
Supplementary information

Quantum correlations between light and the kilogram-mass mirrors of LIGO

In the format provided by the authors and unedited

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Peer Review File

Manuscript Title: Quantum correlations between the light and kilogram-mass mirrors of LIGO

Reviewer Reports on the Initial Version:

Referee #1 (Remarks to the Author):

This is a very interesting study of quantum noise and correlations and their impact on the sensitivity of a laser-interferometric gravitational wave detector (GWD). By injecting a squeezed state of vacuum into one of the LIGO interferometers, the authors implement one of the methods known to allow beating the standard quantum limit (SQL). Comparison of the interferometer noise with and without the squeezed-vacuum input confirms the presence of quantum correlations. The authors show that these correlations would indeed allow beating the SQL in the absence of classical noise, which is subtracted in data post-processing in the present work.

Beating the SQL in displacement measurements through quantum correlations is not new in itself. A recent work (ref [18]) has demonstrated exactly that, even without noise subtraction. For the work at hand, the authors argue, the appeal lies in the fact that the correlations occur in a system where the movable objects are at a human scale (a set of 40-kg mirrors), and in an instrument whose purpose goes far beyond exploring the principles of quantum measurement. While I have some difficulty to grasp the implications of the large mass involved, I do find the latter aspect important. Quantum backaction noise may not yet be dominant in the present data set, but it is only a question of time for GWDs to reach this limit. Thereby, the obtained results clearly demonstrate that quantum engineering techniques can enhance the performance of real-world interferometric sensors. I do believe this is a message of great interest to those concerned more directly with GWDs, but also the wider quantum sensing and metrology community.

Given such broad interest and appeal, I would support publication of the work in Nature.

The experimental work, modeling and data analysis are of very high quality. However, the clarity of the paper should be improved before publication, especially with the broad readership of Nature in mind.

For example, please clarify what the role of the homodyne receiver is. Reference [20], which may have some details on this, is listed as in preparation. Also, could it be shown in figure 1 where/how the squeezing angle ϕ is tuned in practice?

The proper subtraction of classical noise is thoroughly discussed throughout the main manuscript, and in even greater detail in the methods section. It is evidently a crucial part of this experimental work. I therefore think it deserves an explicit mention in the abstract as well. In a similar vein, showing the total measured noise with squeezing engaged (dark green curve of figure 4) in figure 2 would better represent how the results are actually obtained. Furthermore, I think the statement that in this work the SQL is surpassed (l. 136) is misleading. What is shown is that the quantum noise contribution is lower than the

SQL. The total noise in the measurement is still above the SQL. Again, such ambiguity could be easily avoided by showing the measured noise spectrum in figure 2.

The manuscript discusses three models, the simplified one of eqs 1-5, the extended one of eqs. 6-10 and an exact one (l. 444). Please state which one is used to calculate the model M_r of eq. 11, i.e. to derive the main results, the post-subtraction inferred quantum noise. If it is the exact model, how does it differ from eqs. 6-10? Is this the same model than that used for the quantum noise model with unsqueezed vacuum state (blue curve in figures 2-4)? I assume these models take all sources of loss into account?

The error bars in figure 4, according to the figure caption, include the statistical uncertainties of the classical noise from the reference measurement, as well as in the squeezed measurement. Elsewhere (l. 781) it is mentioned that the figure includes the uncertainties of eqn. 12. The latter, in my understanding, also contains systematic noise terms, such as modeling uncertainty. Are those included in the error bars of figure 4? How about the stationarity uncertainties? I would also suggest to state a confidence interval, taking all uncertainties into account, in the abstract.

The general statement of l. 316 (measurements presented here represent long-awaited milestones in verifying the role of quantum mechanics in limiting the measurements of small displacements generally) appears somewhat ignorant of the activities in the field of opto- and electromechanics. The connections and references to the literature in this field are also a bit erratic. For example, the list [15,16] of room-temperature experiments should include the 2017 work from NIST (Quantum correlations from a room-temperature optomechanical cavity, Science). Ref [17] only demonstrates imprecision below the SQL, a situation many experiments have achieved. Quantum correlations play no role for this, which (by the authors' adopted definition) implies that this cannot be a sub-SQL measurement. Ref [18], in contrast, uses a closely related method based on ponderomotive quantum correlations. Indeed the two methods are discussed on the same footing in the prominently cited ref [4]. It also overcomes the SQL, so it is very relevant to this work.

Add a short explanation of what the mechanical degree of freedom is that sets the SQL.

Organizing the very long methods section into subsections with individual headings would make the information there much more accessible.

Typos etc:

l. 594, should this be $\eta_o K^2$?

Caption of figure 2, "Drawn for the cases where the input state is unsqueezed vacuum (dotted blue) and ..." may be more clear

Caption of figure 5 describes a "right" panel which is not visible in the reviewer version of the manuscript

Referee #2 (Remarks to the Author):

This paper shows that the quantum noise contribution in the LIGO detector in Livingston surpasses the standard quantum limit (SQL) after a subtraction of classical noise. Although they have not realized the sub-SQL measurement with the 40 kg mirrors, the sensitivity is quite close to it and the experimental demonstration is impressive.

In fact, it is not clearly written in the abstract that classical noise has been subtracted from the total noise, and some readers may have an impression that LIGO sensitivity has reached and surpassed the SQL. The subtraction is the main part of the paper but it is not introduced until line 154, and the detailed explanation has been moved to a supplementary article (Methods). There are some technical details described in the main body, which should be minimized to make some room to explain the noise subtraction method.

The paper needs to be fully revised to meet the standard quality level of Nature articles. Some technical terms come in without notice: for example, QPRN, output mode cleaner, homodyne readout, optical spring, etc. The paper is like an internal report for people who are already familiar with everything.

I found different forms to refer equations: Eqn.4, Eqn.(10), eq.12, etc., and the same for figures: Fig.2, Fig 2, Figure 3, figure 1, etc., sometimes with and sometimes without a half space before the number. I do not think it is a right way to write the abbreviation at the beginning of a sentence: Eqn.4 -> Equation 4, or to write a mathematical symbol at the beginning of a sentence: L is the... etc. The authors should spend more time checking the manuscript before sending it to a journal.

