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Saturn's Rings as a Seismograph to Probe Saturn's Internal Structure

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Abstract

As it has already done for Earth, the sun, and the stars, seismology has the potential to radically change the way the interiors of giant planets are studied. In a sequence of events foreseen by only a few, observations of Saturn’s rings by the *Cassini* spacecraft have rapidly broken ground on giant planet seismology. Gravity directly couples the planet’s normal mode oscillations to the orbits of ring particles, generating spiral waves whose frequencies encode Saturn’s internal structure and rotation. These modes have revealed a stably stratified region near Saturn’s center, and provided a new constraint on Saturn’s rotation.

Plain Language Summary

Just like measuring earthquakes around the world can tell scientists about Earth’s deep structure, vibrations of gas giant planets can tell us about their deep structure. But these vibrations are very hard to detect. At Saturn, help has come in the form of Saturn’s icy rings, where gravity causes the orbits of ring material to pick up the planet’s steady vibrations. This makes waves in the rings that are now being used as a powerful tool to study the inner workings of Saturn itself. Surprisingly, these waves have shown that the fluid motions in the deepest parts of the planet are relatively tame, compared to the forceful churning motions that were generally expected. They have also provided an estimate of the length of a Saturn day, a tough quantity to measure.

1 Introduction

The structure and makeup of the gas giants are key tracers of the planet formation process. Piecing together this ancient history demands answers to several entangled questions: did the gas giants form around solid planetesimal cores? If so, to what extent do these cores survive the process that then delivers hydrogen and helium, the bulk of these planets’ mass? How are the heavier constituents like ice and rock distributed after formation, and redistributed during the subsequent billions of years of evolution? Are the gas giants convective throughout their interiors, as has usually been assumed?

Within just the past five years or so, the interior mass distributions and rotation profiles of Jupiter and Saturn have been better constrained than ever owing to up-close observations of their gravity fields by spacecraft like *Juno* (Iess et al., 2018; Kaspi et al.,

2018; Guillot et al., 2018) and *Cassini* (Iess et al., 2019; Militzer et al., 2019). However, the gravity fields alone are largely insensitive to the greatest depths in these planets, where precious clues about the planet formation process lie hidden. In the case of Saturn, a totally independent means of peering into the planet’s interior is emerging thanks to information encoded in—of all places—Saturn’s rings.

Like any system in a stable equilibrium, planets respond to small perturbations by oscillating about that equilibrium state. Earth, for example, rings like a bell for days following a major earthquake. Global seismology deciphers the frequencies of these large-scale trapped waves—the normal modes of oscillation—to understand our planet’s internal structure.

The stars, too, vibrate. Helioseismology, the study of our sun’s trapped acoustic wave oscillations, has revealed most of the sun’s internal rotation profile in detail as well as the depth of the solar convection zone (Christensen-Dalsgaard et al., 1991, 1996). These oscillations are excited not by tectonics as on Earth, but by turbulent convection in the sun’s outer layers, just one of several processes that causes stars to vibrate quite generally. Beyond the solar system, tens of thousands of stars from the main sequence through the red giant branch have had their interior oscillation frequencies measured from their rapid brightness variations through time. These data have provided entirely new information about the physics of stellar evolution, rotation, and internal heat transport, and yielded powerful handles on stellar parameters like density, surface gravity, age, and inclination that are vital to studies of exoplanet systems (Chaplin & Miglio, 2013). This field of asteroseismology—the study of stellar interiors using normal mode oscillations—has led to something of a renaissance in stellar astrophysics over the last 15 years as a result of space missions like *COROT*, *Kepler*, and now, *TESS*.

In light of the major advances that normal mode seismology brought to terrestrial, solar, and stellar physics over the last several decades, similar methods hold immense promise for revealing the unseen inner workings of giant planets. Efforts to detect trapped oscillations in the gas giants from ground-based telescopes have been underway for more than 30 years, focusing for the most part on Jupiter (Deming et al., 1989; Schmider et al., 1991). This is because Jupiter’s large angular size and lack of a prominent ring system obscuring its surface make it amenable to seismological study by Doppler imaging, wherein a time series of line-of-sight velocity maps of the planet’s rumbling surface re-

veal the trapped oscillations that are in turn examined in the frequency domain. These studies have so far culminated in an encouraging detection of excess power at mHz frequencies consistent with Jupiter’s trapped acoustic waves (Gaulme et al., 2011). However, the isolation of individual normal mode frequencies—a necessary step to connect measured frequencies with knowledge of the planet’s interior—is stymied by the level of noise in the data gathered so far. Longer continuous coverage provided by observations from several longitudes on Earth may bring ground-based acoustic mode seismology of Jupiter within reach in the coming years (Schmider et al., 2013). In the meantime a very different, and ultimately complementary, method for studying giant planet oscillations has come to light thanks to *Cassini*’s campaign at Saturn.

