**Supplementary Material**

**Crack to pulse transition and magnitude statistics during earthquake cycles on a self-similar rough fault**

Elías R. Heimisson1

1 Seismological Laboratory, California Institute of Technology, Pasadena, California, USA

In the supplementary material I provide details on convergence tests and benchmarking that was done to validate the algorithm described in the main text. In addition, Figure S4, which is referenced in the main text is shown. All references in the Supplementary Materials are found in the main text.

**Convergence tests:**

Here I show convergence tests carried out for the fault slip, once 6 seismic events have occurred. Slip velocity during the first and largest slip event, and for the timing when the slip speed first exceeds 1 m/s. The convergence is shown with decreasing epsilon (which scales the adaptive time-step), while maintaining the same spatial discretization as in the paper. Since poorly resolved cohesive zones typically generate random fluctuation at the propagating tip of a rupture, the last test of checking the convergence of the time at which the rupture reaches 1 m/s should reveal if the cohesive zone is well resolved. The convergence test is carried out only using the center 200 m of the fault shown in the main text (Figures 1-4). This allows for generating a highly refined manufactured solution with epsilon = 1/1024 (compared to 1/64 used in the paper). Less refined solutions are compared to the manufactured solution using a L1 norm. Note all L1 norms represent absolute error, not relative error, which in all cases would be much smaller.

|  |
| --- |
|  |
| **Figure S1: a** Slip profiles at fixed time as epsilon is decreased, darkness of the grey indicates smaller epsilon showing convergence. **b** Convergence test for the last slip profile in **a**, as a function of relative change and absolute epsilon. Vertical axis is the L1 norm between a manufactured solution slip profile at every point and the less refined solutions. Plot reveals a slightly better than 1 order convergence. |

|  |
| --- |
|  |
| **Figure S2: a** Slip speed profiles in the simulation shown in Figure **S1a,** at the time when the solution first exceeds a slip speed of 1 m/s. Darkness of the lines indicates refining of epsilon and reveals convergence of the solution. **b** Similar to Figure **S1b**, but showing convergence of the slip speed profiles in **a**, again an approximately order 1 convergence is attained. **c** Convergence for the absolute time that slip speed exceeds 1 m/s in **S2a**. The event happens at approximate 10 million seconds and thus the relative error is numerically very small. Clear convergence is seen for all values of epsilon, indicating that the cohesive zone is well resolved and not causing random fluctuations at high slip speeds. |

**Conclusions of convergence tests:**

For all the investigated metrics for convergence we see that we attained a clear convergence at the epsilon value and spatial refinement shown in the manuscript. This strongly suggests that the numerical method is robust.

**Benchmarking:**

Here I describe a benchmarking test, the results of which are now briefly described in the main text.

Since there is no established benchmark for rough faults, I tested the algorithm on anti-plane cycle simulations benchmark problem described in Erickson et al., (2020). It is worth noting that the benchmark uses regularized rate-and-state friction, whereas the here I use non-regularized rate-and-state friction. Thus very small differences are expected in frictional strength at very small slip speeds.

Comparison of the BP2 problem (Erickson et al. 2020) simulation using the approach in the manuscript and using BICyclE (e.g. Lapusta and Liu 2009) shows clear agreement (Figure 9) in spite of the fact that cohesive zone is only resolved by 1.5 element in my implementation, whereas the BICyclE simulation resolves the cohesive zone by 6 element. For reference the BICyclE simulations reported by Erickson et al. (2019) in Figure 3c which resolves the cohesive zone by 1.5 cells (Figure 10) reveals significant differences, this indicates the Spectral boundary integral approach requires a better resolved cohesive zone than the boundary integral approach implemented in this paper.

|  |
| --- |
|  |
| **Figure S3:** Comparison of the BP2 benchmark from Erickson et al., 2020. (Top) simulation using the algorithm described in the manuscript by resolving the cohesive zone by 1.5 elements (dz = 100 m, epsilon = 1/64), bottom: compared to a simulation reported by Erickson et al., (2020), which used BICyclE and resolves the cohesive zone by 6 elements. Black vertical lines are to guide the eye, horizontal lines are all of the same length and represent co-seismic slip in the larger event in the two event cycle. Simulations agree in great detail, both in terms of the stable two cycle the evolves, but also in how the initial activity is slightly different. With time the timing of the large event becomes slightly off-set. This might be due to cumulative error that arises from one simulation using regularized friction law, whereas the other uses non-regularized friction law. However, it was noted by Erickson et al., (2020) that such divergence was common with time for different codes, even when the friction law is identical. |

**Conclusion of benchmarking:**

Figure S3 (two event cycle simulations benchmark (BP2) by Erickson et al., (2020)) strongly suggests that the resolving the cohesive zone by 3 elements in the rough fault simulations is more than enough since 1.5 elements appears sufficient in an anti-plane simulation, this conclusion is also supported by previously reported convergence tests. The comparison to the benchmark reveals further that the time-stepping scheme is robust and can resolve complex and stable cycles over the duration of multiple cycles without aggregating error that disrupts the cycle in some way.

|  |
| --- |
|  |
| **Figure S4:** Stem plot showing all events for all simulations at once, showing both all events and the total time-span of all the simulations. |