There are also some typos and grammatical errors:

- ponderomotive squeezing became pondermotive squeezing after some points
- discussion of Figure 3 below -> discussions in the caption of FIG.3 (L261)
- due -> due to (L291)
- signal cavity -> signal recycling cavity (L292)
- quantum limited -> quantum noise limited (L490)
- The sentence starting in L538 is strange.
- The sentence starting in L548 is strange.

I have some more comments.

(1) A mathematical expression of ψ is missing. It is also confusing that the authors sometime call this variable the detuning of the signal recycling cavity.

(2) Some description about the homodyne detector is needed. Can one choose the

homodyne angle? If it is fixed, is it on the phase quadrature? A possible detuning from the phase quadrature measurement seems to be missing in the noise estimate.

(3) The discussion in Method is lengthy. Consider making it shorter. Adding a diagram for the explanation would be helpful.

Referee #3 (Remarks to the Author):

In this work, quantum correlations between the phase of the high power laser beam circulating in the gravitational wave (GW) detector LIGO and the 40 kg mirrors of this device, has been experimentally demonstrated.

Vacuum fluctuations entering through the interferometer dark port give rise to ponderomotive squeezing. This effect, together with the injection of the squeezed vacuum injected into the interferometer, allows to surpass the Standard Quantum Limit (SQL), that is a limit to the precise continuous measurement of position of an object.

This result proves the existence of quantum correlations involving the position of a human-scale object. Moreover, this work presents the first direct observation of the Quantum Radiation Pressure Noise (QRPN) on a macroscopic object that arises from the quantum back action due to the Heisenberg uncertainty principle.

In this work the QRPN is measured for the first time for macroscopic objects at room temperature. This effect have been already observed for microscopic objects, mostly at cryogenic temperatures, but also at room temperature. Proper references have been provided.

My suggestion is to underline the fact that, all the cited works contain the experimental demonstration of QRPN for microscopic objects and this particular work presented in this paper describes the first ever measurement of QRPN performed on macroscopic objects, at low frequency and at room temperature. Finally, it has been presented that the QRPN observation has been done also in Advanced Virgo GW detector and a reference to the related work (in preparation) is cited.

The other milestone that has been reported is the overtaking of the SQL using squeezing and ponderomotive effect. This is an original result achieved and, as the authors specify, this represents the first realization of a quantum nondemolition technique in GW detectors.

The reported results will be useful for the improvement of future generation of GW detectors. In the abstract, it has been underlined that, the presented work can also improve all type of measurements in future. In the text it has been specified that the performed measurement verify the role of the quantum mechanics in limiting the measurement of small displacements. This sentence could be better supported with the particular kind of effect that can benefit scientific measurements using quantum mechanics.

The approach used and the quality of data are very good. The results are presented in a good way mostly in the supplementary part, where the methods are very well described and particular attention has been drawn towards the statistics and the treatment of the uncertainties.

In the abstract, the context and the importance of the measurements performed are very well presented. The paper is not yet divided into sections. But I can clearly distinguish introduction, theoretical background, experimental demonstration, discussion and the conclusions.

I particularly appreciate the introduction where the concept of Standard Quantum Limit is very well explained.

In the conclusion, the importance of the performed measurement has been mentioned and the two milestones achieved have been clearly underlined. In the last part, the importance of the measurement in the GW detection field has been stressed.

The paper presents robust, valid and reliable results. This is more evident in the supplementary material. In conclusion, I consider this work to be very good and useful for improving quantum noise limited scientific measurements.

Suggested improvements:

I think that the authors should better clarify what other kind of measurements this experimental demonstration can be useful for.

Formulas (1) and (2): following the given reference, I was not able to derive these formulas. Maybe you can guide better the reader towards this derivation. Moreover, in order to make dimensionless the parameters in equation (2), γ should be in Hz instead of rad/s.

Line 445 space missing between the number and the units.

Line 638 the same as for line 445

Line 689 contribution --> contribution

Line 795 yellow, blue and purple --> pink, blue and orange (as it is written in the caption of Fig.5)

No need for experiment or data revision.

Author Rebuttals to Initial Comments:

Referee #1

Remarks to the Author:

This is a very interesting study of quantum noise and correlations and their impact on the sensitivity of a laser-interferometric gravitational wave detector (GWD). By injecting a squeezed state of vacuum into one of the LIGO interferometers, the authors implement one of the methods known to allow beating the standard quantum limit (SQL). Comparison of the interferometer noise with and without the squeezed-vacuum input confirms the presence of quantum correlations. The authors show that these correlations would indeed allow beating the SQL in the absence of classical noise, which is subtracted in data post-processing in the present work.

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set, but it is only a question of time for GWDs to reach this limit. Thereby, the obtained results clearly demonstrate that quantum engineering techniques can enhance the performance of real-world interferometric sensors. I do believe this is a message of great interest to those concerned more directly with GWDs, but also the wider quantum sensing and metrology community.

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For example, please clarify what the role of the homodyne receiver is. Reference [20], which may have some details on this, is listed as in preparation.

Response: We have changed the naming in Figure 1. and future references to a “GW readout” (lines 178, 189, 209). We have removed references to homodyne readout in the methods. We note that the described fringe-offset readout method is indeed a form of homodyne detection that is constrained to measure only the phase quadrature of the interferometer. Earlier drafts of the paper had language elaborating this, but we failed to fully simplify the figure along with the text. We choose now to avoid the usage of “homodyne” entirely as it is not strongly relevant to the discussion. Furthermore, LIGO’s peculiar usage of two photodetectors for technical reasons leads one to see the figure and assume it is a form of balanced homodyne detector, which it is not. Its local oscillator is carried with the signal and it uses the sum-channel rather than difference. While the two photodetectors could be merged into a single one in Figure 1, we choose to show both as they are needed for the cross correlation described in the Methods section to be performed.

Also, could it be shown in figure 1 where/how the squeezing angle ϕ is tuned in practice?