2 Kronoseismology

The very rings that so inconveniently obscure part of Saturn’s disk on the sky turn out to offer the so far singular window into the individual normal-mode oscillations of a giant planet. Confirming a decades-old hypothesis (Stevenson, 1982) and a pioneering body of theoretical work that followed (Marley, 1990, 1991; Marley & Porco, 1993), NASA’s *Cassini* mission to Saturn has decisively shown that the periodic variations in Saturn’s gravity field caused by the planet’s internal oscillations in turn disturb the typically well-ordered orbits of particles in Saturn’s icy rings (Hedman & Nicholson, 2013, 2014; French et al., 2016, 2019; Hedman et al., 2019). This regular forcing stirs up waves that are wound into spiral patterns by the rings’ differential rotation—the same process by which a rotating bar structure in the center of a galaxy can organize the stellar, gas, and dust mass into spiral arms. A key difference in Saturn’s rings is that there the waves are very tightly wound around the planet, a result of Saturn’s immense mass compared to the mass in the rings themselves. As a result, the radial wavelength of these waves is of order a mere kilometer, versus the whopping 70,000 km scale of the main rings overall. The effect of the waves is therefore invisible from afar; their detection requires an up-close view the likes of which only a spacecraft mission can provide.

Spiral waves in Saturn’s rings were first studied intensely during the *Voyager* era, when it became clear that periodic gravitational perturbations from Saturn’s satellites launch an abundance of spiral waves throughout the rings (Cuzzi et al., 1981; Shu et al., 1983). Each wave falls into one of two classes: density waves are alternating compressions and rarefactions of orbits confined to the ring plane, whereas bending waves are

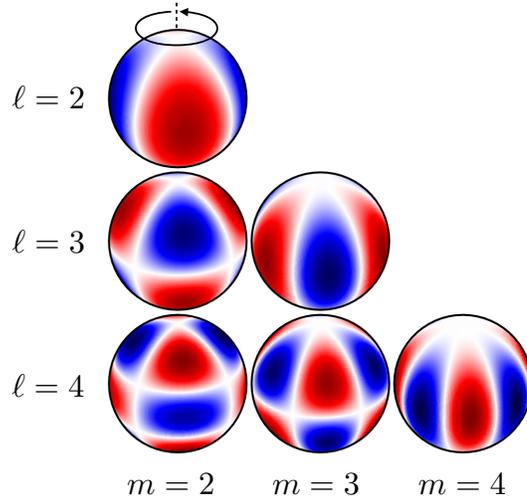


Figure 1. A visualization of some of the spherical harmonics relevant for Saturn ring seismology, labeled by their angular degree (ℓ) and azimuthal order (m). The color map corresponds to the magnitude of the perturbations—e.g., to the density and gravity field—as a result of the oscillation. An inertial observer sees each $m \neq 0$ pattern rotating as a function time, the combined effect of the planet’s rotation and the steady propagation of the wave pattern around the planet.

107 alternating vertical departures above and below the ring plane. Density waves are forced
 108 by eccentric satellite orbits; bending waves by inclined orbits. The physical description
 109 of the ring response to this slow periodic forcing by satellites applies equally well to the
 110 faster forcing by normal mode oscillations inside Saturn, which indeed create their own
 111 density and bending waves in the rings. In the language of the spherical harmonics—
 112 a convenient language for separating the components of the complicated overall planet
 113 oscillation (Figure 1)—modes with even $\ell - m$ can drive density waves, and modes with
 114 odd $\ell - m$ can drive bending waves.

