Reply to "Comment on 'Performance of different synchronization measures in real data: A case study on electroencephalographic signals' "

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(Received 28 September 2005; published 13 December 2005)

We agree with the Comment by Nicolaou and Nasuto about the utility of mutual information (MI) when properly estimated and we also concur with their view that the estimation based on k nearest neighbors gives optimal results. However, we claim that embedding parameters can indeed change MI results, as we show for the electroencephalogram data sets of our original study and for coupled chaotic systems. Furthermore, we show that proper embedding can actually improve the estimation of MI with the k nearest neighbors algorithm.

DOI: 10.1103/PhysRevE.72.063902 PACS number(s): 87.19.Nn, 05.45.Tp, 87.90.+y, 05.45.Xt

In a previous study we compared the performance of several synchronization measures using three short segments of electroencephalographic (EEG) data recorded in two channels [1]. All measures but mutual information (MI) gave qualitatively similar results in the sense that they ranked synchronization values in the same way, namely, example B > example A > example C. Interestingly, nonlinear interdependency measures, which rely on a phase space reconstruction of the signals and then quantify the similarity between the resulting attractors, showed a larger sensitivity in comparison with the linear measures. For MI the ranking of the three examples was less robust and depended heavily on the parameters used for its implementation. This was due to the short duration of the data sets (5 s), a frequent limitation in EEG data due to nonstationarity [2], and due to the method used to estimate the MI. Estimation of MI is far from straightforward. In our original study [1], it was based on the first order correlation integral, using fixed size neighborhoods around each point. This offers a better sampling of the distribution than the use of naive box counting with fixed partitions [3]. Subsequently, Duckrow and Albano [4] showed in an interesting Comment that more robust results are obtained for this data set [5] by means of the adaptive Fraser-Swinney algorithm [6], combined with a novel embedding technique. In our Reply [7] we pointed out that the robustness was indeed an effect of this special embedding, and we proposed that k nearest neighbors (k-NN) instead of fixed neighborhoods would provide an adaptive and optimal estimation of MI. For a recent systematic study of k-NN estimators and for their application to independent component analysis, see Refs. [8,9].

We welcome the reanalysis of the data done by the authors of the preceding Comment [10] and we agree with most of their claims. However, we disagree with their claim that MI is basically independent of the details of the embedding. Theoretically, it is clear that the true MI must increase with embedding dimension m. For chaotic or stochastic signals it should scale linearly with m, in the limit $m \to \infty$. For most algorithms, however, this increase is compensated by the increasing lack of details which can be resolved when m increases.

In the following we reanalyzed the EEG data of Ref. [1] by means of the k-NN algorithm with $k=10$, using a conventional delay embedding. We varied the time lag τ from 1 to 20, and m from 1 to 10. Figure 1 shows the MI for three values of τ and increasing m. The estimates indeed increase with m, but for large m and τ (and in particular for data set C) the distance to the tenth neighbor is already so large that

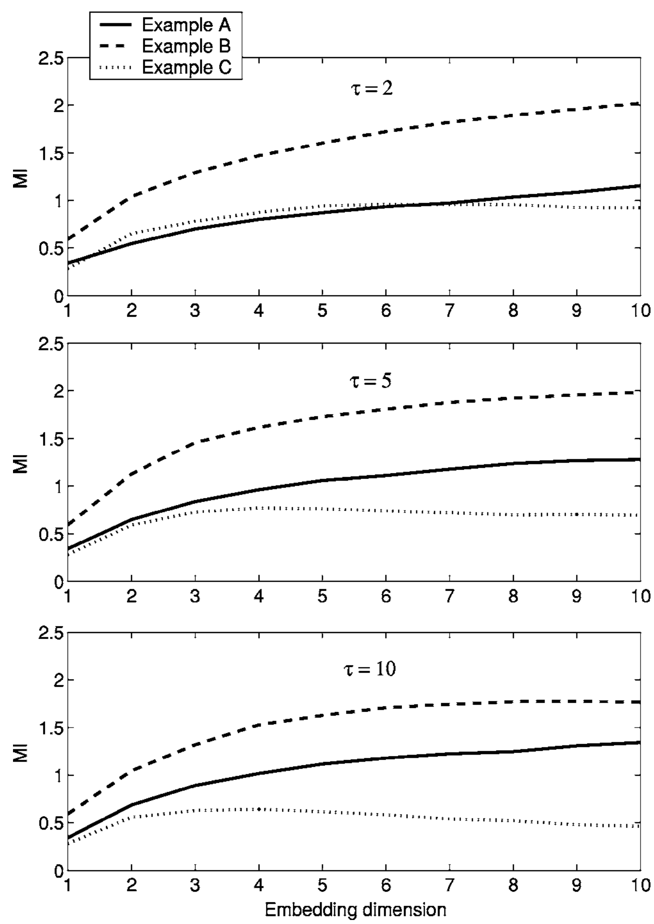

FIG. 1. MI values for the three examples using the k-NN estimation and different embedding parameters.