Response: The squeezing angle is controlled using a frequency servo of a separate laser that generates the squeezer OPO pump field and is described in citation [19]. For the simplicity of Fig 1, we are omitting the pump field, but have added this text to the caption to resolve this question for readers “A frequency-shifted control field (orange) is used to sense the squeeze angle to control it using the phase of the squeezer optical pump field (not shown)[19].”

The proper subtraction of classical noise is thoroughly discussed throughout the main manuscript, and in even greater detail in the methods section. It is evidently a crucial part of this experimental work. I therefore think it deserves an explicit mention in the abstract as well.

Response: We agree with the referee, and added an explicit mention in the abstract in the new revised manuscript as follows: “After subtracting out classical noise, our measurements show that. . .” (lines 85-86)

In a similar vein, showing the total measured noise with squeezing engaged (dark green curve of figure 4) in figure 2 would better represent how the results are actually obtained.

Response: Thanks for the referee’s suggestion. We updated Figure 2 by adding in the brown trace of total measured noise spectrum with squeezing (dark green curve of original figure 4). The figure caption is also revised to say (“The grey and brown traces show the measured total noise level of the interferometer with unsqueezed vacuum state (i.e. the reference) and injected squeezing at 35° , respectively.”). Our original plot omitted this curve as we consider this plot considerably more busy.

Furthermore, I think the statement that in this work the SQL is surpassed (l. 136) is misleading. What is shown is that the quantum noise contribution is lower than the SQL. The

total noise in the measurement is still above the SQL. Again, such ambiguity could be easily avoided by showing the measured noise spectrum in figure 2.

Response: i) We have revised the statement as: “. . . , and use the optomechanically-induced correlations of ponderomotive squeezing to show quantum noise below the SQL.” (lines 132-133)
ii) Figure 2 is updated by adding the trace of total measured noise spectrum with squeezing.

The manuscript discusses three models, the simplified one of eqs 1-5, the extended one of eqs. 6-10 and an exact one (l. 444). Please state which one is used to calculate the model M_r of eq. 11, i.e. to derive the main results, the post-subtraction inferred quantum noise. If it is the exact model, how does it differ from eqs. 6-10? Is this the same model than that used for the quantum noise model with unsqueezed vacuum state (blue curve in figures 2-4)? I assume these models take all sources of loss into account?

Response: We have changed the text in lines 561-565 to make our choice more explicit: “For our analysis, we use the full coupled cavity equations [1, 2] including all losses and optical spring effects, but reiterate that the deviation between the exact model and equations (6)-(10) is small for our operating parameters.”

The error bars in figure 4, according to the figure caption, include the statistical uncertainties of the classical noise from the reference measurement, as well as in the squeezed measurement. Elsewhere (l. 781) it is mentioned that the figure includes the uncertainties of eqn. 12. The latter, in my understanding, also contains systematic noise terms, such as modeling uncertainty. Are those included in the error bars of figure 4? How about the stationarity uncertainties? I would also suggest to state a confidence interval, taking all uncertainties into account, in the abstract.

Response: Thank the referee for raising this point. The plot has been updated, and the error bars in revised Figure 4 include all the uncertainties of eqn. 12: the optical sensitivity calibration δG , the servo loop calibration δC , statistical fluctuations δD_r and δD_s , the modeling uncertainty δM_s , and the relative stationarity uncertainty terms δN_t . δN_m is ignored because it is considered as small supporting by l. 729-743 and Figure 6. The source of quoted frequency-dependent calibration uncertainty $\delta G^2 + \delta C^2$ is updated referring to the latest paper: Sun, L. et al. *The role of systematic error in Advanced LIGO calibration during the first half of the third observing run. (2020)* This is also revised in the reference.

The modeling uncertainty δM_s is shown with the purple curve. The reference curve modeling uncertainty is included with the blue curve (though small), and the separated dashed blue QRPN curve. It is also now included in the error bars of the inferred quantum noise for consistency with the other uncertainties. The caption of the figure is revised as “The green inferred quantum noise curve and error bars include all uncertainty terms present in equation (12) as estimated in the methods section, including frequency dependence.”

The general statement of l. 316 (measurements presented here represent long-awaited milestones in verifying the role of quantum mechanics in limiting the measurements of small displacements generally) appears somewhat ignorant of the activities in the field of opto- and electromechanics. The connections and references to the literature in this field are also a bit erratic. For example, the list [15,16] of room-temperature experiments should include the 2017 work from NIST (Quantum correlations from a room-temperature optomechanical cavity, Science). Ref [17] only demonstrates imprecision below the SQL, a situation many experiments have achieved. Quantum correlations play no role for this, which (by the authors’ adopted definition)

implies that this cannot be a sub-SQL measurement. Ref [18], in contrast, uses a closely related method based on ponderomotive quantum correlations. Indeed the two methods are discussed on the same footing in the prominently cited ref [4]. It also overcomes the SQL, so it is very relevant to this work.

Response: We agree with the reviewer on all counts, and have addressed this by including a reference to NIST paper as follows: “[Previous demonstrations of QRPN have involved cryogenically pre-cooled, pico- to micro-gram scale mechanics \[10–14\], with three exceptions \[15–17\]. Similarly, a previous sub-SQL measurement of displacement has been performed on a cryogenically pre-cooled mechanical oscillators at the nano-gram mass scale \[18\].](#)” (lines 148 - 154)

Add a short explanation of what the mechanical degree of freedom is that sets the SQL.

Response: We address this by adding the text “...and injected squeezing allows us to surpass the SQL [for the differential arm motion.](#)” (lines 193-194)

Organizing the very long methods section into subsections with individual headings would make the information there much more accessible.

Response: The Methods section is re-organized into subsections with individual headings in the revised manuscript.

Typos etc:

I. 594, should this be $\eta_0 k^2$?

Response: Corrected.

Caption of figure 2, “Drawn for the cases where the input state is unsqueezed vacuum (dotted blue) and . . .” may be more clear

Response: Corrected.

Caption of figure 5 describes a “right” panel which is not visible in the reviewer version of the manuscript

Response: Corrected.

Referee #2

Remarks to the Author:

This paper shows that the quantum noise contribution in the LIGO detector in Livingston surpasses the standard quantum limit (SQL) after a subtraction of classical noise. Although they have not realized the sub-SQL measurement with the 40 kg mirrors, the sensitivity is quite close to it and the experimental demonstration is impressive.