115 However, while Saturn vibrates, most parts of the rings experience a negligible re-
 116 sponse. For a given oscillation frequency in the planet, the vast majority of ring orbits
 117 couple to it quite poorly because ring particles will experience extrema of the planet os-
 118 cillation at random orbital phases; the forcing tends to cancel, and no coherent response
 119 can develop. But in the special case wherein the local orbital frequency happens to nearly
 120 coincide with the frequency of forcing by the planet oscillation, then each extremum in
 121 the planet oscillation forcing takes place at about the same orbital phase for the ring par-
 122 ticles, and a coherent response will develop. This is the condition of resonance: a com-

123 measurability of the forcing planet frequency and the natural frequency of a ring orbit.
124 Ring seismology is thus sensitive only to the range frequencies that are occupied by ring
125 orbits, setting intrinsic limits on the type of oscillation within Saturn that this method
126 can probe.

127 Because the frequencies of ring orbits decrease steeply with distance from Saturn,
128 distinct planet oscillation modes excite waves at distinct locations in the rings. This means
129 that when these waves can be detected, they are spatially separated according to the fre-
130 quency and geometry (m value) of the corresponding normal mode in Saturn. Saturn's
131 rings thus, incredibly, form a natural frequency-domain seismograph for the planet's nor-
132 mal mode oscillations.

133 *Cassini* was able to realize these ideas by peering through Saturn's rings toward
134 bright stars and recording the variation in transmitted light as the spacecraft moved in
135 its orbit. As the line of sight passes through a wave in a translucent part of the rings,
136 the transmitted starlight varies sinusoidally, and the wave pattern can be reconstructed
137 to obtain the precise location of the resonance and thus the frequency of the perturb-
138 ing planet mode. Furthermore, by making repeated passes as *Cassini* orbited Saturn for
139 longer than a decade, scientists have been able to observe each wave from multiple per-
140 spectives. This broader view allowed them to count the number of spiral arms in each
141 spiral wave pattern, a crucial piece of information for discriminating which mode of the
142 planet's oscillation is responsible. (An $m = 2$ mode in Saturn creates a two-armed spi-
143 ral, an $m = 3$ mode a three-armed spiral, and so on; see Figure 2.) A spate of recent
144 *Cassini* results (Hedman & Nicholson, 2013, 2014; French et al., 2016, 2019; Hedman et
145 al., 2019) has characterized about two dozen spiral waves associated with normal modes
146 oscillations inside Saturn, providing for the first time a power spectrum suitable for nor-
147 mal mode seismology of a giant planet. Hedman, Nicholson and their collaborators termed
148 this field Kronoseismology, after the Greek name for Saturn. As it turns out, even as the
149 waves that emerged from these data validated the hypothesis of the rings as a natural
150 seismograph, they also revealed surprises of profound consequence for studying Saturn's
151 interior.

2.1 Deep interior structure

The expectation from Marley and Porco’s theory was that ring waves would be seen at resonances with Saturn’s fundamental mode oscillations, i.e., trapped surface gravity waves. They showed that these resonances would lie almost entirely in an inner region of the rings known as the C ring, a fortuitous alignment because the translucent C ring transmits enough starlight to make these experiments possible. (The heftier A and B rings that dominate the rings’ visual appearance are generally opaque to starlight.) They predicted an ordered pattern of resonances at distinct locations, and that the normal mode of Saturn responsible for each observed wave feature would be readily apparent based on the observed number of spiral arms. Instead, what Hedman and Nicholson discovered were *clusters* of waves (a pair of $m = 2$ waves; a triplet of $m = 3$ waves) in the proximity of the strongest fundamental mode resonances, an impossibility if the detailed model that Marley and Porco had proposed 20 years earlier represented the whole truth. What the data showed was unambiguous; what they demanded was a reexamining of the assumptions that had been made so far about the physics at work in Saturn’s interior.

The origin of these unexpected waves did not stay mysterious for long: it was soon demonstrated that they could be naturally produced if Saturn’s interior hosts not only the expected fundamental modes, but also gravity modes—trapped internal gravity waves (Fuller, 2014).

The implication that Saturn supports internal gravity waves is profound because their presence requires part of Saturn’s fluid interior to be stably stratified, a stark departure from the common assumption that Saturn’s interior is fully convective. A stable stratification means that a vertically displaced fluid parcel will tend to return to its starting position, enabling oscillations. By contrast, in a convective environment, a similarly displaced fluid parcel would simply continue to accelerate away from its starting position, so that no periodic fluid motion could be sustained.

