$ of a single moving electron is found; it has the form $d\sigma_e = d\sigma_T + d\sigma_v$ where $d\sigma_T$ is the conventional Thomson cross-section and $d\sigma_v$ is a correction term arising from the initial velocity of the electron. The scattering cross-section $d\sigma_N$ of N electrons per unit volume is calculated under the assumption that the electrons are in thermal equilibrium and hence have a Maxwellian distribution of velocities. That is, $d\sigma_N$ is found by multiplying $d\sigma_e$ by the

Maxwellian distribution function and integrating the product over all velocities. Thus the result $d\sigma_N = N d\sigma_T$ is obtained. This result constitutes the theoretical basis of recent experiments on the incoherent scattering of radio waves by the ionosphere. Since the frequency of the field scattered by a single moving electron is shifted from the frequency of the incident wave in accord with the Doppler effect, clearly the power scattered by a collection of electrons in thermal motion spreads over a band of frequencies. This power spectrum is computed. The bandwidth of the spectrum turns out to be proportional to $T^{1/2}$ and can be used as a means of deducing the electronic temperature T from spectrum measurements. This type of incoherent scattering from free electrons plays a dominant role in ionospheric (or plasma) scattering when $\omega \gg \omega_p$ and $\lambda \ll \lambda_D$, where ω and λ are the angular frequency and wavelength of the incident wave, ω_p is the plasma frequency, and λ_D is the Debye length.

References

1. W. E. Gordon, Proc. Inst. Radio Engrs. 46, 1824 (1958).
2. G. M. Avakiants, Izvestiia AN UzbSSR, Fiz. Mate. Nauk, No.4, 1960, pp. 89-92.
3. K. L. Bowles, Jour. Res. of the National Bureau of Stds., D. Radio Propagation, Vol. 65D, No. 1, 1961, pp. 1-14.
4. J. A. Fejer, Canadian J. Phys. 38, 1114 (1960).

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