In fact, it is not clearly written in the abstract that classical noise has been subtracted from the total noise, and some readers may have an impression that LIGO sensitivity has reached and surpassed the SQL. The subtraction is the main part of the paper but it is not introduced until line 154, and the detailed explanation has been moved to a supplementary article

(Methods).

Response: We agree with the referee, and added an explicit mention in the abstract abstract in the new revised manuscript as follow: “After subtracting out classical noise, our measurements show that...”(lines 85-86)

There are some technical details described in the main body, which should be minimized to make some rooms to explain the noise subtraction method.

The paper needs to be fully revised to meet the standard quality level of Nature articles. Some technical terms come in without notice: for example, QRPN, output mode cleaner, homodyne readout, optical spring, etc. The paper is like an internal report for people who are already familiar with everything.

Response: Each of these technical terms are clarified as follows:

i) QRPN: The full form “quantum radiation pressure noise” is added to the revised manuscript in lines 110-111.

ii) Output mode cleaner: This term indeed has no followup in the article and is not relevant to the discussion, so we have removed it.

iii) Homodyne readout: Now removed per the later comment (2).

iv) Optical spring: The original reference to this term is now removed (original in line 128 and 313), deeming it unnecessary to bring up only to describe it as insignificant. In methods, we now introduce the term as an interaction between radiation pressure and the cavity.

Lines 475-481 of methods now read “Notably absent from this non-ideal model but present in [1] is the interaction between radiation pressure and the signal recycling cavity detuning, $\xi \neq 0$, typically labeled an “optical spring”. This interaction is accounted for in the calibration and exact model curves included in our plots but, at this ξ and ψ , is not significant for the analysis. We note that the above non-ideal model is accurate to 1% in the zero detuning case $\psi = \xi = 0$. While strong optical springs are an alternative method of achieving sub-SQL quantum noise sensitivity, the accuracy of the above simplified model indicates that the spring contribution is not significant for this analysis.”

I found different forms to refer equations: equation 4, Eqn.(10), eq.12, etc., and the same for figures: Fig.2, Fig 2, Figure 3, figure 1, etc., sometimes with and sometimes without a half space before the number. I do not think it is a right way to write the abbreviation at the beginning of a sentence: Eqn.4 → Equation 4, or to write a mathematical symbol at the beginning of a sentence: L is the... etc. The authors should spend more time checking the manuscript before sending it to a journal.

Response: All of the forms to refer figures are unified to “Fig. X” if in text, and “Figure X” if at the beginning of a sentence. All of the forms to refer equations are unified to “equation (X)” everywhere.

There are also some typos and grammatical errors:

- ponderomotive squeezing became pondermotive squeezing after some points.

Response: All corrected.

- discussion of Figure 3 below → discussions in the caption of FIG.3 (L261)

Response: Corrected.

- due → due to (L291)

Response: Corrected.

- signal cavity → signal recycling cavity (L292)

Response: Corrected.

- due → due to (L291)

Response: Corrected.

- quantum limited → quantum noise limited (L490)

Response: Corrected.

- The sentence starting in L538 is strange.

Response: This sentence is revised as: “All of these uncertainties are frequency-dependent, but the full functional forms are omitted for brevity. ”

- The sentence starting in L548 is strange.

Response: This sentence is revised as: “ D_r , D_s , and M_r , M_s stand for the frequency-dependent data and model spectral densities for the reference and squeezing operating cases.”

I have some more comments.

(1) A mathematical expression of ψ is missing. It is also confusing that the authors sometime call this variable the detuning of the signal recycling cavity.

Response: We have adjusted the language in lines 315-317 to describe ψ as a squeezing angle shift from detuning, rather than calling it the detuning, we also now refer to methods to indicate where expressions involving ψ can be found. We include the relation that $\psi = 10.7\xi$ in line 474 to relate it to the physical phase shift. In the event the reviewer is looking for a validation of this term in the model expressions, we derive this relation attached below, but consider the derivation verbose and unnecessary for the results of the paper given the length of Methods.

(2) Some description about the homodyne detector is needed. Can one choose the homodyne angle? If it is fixed, is it on the phase quadrature? A possible detuning from the phase quadrature measurement seems to be missing in the noise estimate.

Response: We have changed the naming in Figure 1. and future references to a “GW readout” (lines 178, 189, 209). We have removed references to homodyne readout in the methods. We note that the described fringe-offset readout method is indeed a form of homodyne detection that is constrained to measure only the phase quadrature of the interferometer. Earlier drafts of the paper had language elaborating this, but we failed to fully simplify the figure along with the text. We choose now to avoid the usage of “homodyne” entirely as it is not strongly relevant to the discussion. Furthermore, LIGO’s peculiar usage of two photodetectors for technical reasons leads one to see the figure and assume it is a form of balanced homodyne detector, which it is not. Its local oscillator is carried with the signal and it uses the sum-channel rather than difference. While the two photodetectors could be merged into a single one in Figure 1, we choose to show both as they are needed for the cross correlation described in the Methods section to be performed.

In short, the GW readout we employ is merely a form of homodyne readout, not a balanced homodyne readout. Its readout angle is fixed to the phase quadrature and so does not contribute an uncertainty term.

(3) The discussion in Method is lengthy. Consider making it shorter. Adding a diagram for the explanation would be helpful.

Response: The Methods section is re-organized into subsections with individual headings as in the revised manuscript. Our understanding is that the journal format does not allow figures in methods, instead those are in a separate extended data section currently appended to the paper. We agree that Methods is currently long, but consider all of its content pertinent to justify and validate subtraction in our data analysis and simplify the theoretical discussion using the phenomenological models. We have written it to be as efficient as possible.