This stable stratification suggests that Saturn’s deep interior has a significant composition gradient wherein molecular weight increases toward the planet’s center, mitigating the unstable temperature gradient that if left to its own devices would trigger convection and large-scale mixing of material. Instead, the gravity modes suggest a relatively

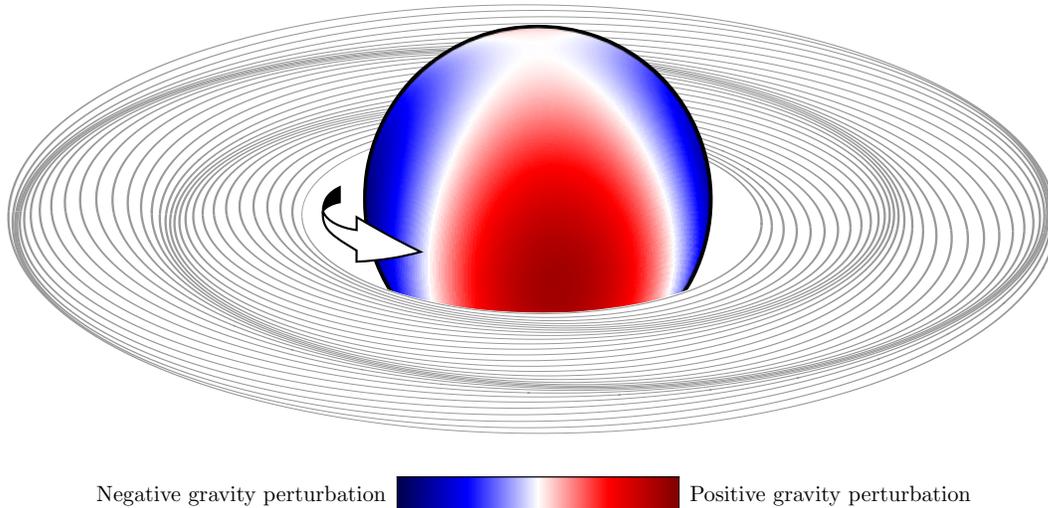


Figure 2. A schematic of an $\ell = 2, m = 2$ normal mode of oscillation inside Saturn generating a two-armed spiral density wave in the rings. In reality spiral patterns in the rings are much more tightly wound, and are only evident near a resonance.

183 quiet, extended, smooth transition between a dense rock- and ice-dominated core and
 184 the less dense hydrogen-dominated envelope.

185 While Fuller presented strong evidence that the mixture of the fundamental modes
 186 and gravity modes was responsible for the complicated spectrum of waves observed in
 187 the rings up to that point, the model was effectively a proof of concept: the ideas have
 188 yet to be leveraged into quantitative knowledge of Saturn’s deep interior. Updated anal-
 189 yses that address the Saturn-associated ring waves discovered in more recent years—and
 190 that apply more detailed and realistic models for Saturn’s interior structure—will offer
 191 meaningful constraints on the location and extent of Saturn’s deep stable stratification.
 192 Because of the sensitivity of these waves to the deepest regions inside Saturn, these new
 193 constraints will serve as an invaluable complement to the gravity science (Militzer et al.,
 194 2019; Galanti et al., 2019) that has come out of the end of the *Cassini* mission.

195 2.2 Rotation

196 The second major advance to come of ring seismology is the window it offers into
 197 Saturn’s interior rotation. One of the major historical unknowns about the Saturnian
 198 system is just how quickly Saturn rotates, a quantity of fundamental importance but one
 199 that is difficult to measure. Meteorological features can be tracked as Saturn rotates,

200 but as on Jupiter or Earth, flows associated with the weather do not track the rotation
201 of the bulk of the planet’s mass. Even among planets with no solid surface, Saturn’s ro-
202 tation is exceptionally difficult to pin down. The virtually perfect alignment of its mag-
203 netic dipole axis with its rotation axis (Dougherty et al., 2018) means that no obvious
204 trace of the planet’s rotation is visible from afar; this stands in contrast to Jupiter, where
205 the rotating magnetic field produces a strong periodic radio emission that is ideal for track-
206 ing the planet’s spin. As a result, Jupiter’s spin period has long been known to the level
207 of milliseconds, while estimates for Saturn’s spin period historically vary anywhere from
208 10 hours 30 minutes to 10 hours 50 minutes. While this spread is only a few percent of
209 a Saturn day, it significantly muddies the waters when it comes to understanding Sat-
210 urn’s atmospheric and interior flows, its overall interior structure, and consequently the
211 formation and evolution pathway that Saturn has undergone. With this historical chal-
212 lenge in mind, *Cassini* was tasked with finding new means of constraining Saturn’s in-
213 terior rotation. Indeed, unexpectedly, Saturn ring seismology has proven to be one such
214 path.