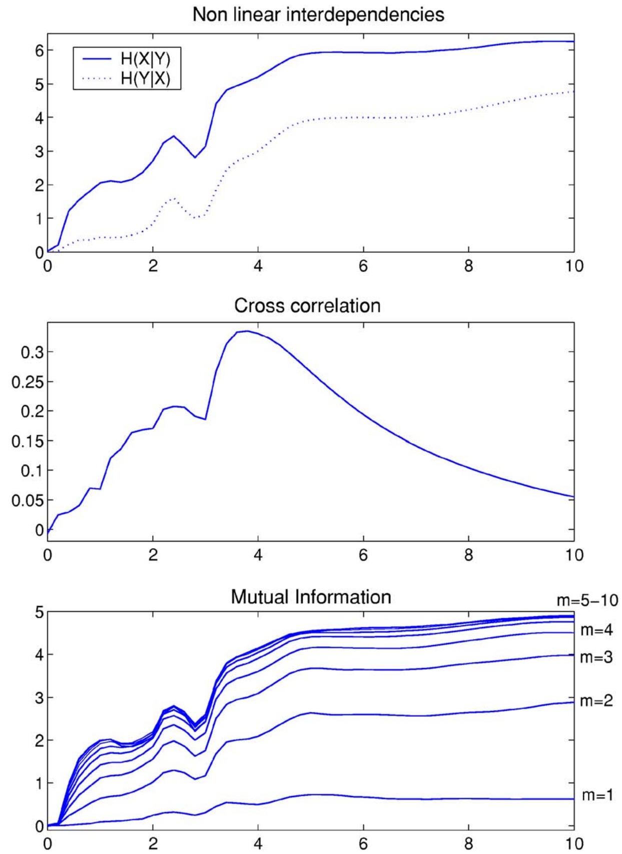

FIG. 2. (Color online) Different synchronization measures for a coupled Rössler-Lorenz system as a function of the coupling strength. Top plot, nonlinear interdependence H; middle plot, cross-correlation; bottom plot, MI with $k=10$ nearest neighbors and different embedding dimensions.

most details are missed and the increase is stopped or even inverted. In spite of this, the ranking implied by the estimates shown in Fig. 1 is very robust. Except for very small delays ($\tau=2$) *and* embedding dimensions $m<7$, we always see the ranking B > A > C obtained also from other interdependence measures. Results are less clear-cut for smaller k due to larger statistical fluctuations, but even for $k=1$ more than half of all pairs (τ, m) give this ranking.

In order to show that embedding with sufficiently large m makes MI estimation more robust, we also studied two coupled chaotic systems already described in Ref. [11]. These are a Rössler system driving via unidirectional and nonlinear coupling a Lorenz system. Implementation details were the same as in Ref. [11]. The parameter determining the relative frequency between the two systems is set to $\alpha=10$. Figure 2 shows results for increasing coupling C of the nonlinear interdependence measures $H(X|Y)$ and $H(Y|X)$ (upper plot); the cross-correlation function (middle plot); and the MI (lowest plot). Implementation details for H were the same as the ones used in Fig. 2(d) of Ref. [11]. We used $k=10$ nearest neighbors, $\tau=1$, and varied the embedding dimension between $m=1$ and 10. We observe that MI with $m>1$ reproduces the results obtained with H, in spite of their completely different definitions. Results for $k=1$ (not shown) are qualitatively similar but with larger MI values. Without embedding ($m=1$), MI would be estimated as nearly independent of the coupling strength. Note that for a linear measure, such as the cross correlation, the performance degrades for strong couplings. This reinforces the view that MI may be preferable to linear measures since it is sensitive to nonlinear interactions.

In conclusion, we showed that *true* MIs increase with the embedding dimension m. But for all practical algorithms, the increase of *estimated* MI stops for large m. The latter is not related to the finiteness of the attractor dimension, but is due to a limited power of any practical algorithm to reslove details. Although this means that MI between high dimensional delay vectors is underestimated, these estimates proved more useful for ranking data according to their synchronization level than estimates obtained without any embedding.

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