Requested Derivations

1. Equivalence of Equation (2) to Formulas of [KLMTV 4]

The following expressions are from the reference [KLMTV 4].

$$\mathcal{K}(\Omega) = \frac{2I_0\gamma^4}{I_{\text{SQL}}\Omega^2(\gamma^2 + \Omega^2)} \quad \text{from table 1 of [4]} \quad (\text{R1})$$

$$I_{\text{SQL}} = \frac{mL^2\gamma^4}{4kc} \quad \text{from equation (19) [4]} \quad (\text{R2})$$

$$P_{\text{arm}} = \frac{cI_0}{2\gamma L} \quad \text{from equation (B16) [4]} \quad (\text{R3})$$

These may be combined, expanded, and then factorized to construct equations (2) of our paper.

$$\mathcal{K}(\Omega) = \frac{2I_0\gamma^4}{I_{\text{SQL}}\Omega^2(\gamma^2 + \Omega^2)} = \frac{8I_0kc}{mL^2\Omega^2(\gamma^2 + \Omega^2)} = \frac{16kP_{\text{arm}}\gamma}{mL\Omega^2(\gamma^2 + \Omega^2)} \quad (\text{R4})$$

$$= \frac{32kP_{\text{arm}}}{m\Omega^2c} \left(\frac{\gamma c}{2L} \frac{1}{(\gamma^2 + \Omega^2)} \right) = \frac{32k|G(\Omega)|^2 P_{\text{arm}}}{m\Omega^2c} \quad (\text{R5})$$

Again, we note that the choice of $G(\Omega)$ through this factorization technique is arbitrary, and we choose a $G(\Omega)$ to be an optical gain most related to the interferometer sensing function. We should note a subtlety of this relation/derivation in that the interferometer with power and signal recycling has three time-constants, γ , one for common power buildup, one for differential sensing, and one for the arms that relates to the other two. The reference [KLMTV 4] only uses the arm time constant, but our paper uses only the differential signal time constant. However, this derivation is further validated during the computation of the input and output losses below, which only need the differential signal time constant.

2. Equivalence of Equation (1) to Formulas of [KLMTV 4]

Starting with definitions of equation (28 [4]) and (46 [4]).

$$h_{\text{SQL}} = \sqrt{\frac{8\hbar}{m\Omega^2 L^2}} \quad S_h = \frac{h_{\text{SQL}}^2}{2} (\mathcal{K}^{-1} + \mathcal{K}) \left(\cosh(2R) - \cos(2(\lambda + \Theta)) \sinh(2R) \right) \quad (\text{R6})$$

To derive the equations for this paper, we must apply the mappings, $R \rightarrow r$, $\lambda \rightarrow \frac{\pi}{2} + \phi$, $\Theta \rightarrow \frac{\pi}{2} - \theta$. The last two mappings arise from our choice of phase convention to have $\phi = 0$ be phase squeezing and from our usage of arctan of equation (5) rather than arccot for equation (47 [4]). Applying those mappings and then simplifying gives:

$$S_x = L^2 S_h \quad (\text{R7})$$

$$= \frac{L^2 h_{\text{SQL}}^2}{2} (\mathcal{K}^{-1} + \mathcal{K}) \left(\cosh(2r) - \cos(2(\phi - \theta)) \sinh(2r) \right) \quad (\text{R8})$$

$$= \frac{4\hbar}{\mathcal{K}m\Omega^2} (1 + \mathcal{K}^2) \left(\cosh(2r) - (\cos^2(\phi - \theta) - \sin^2(\phi - \theta)) \sinh(2r) \right) \quad (\text{R9})$$

$$= \frac{\hbar c}{8k|G|^2 P_{\text{arm}}} \frac{1 + \mathcal{K}^2}{2} \left(e^{2r} (1 - \cos^2(\phi - \theta) + \sin^2(\phi - \theta)) \right. \\ \left. + e^{-2r} (1 + \cos^2(\phi - \theta) - \sin^2(\phi - \theta)) \right) \quad (\text{R10})$$

$$= \frac{\hbar c}{8k|G|^2 P_{\text{arm}}} (1 + \mathcal{K}^2) \cdot \left(e^{2r} \sin^2(\phi - \theta) + e^{-2r} \cos^2(\phi - \theta) \right) \quad (\text{R11})$$

$$= S(\phi, \theta) (1 + \mathcal{K}^2) \frac{\hbar c}{8k|G|^2 P_{\text{arm}}} \quad (\text{R12})$$

3. Input Output Losses for equations (6) - (10)

While the above derivations link the exact calculations of [KLMTV 4] to the expressions used in this paper, here I will validate the equations (6)-(10) using more concise but complex matrix machinery than present in [4]. To do this, I will express the input-output relations for the input squeezing/vacuum operators \hat{a}_i in reflection of the signal recycling cavity \hat{a}'_i . This expression implicitly includes the cavity effects (ignoring subdominant cavity losses) inside of $\mathcal{K}(\Omega)$ and $G(\Omega)$. To save space, arguments indicating frequency dependence Ω of variables are omitted.

The input-output relations are the following two separate equations, compacted to use vector notation.

$$\underbrace{\begin{bmatrix} \hat{a}'_1 \\ \hat{a}'_2 \end{bmatrix}}_{\vec{\hat{a}}'} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\mathcal{K} & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}}_{\vec{\hat{a}}} + \underbrace{\begin{bmatrix} 0 \\ 2kG\sqrt{\frac{P}{\hbar kc}} \end{bmatrix}}_{\vec{c}} \hat{x} \quad (\text{R13})$$

Including squeezing applies the following intermediate transformation between the vacuum state and mirror reflection transformation:

$$\begin{bmatrix} \hat{a}_1(\Omega) \\ \hat{a}_2(\Omega) \end{bmatrix} = \mathbf{S}(r, \phi) \begin{bmatrix} \hat{a}_1^{\text{vac}}(\Omega) \\ \hat{a}_2^{\text{vac}}(\Omega) \end{bmatrix} \quad (\text{R14})$$

$$\mathbf{S}(r, \phi) = \mathbf{R}(\phi) \begin{bmatrix} e^r & 0 \\ 0 & e^{-r} \end{bmatrix} \mathbf{R}(-\phi) \quad (\text{R15})$$

$$\mathbf{R}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \quad (\text{R16})$$