215 Putting aside the subset of C ring waves complicated by the mixture of fundamen-
216 tal and gravity mode oscillations, the remainder of Saturn-associated waves detected to
217 date—14 out of a total of 21—are well understood as resonances with simple fundamen-
218 tal mode oscillations of Saturn, the likes of which Marley and Porco had anticipated. These
219 planet modes have higher frequencies and angular degrees ℓ , and are consequently con-
220 fined somewhat closer to the surface, diminishing their value for constraining the struc-
221 ture of Saturn’s deep interior. However, there is a tradeoff at play: these shallower, higher-
222 ℓ modes also intrinsically possess more angular structure, and as a result are dramat-
223 ically more sensitive to Saturn’s rotation. Detailed calculations show that even account-
224 ing for the significant uncertainties in modeling Saturn’s interior structure, the quality
225 of fit to the ensemble of ring wave frequencies is dominated by the rotation rate assumed
226 for Saturn’s interior. To leading order this sensitivity is due to the Doppler shift relat-
227 ing a frequency in the planet’s rotating reference frame to the inertial reference frame
228 appropriate for studying the ring response. Rotation also subtly modifies mode frequen-
229 cies by inducing Coriolis forces and rendering the planet oblate, adding significant com-
230 plexity to the frequency calculation. This sensitivity forms a basis for a recent seismo-
231 logical measurement of Saturn’s rotation rate (Mankovich et al., 2019). The resulting
232 period is fast compared to radiometric and magnetic periods observed by spacecraft and

233 long used as a proxy for the planet’s interior rotation (Desch & Kaiser, 1981); the faster
234 seismological estimate is instead consistent with recent estimates based on Saturn’s shape
235 and gravity field (Helled et al., 2015; Militzer et al., 2019) and the stability of its jet streams
236 (Read et al., 2009), strengthening the growing consensus that periodic modulations as-
237 sociated with Saturn’s magnetosphere are not well coupled to the rotation of Saturn it-
238 self.

239 The full power of the seismological probe of Saturn’s rotation has yet to be lever-
240 aged, however. In contrast to the extraordinarily precise frequencies provided by ring
241 seismology, the theoretical methods employed so far to predict mode frequencies from
242 an interior structure model are significantly imprecise as a result of their approximate
243 treatment of rotation effects. Even with perfect knowledge of Saturn’s interior structure,
244 these methods can only predict fundamental mode frequencies with a relative precision
245 of order 10^{-3} at best; by comparison the observations by *Cassini* yield wave frequencies
246 with a typical relative precision of 10^{-5} . In particular, the seismology delivers Saturn’s
247 rotation period to a precision of about 1.5 minutes, an uncertainty comparable with the
248 more model-dependent constraints based on Saturn’s shape and gravity field, but sig-
249 nificantly larger than that derived from the stability of atmospheric flows. Whether the
250 seismology, gravity-shape, and atmospheric dynamics constraints will converge on a con-
251 sistent rate for Saturn’s bulk rotation thus awaits improved theoretical methods for the
252 seismological forwarding modeling; these will take the form of higher-order asymptotic
253 treatments of rotation, or hydrodynamical models that can solve for eigenfrequencies em-
254 pirically. Because the fundamental mode frequencies scale roughly linearly with Saturn’s
255 rotation rate, if the theory can match the data at a relative precision of 10^{-5} , the ex-
256 isting seismology data could in principle yield Saturn’s rotation period to within a sec-
257 ond. In reality, at this level, matters are complicated by *differential* rotation: rather than
258 measuring any single rotation rate, it is more appropriate to speak of quantifying Sat-
259 urn’s rotation *profile*.