The above vector operator expressions will be used to calculate the power spectral density using homodyne detection of the phase quadrature, calibrated into units of displacement. This density is calculated from the expression:

$$S_x \delta(\nu) = \left| \frac{\partial \hat{a}'_2}{\partial \hat{x}} \right|^{-2} \langle \hat{a}'_2(\Omega) \hat{a}'_2(\nu - \Omega) \rangle_{\text{sym}} \quad (\text{R17})$$

abusing vector and expectation notation slightly to be concise, this leads to

$$S_x \delta(\nu) = \left| \frac{\partial \hat{a}'_2}{\partial \hat{x}} \right|^{-2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \langle \vec{\hat{a}}'(\Omega) \vec{\hat{a}}'^{\dagger}(\Omega - \nu) \rangle_{\text{sym}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (\text{R18})$$

Which will lead to simpler equations using algebraic relations founded from:

$$\langle \vec{\hat{a}}_{\text{vac}}(\Omega) \vec{\hat{a}}_{\text{vac}}^{\dagger}(\Omega - \nu) \rangle_{\text{sym}} = \frac{\hbar kc}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \delta(\nu) \quad (\text{R19})$$

The two vector expressions above in the symmetrized expectation allows us to then switch to concise matrix expressions in the two-photon formulism. From this we can relate the matrix expressions to the displacement spectrum calculation. While this so far is entirely redundant with the work of [KLMTV 4], the machinery will be used in a moment to incorporate and simplify input and output losses to the expressions (6)-(10).

$$\frac{2}{\hbar kc} \left| \frac{\partial \hat{a}'_2}{\partial \hat{x}} \right|^2 S_x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{K} \mathbf{S}(r, \phi) \mathbf{S}^{\dagger}(r, \phi) \mathbf{K}^{\dagger} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{K} \mathbf{S}(r, \phi) \right\|^2 \quad (\text{R20})$$

$$= e^{-2r} \left(\cos^2(\phi) \mathcal{K}^2 - 2 \sin(\phi) \cos(\phi) \mathcal{K} + \sin^2(\phi) \right) + e^{2r} \left(\sin^2(\phi) \mathcal{K}^2 + 2 \sin(\phi) \cos(\phi) \mathcal{K} + \cos^2(\phi) \right) \quad (\text{R21})$$

$$= e^{-2r} \left(\cos(\phi) \mathcal{K} + \sin(\phi) \right)^2 + e^{2r} \left(\sin(\phi) \mathcal{K} - \cos(\phi) \right)^2 \quad (\text{R22})$$

using formulas for $\sin(\arctan(\mathcal{K}))$ we simplify into a form using θ

$$= (1 + \mathcal{K}^2)e^{-2r} \left(\cos(\phi) \cos(\theta) + \sin(\phi) \sin(\theta) \right)^2 + (1 + \mathcal{K}^2)e^{2r} \left(\sin(\phi) \cos(\theta) - \cos(\phi) \sin(\theta) \right)^2 \quad (\text{R23})$$

$$= (1 + \mathcal{K}^2)S(\phi, \theta) \quad (\text{R24})$$

This makes S_x equivalent to equation (1).

Now, having established the above relations, we can concisely add loss using vectors of the two-photon quantum operators, and then derive the expectation.

$$\vec{a}' = \sqrt{\eta_o} \mathbf{K} \left(\sqrt{\eta_i} \mathbf{S}(r, \phi) \vec{a}_{\text{vac}} + \sqrt{1 - \eta_i} \vec{b}_{\text{vac}} \right) + \sqrt{1 - \eta_o} \vec{c}_{\text{vac}} + \sqrt{\eta_o} \vec{C} \hat{x} \quad (\text{R25})$$

This loss-inclusive expression R25 is then applied to equation R18. Afterwards, the above trigonometric reductions of equation R20-R24 are applied, and then the expression is further simplified to use the factorizations of (6)-(10).

$$\frac{2}{\hbar} \left| \frac{\partial \vec{a}'_x}{\partial \hat{x}} \right|^2 S_x = \eta_o \eta_i \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{K} \mathbf{S}(r, \phi) \right\|^2 + \eta_o (1 - \eta_i) \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \mathbf{K} \right\|^2 + (1 - \eta_o) \quad (\text{R26})$$

$$= \eta_o \eta_i (1 + \mathcal{K}^2) S(\phi, \theta) + \eta_o (1 - \eta_i) (1 + \mathcal{K}^2) + (1 - \eta_o) \quad (\text{R27})$$

$$= \eta_o (1 + \mathcal{K}^2) \left(\eta_i S(\phi, \theta) + (1 - \eta_i) + \frac{(1 - \eta_o)}{\eta_o (1 + \mathcal{K}^2)} \right) \quad (\text{R28})$$

$$= \eta_o (1 + \mathcal{K}^2) \frac{1 + \eta_o \mathcal{K}^2}{1 + \eta_o \mathcal{K}^2} \left(\eta_i S(\phi, \theta) + (1 - \eta_i) + \frac{(1 - \eta_o)}{\eta_o (1 + \mathcal{K}^2)} \right) \quad (\text{R29})$$

$$= (1 + \eta_o \mathcal{K}^2) \left(\eta_i \frac{\eta_o + \eta_o \mathcal{K}^2}{1 + \eta_o \mathcal{K}^2} S(\phi, \theta) + \frac{\eta_o + \eta_o \mathcal{K}^2}{1 + \eta_o \mathcal{K}^2} (1 - \eta_i) + \frac{(1 - \eta_o)}{1 + \eta_o \mathcal{K}^2} \right) \quad (\text{R30})$$

$$\approx (1 + \eta_o \mathcal{K}^2) (\eta_e S(\phi, \theta) + (1 - \eta_e)) \quad (\text{R31})$$

This final approximation makes S_x equivalent to equation (6) of Methods. If you inspect these simplifications and approximations, you might initially like to use equation R28 to define the effective loss η_e of Eq (7) and the factors in Eqs (6)-(10), but we decided instead to use additional approximations because otherwise the most natural definition of S^* arising from R28 can be greater than 1 even with no squeezing, due to the η_o in a denominator. Our version preserves $S^* = 1$ in the case of no squeezing. Instead, we continue with an ad-hoc additional simplification using the factor $(1 + \eta_o \mathcal{K}^2)$ leading to equation R30. One can check the limiting cases $\mathcal{K} \rightarrow 1$ and $\mathcal{K} \rightarrow \infty$ to see that the final form of equation R31 is accurate in the reasonably low-loss regime where $\eta_i \eta_o \approx \eta_i + \eta_o - 1$ for both the no squeezing case $S(\phi, \theta) = 1$ and high squeezing case $S(\phi, \theta) = 0$. For the intermediate cases the expression for the error is complex, but we have ultimately checked the accuracy of the expressions (6)-(10) in the paper to be numerically accurate to 1% to 5% to the exact models, and the inaccuracies are due primarily the omission of the complex optical spring rather than from the above treatment of loss. The following section analyzes how the ψ term of equation 10 arises. While incorporating ψ is ad-hoc with respect to the derivation above, we have checked that it is numerically accurate compared to the exact models.

4. SRC Detuning Reduction

The interferometer with balanced arms and at perfect destructive interference on the beamsplitter may be modeled as two separate coupled cavities (power and signal) rather than an interferometer. Because the signal recycling cavity is short relative to the arms, it may be adiabatically eliminated, at which point the transmissivity of the arm mirrors, T_a , and signal recycling mirror T_s , become an effective transmissivity T_e . Similarly, a detuning of the signal recycling round trip phase ξ_s is enhanced by the signal recycling cavity to become an effective phase shift ξ_e in the reduced arm cavity. The physical and effective transmissivity and reflectivities are expressed:

$$r_a = \sqrt{1 - T_a} \qquad r_s = \sqrt{1 - T_s} \qquad (\text{R32})$$

$$T_e = \frac{T_a T_s}{(1 - r_a r_s)^2} \qquad r_e(\xi_s) = \frac{r_a - r_s e^{i\xi_s}}{1 - r_a r_s e^{i\xi_s}} \qquad (\text{R33})$$

Note that for the LIGO detectors, the effective transmissivity is sufficiently high that the approximate calculation of the signal bandwidth, $\gamma \approx T_e c / 4L$, is biased low by $\sim 7\%$ from an exact calculation.

The effective reflectivity and its cavity-modified phase shift can then be converted to the total phase shift from the combined signal + arm cavity. The factor $\frac{4}{T_e}$ includes the arm cavity enhancement of the phase signal, and the derivative calculates the relationship between the internal SRC phase shift and effective SRC-as-mirror phase shift.

$$\psi = \frac{4}{T_e} \frac{1}{ir_e(\xi_s)} \left. \frac{\partial r_e(\xi_s)}{\partial \xi_s} \right|_{\xi_s=0} \xi_s = \left(\frac{1}{1 - r_a r_s} - \frac{r_a}{r_a - r_s} \right) \xi_s \approx 10.7 \xi_s \qquad (\text{R34})$$

This ψ then is the total rotation experienced by a field between $\Omega = 0$ and $\Omega \gg \gamma$. The functional form for the frequency dependence of equation (10) is from equation 4 of [Kwee et al. 7].

Referee #3

Remarks to the Author:

In this work, quantum correlations between the phase of the high power laser beam circulating in the gravitational wave (GW) detector LIGO and the 40 kg mirrors of this device, has been experimentally demonstrated. Vacuum fluctuations entering through the interferometer dark port give rise to ponderomotive squeezing. This effect, together with the injection of the squeezed vacuum injected into the interferometer, allows to surpass the Standard Quantum Limit (SQL), that is a limit to the precise continuous measurement of position of an object. This result proves the existence of quantum correlations involving the position of a human-scale object. Moreover, this work presents the first direct observation of the Quantum Radiation Pressure Noise (QRPN) on a macroscopic object that arises from the quantum back action due to the Heisenberg uncertainty

principle. In this work the QPRN is measured for the first time for macroscopic objects at room temperature. This effect has been already observed for microscopic objects, mostly at cryogenic temperatures, but also at room temperature. Proper references have been provided. My suggestion is to underline the fact that, all the cited works contain the experimental demonstration of QPRN for microscopic objects and this particular work presented in this paper describes the first ever measurement of QPRN performed on macroscopic objects, at low frequency and at room temperature.

Response: This is already stated in lines 154-156 as "[The present measurements are performed on the room-temperature, 40 kg mirrors of Advanced LIGO using 200 kW of laser light,...](#)". For additional emphasis, we have added some words in line 136. "First, we directly observe QPRN contributing to the motion of kg-mass objects [at room temperature,...](#)"

Finally, it has been presented that the QPRN observation has been done also in Advanced Virgo GW detector and a reference to the related work (in preparation) is cited. The other milestone that has been reported is the overtaking of the SQL using squeezing and ponderomotive effect. This is an original result achieved and, as the authors specify, this represents the first realization of a quantum nondemolition technique in GW detectors. The reported results will be useful for the improvement of future generation of GW detectors. In the abstract, it has been underlined that, the presented work can also improve all type of measurements in future. In the text it has been specified that the performed measurement verify the role of the quantum mechanics in limiting the measurement of small displacements. This sentence could be better supported with the particular kind of effect that can benefit scientific measurements using quantum mechanics.

Response: The lines at line 318-322 have been rewritten to emphasize macroscopic objects and GWs rather than precision displacements measurement generally, as follows: "[The measurements presented here represent long-awaited milestones in verifying the role of quantum mechanics in limiting the precision of position measurement even for macroscopic objects, and thereby limiting the sensitivity of GW detectors.](#)"

The approach used and the quality of data are very good. The results are presented in a good way mostly in the supplementary part, where the methods are very well described and particular attention has been drawn towards the statistics and the treatment of the uncertainties. In the abstract, the context and the importance of the measurements performed are very well presented. The paper is not yet divided into sections. But I can clearly distinguish introduction, theoretical background, experimental demonstration, discussion and the conclusions. I particularly appreciate the introduction where the concept of Standard Quantum Limit is very well explained. In the conclusion, the importance of the performed measurement has been mentioned and the two milestones achieved have been clearly underlined. In the last part, the importance of the measurement in the GW detection field has been stressed. The paper presents robust, valid and reliable results. This is more evident in the supplementary material. In conclusion, I consider this work to be very good and useful for improving quantum noise limited scientific measurements.