260 The discovery of deep differential rotation in Saturn was a major advance to come
261 out of *Cassini* gravity science (Iess et al., 2019; Galanti et al., 2019), echoing a similar
262 discovery at Jupiter by the *Juno* spacecraft reported only months earlier (Kaspi et al.,
263 2018; Guillot et al., 2018). It had been understood for some time that the electrically
264 conductive deep interiors of both planets—for Saturn, roughly the inner half by radius—
265 should be kept rigidly rotating by electromagnetic forces. What has not become clear

266 until recently is how the interior flows are organized between that rigid fluid metallic in-
 267 terior and the east-west zonal flows apparent on Saturn’s surface: are the surface flows
 268 a shallow atmospheric phenomenon, or are they deep-seated? Structure in Saturn’s grav-
 269 ity field as observed at the end of the *Cassini* mission has rapidly shed light on this ques-
 270 tion, showing that the east-west zonal winds evident on Saturn’s surface indeed pene-
 271 trate to significant depth in the interior, to approximately 9,000 km—15% of the planet’s
 272 radius—below the surface (Iess et al., 2019; Galanti et al., 2019). Such a deep flow pat-
 273 tern must indelibly alter the frequencies of the fundamental mode oscillations, an effect
 274 studied by Marley and Porco (1993) but one that has yet to be considered in the detailed
 275 numerical calculations used to interpret the glut of mode frequencies now available. No-
 276 tably, the fundamental modes present in the data have angular degrees covering almost
 277 all values from $\ell = 2$ to $\ell = 14$, meaning that they probe a wide range of depths in
 278 Saturn and thus, when taken together, offer a sensitive handle on the differential rota-
 279 tion. Realizing this potential will require the kind of theoretical improvements described
 280 above to accurately account for Saturn’s rapid rotation, an endeavor that will enable an
 281 independent confirmation of the rotation profiles derived from gravity science. Of course,
 282 in pursuing this brand-new line of observational evidence, there is also the potential to
 283 uncover surprises.

284 **3 Conclusions & Outlook**

285 The frequencies of 21 normal modes of oscillation in Saturn have been measured
 286 from waves in high-resolution profiles of ring-occulted starlight.

287 Seven of these modes (those with $m = 2$ and $m = 3$) appear to be rooted in mixed
 288 gravity-fundamental modes. Their gravity mode character requires that a significant frac-
 289 tion of Saturn’s deep interior—potentially most of the inner half by radius—is stabilized
 290 against convection by composition gradients. The detection of these modes is the strongest
 291 evidence to date that the fluid envelope of Saturn is not fully convective, supporting in-
 292 dependent indications from *Cassini* magnetic data. This conclusion fundamentally al-
 293 ters the picture of the planet’s deep interior. These modes are the most direct probes
 294 of the deepest inner workings of Saturn yet available, and a quantitative understanding
 295 of the deep distributions of hydrogen, helium, rocks and ices—of central importance to
 296 formation models—awaits the systematic application of more realistic interior models
 297 to the seismology data.

298 The remaining modes (those with $m \geq 4$) correspond to pure fundamental modes.
 299 They carry less information about Saturn’s interior structure and more about its rota-
 300 tion profile, allowing the first seismological measurement of Saturn’s bulk rotation rate.
 301 The current data will provide stringent constraints on differential rotation within Sat-
 302 urn, but only after the theory is extended to more accurately treat Saturn’s rapid ro-
 303 tation, including its dependence on depth and latitude within the planet.

304 Summarizing, the current moment leaves a few important gaps to be bridged:

- 305 1. Theoretical Saturn mode frequencies computed so far are imprecise, while the ob-
 306 served frequencies are extremely precise.
- 307 2. The low- m mixed modes and high- m fundamental modes have yet to be addressed
 308 jointly in a single Saturn model.
- 309 3. The seismology, gravity, and magnetic data from *Cassini* have not been addressed
 310 jointly in a single Saturn model. For a start, the normal mode eigenfrequencies
 311 and zonal gravity harmonics should be retrieved on simultaneously to provide better-
 312 constrained Saturn interior models. This will significantly diminish degeneracies
 313 inherent to each dataset taken in isolation.

314 Finally, the most basic puzzle that remains is how normal mode oscillations in gi-
 315 ant planets are excited in the first place. Turbulent convection, the mechanism power-
 316 ing the solar oscillations, is almost certainly ineffective in the dimmer Jupiter and Sat-
 317 urn. Some imaginative ideas have appealed to rock storms (Markham & Stevenson, 2018)
 318 and ancient giant impacts (Wu & Lithwick, 2019), but neither theory provides a com-
 319 pletely satisfactory fit to the Saturn ring wave amplitudes reported by Hedman et al. (2019).
 320 In this arena as in the others, it appears that theory has some catching up to do.

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 323 the aid of code by Keaton Bell.

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