Suggested improvements: I think that the authors should better clarify what other kind of measurements this experimental demonstration can be useful for. Formulas (1) and (2): following the given reference, I was not able to derive these formulas. Maybe you can guide better the reader towards this derivation. Moreover, in order to make dimensionless the parameters in equation (2), γ should be in Hz instead of rad/s. Line 445 space missing between the

number and the units.

Response: Given the lengthy extent and technical nature of [KLMTV 4], we think any attempt to relay these relations is beyond the scope of this experimental paper, particularly the already-long Methods section. We certainly understand the desire to validate the relationship between the formulas of this paper and that reference, so we have included a section with the derivations for those relations.

This derivation comes from a considerable trigonometric simplification combining eqs. (52), (47) and (28). of [KLMTV 4]. The existence of such a simplification must be true from the statement following Eq. (48 [4]) given the choice of equation (47 [4]): "This says that the squeezed-input interferometer has the same noise spectral density as the conventional interferometer except for an overall reduction by e^{-2R} ." This statement is mirrored in the factorization of $S(\Omega, \phi)$ chosen for this paper, which more concisely shows the preservation of the spectral shape for the ideally chosen squeezing angle.

If $G(\Omega)$ is expanded, the factor $K(\Omega)$ in our equation (2) can be equated to K from Table 1 of [4] and I_{SQL} of eq. (19 [4]). $G(\Omega)$ as a factor has some freedom, and the choice of convention is explained in our lines 206-209. The expression for $G(\Omega)$ can be calculated from the optical sensitivity of a Fabry-Perot cavity using the mirror transmissivity and Eq. (11 [4]). The factor of $1/\sqrt{2}$ in $G(\Omega)$ is from the beamsplitter in the FP Michelson. Ultimately $|G(\Omega)|^2$ is related to equation (B16 [4]), which works even though [KLMTV 4] does not anticipate the addition of the signal recycling mirror. Other than this length response, we acknowledge that it is not obvious to arrive at G or K from the cited references given the manner in which they factorize the equations. This is one reason we desire to publish using the simpler form we have chosen, as it can be more easily related to experimental measurables such as the sensing function.

The authors do not understand the comment that gamma should be in units of rad/s as radians are unitless in this context. We use rad/s consistently through the paper, so gamma and Omega are consistent in the denominator. It happens to be a coincidence that the formula for $G(\Omega)$ does not require any factors of π when using rad/s units for gamma. The equation otherwise requires the factor 2π to express with gamma in Hz units. The derivations below confirm that our choice is consistent with the equations of [KLMTV 4].

Line 445 space missing between the number and the units.

Response: All corrected.

Line 638 the same as for line 445

Response: Corrected.

Line 689 contibution → contribution

Response: Corrected.

Line 795 yellow, blue and purple → pink, blue and orange (as it is written in the caption of Fig.5)

Response: Corrected.

No need for experiment or data revision.

Reviewer Reports on the First Revision:

Referee #1 (Remarks to the Author):

Thanks to the authors for their detailed answer letter and thoughtful revision of the manuscript.

I think this is an excellent work, and in this revised form it is suited for publication in Nature.

There is only one point left. To be consistent with the rest of the changes, l. 139 should also read "Second, we demonstrate quantum noise below the SQL, ..." (instead of "Second, we surpass the SQL ...")

Referee #2 (Remarks to the Author):

The manuscript has been refined very well. I think it is now ready to be published from Nature.

Referee #3 (Remarks to the Author):

I read the revised manuscript where the suggestions/corrections of the referees have been implemented.

Concerning the points I raised in the previous review, apart for the corrections of few typos, that have been corrected, I mainly suggested to emphasize the importance of the presented work with respect to previous works. The authors added appropriate sentences, in order to accomplish this, as we can see in the following responses:

Response: This is already stated in lines 154-156 as "The present measurements are performed on the room-temperature, 40 kg mirrors of Advanced LIGO using 200 kW of laser light,...". For additional emphasis, we have added some words in line 136. "First, we directly observe QRPN contributing to the motion of kg-mass objects at room temperature,..."

Response: The lines at line 318-322 have been rewritten to emphasize macroscopic objects and GWs rather than precision displacements measurement generally, as follows: "The measurements presented here represent long-awaited milestones in verifying the role of quantum mechanics in limiting the precision of position measurement even for macroscopic objects, and thereby limiting the sensitivity of GW detectors."

Moreover, the responses about the formulas were satisfactory to me.

In particular, I asked to guide the reader towards the derivation of formulas (1) and (2) starting from the reference given in the text.

The authors responded that, the choice to don't relay these relations is due to the fact that this is beyond the scope of this experimental paper and that this would have made the Methods sections much longer.

Response: Given the lengthy extent and technical nature of [KLMTV 4], we think any attempt to relay these relations is beyond the scope of this experimental paper, particularly the already-long Methods section. We certainly understand the desire to validate the relationship between the formulas of this paper and that reference, so we have included a section with the derivations for those relations.

After I have been gone through all the calculations they kindly provided, I agree they did the better choice.

The response about the unit of measurement for gamma:

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also convinced me.

In conclusion, I would express my satisfaction for all the responses and, in general, for the whole paper that I consider very good.

Author Rebuttals to First Revision:

Response to the referees

We thank the referees for their reviews and approvals. One remaining issue raised by Referee #1 is addressed as below. This change is also highlighted in the revised manuscript

Referee #1

Thanks to the authors for their detailed answer letter and thoughtful revision of the manuscript. I think this is an excellent work, and in this revised form it is suited for publication in Nature. There is only one point left. To be consistent with the rest of the changes, l. 139 should also read "Second, we demonstrate quantum noise below the SQL, ..." (instead of "Second, we surpass the SQL ...")

Response: We have revised the text in original line 139 as: "[Second, we demonstrate quantum noise below the SQL, ...](#)"

Referee #2

The manuscript has been refined very well. I think it is now ready to be published from Nature.

Referee #3

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In conclusion, I would express my satisfaction for all the responses and, in general, for the whole paper that I consider very good.