lated that branching ratio to be 9.5×10^{-4}.[4] The agreement is reasonable.

A total of 5 radiative τ^+ decays have been reported by other experimenters: Daniel and Pal[5] report one event with γ energy 30.2 ± 1.5 MeV; O'Halloran *et al.*[6] report one event with γ energy 32.6 ± 1.0 MeV from ~3000 τ decays; and Puschel *et al.*[7] report three events with γ energies of 14.7 ± 0.5, 10.3 ± 0.6, and 10.5 ± 5.2 MeV from 1389 τ decays.

[4] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).
[5] R. R. Daniel and Yash Pal, Proc. Indian Acad. Sci. **A40**, 114 (1954) and Yash Pal, in *Proceedings of the Fifth Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1955).
[6] T. O'Halloran, G. Goldhaber, and S. Goldhaber, Bull. Am. Phys. Soc. **6**, 509 (1961).
[7] W. Puschel, J. Tietge, D. Monti, G. Giacomelli, and A. Quareni Vignudelli, Phys. Letters **2**, 96 (1962).

TABLE I. Data on the radiative τ events. The $T_{i\pm}$ are the energies of the positive and negative secondary pions. E_γ and p_γ are the calculated energy and momentum of the γ. ϕ is the coplanarity measure defined in the text.

Event No.	T_{1+} (MeV)	T_{2+} (MeV)	T_{3-} (MeV)	$\Sigma T_{i\pm}$ (MeV)	$10^2\phi$	E_γ (MeV)	p_γ (MeV/c)
567	4.8	21.6	42.4	68.8	0.66	6.3 ± 1.5	9.1 ± 3.0
1866	13.1	20.5	23.7	57.3	5.6	17.8 ± 1.3	21.4 ± 3.2
1886	5.0	35.3	20.6	60.9	7.1	14.2 ± 1.4	13.0 ± 3.3
1903	3.5	28.5	29.7	61.7	3.4	13.4 ± 1.4	14.6 ± 4.3
1937	16.1	27.3	20.2	63.7	6.2	11.4 ± 1.4	8.6 ± 5.5
2146	9.3	13.8	18.2	41.3	33	34.0 ± 0.9	31.6 ± 1.4
2190	7.6	27.3	12.7	47.6	8.2	27.5 ± 1.0	28.1 ± 4.2

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Parity Conservation and Bootstraps*

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It is suggested that parity conservation in the strong interactions may be a consequence of the bootstrap hypothesis. A model is presented to illustrate how this might come about.

I. INTRODUCTION

In an earlier epoch, the invariance of interactions under space reflection was, like their invariance under other space-time transformations, "obvious" because the transformation related observers in equivalent inertial frames. The work of Yang and Lee clarified the reliability of such arguments, leaving the true origin of parity conservation in strong and electromagnetic interactions obscure.

We suggest that the invariance of the strong interactions under parity, like isospin invariance and other internal symmetries, be viewed as a dynamical consequeuce of the bootstrap philosophy. Thus, parity conservation should follow as a consequence of a sufficiently accurate dynamical calculation (since it seems to be valid in nature), rather than be put in as an initial hypothesis. On the other hand, parity nonconservation might well emerge from an inaccurate calculation. To put this suggestion into context, we recall three basic features of the bootstrap philosophy, as they apply to internal symmetries[1]:

(i) There is, first of all, the presumption—or faith—that "dynamics" exists. By dynamics, we mean a set of rules such that, given input data about a physical system, physical consequences can be calculated. The input data consist, presumably, of dimensionless quantities such as the numbers of particles of various spins, the ratios of their masses, interaction coupling constants, and whatever else might be necessary to define the system. The output will include, among other things, an enumeration of the number of bound states produced, their spins, the ratios of their masses, and, in general, other numbers of the same character as the input numbers.

(ii) The bootstrap principle is that the output num-

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[1] The connection between bootstraps and symmetries has, for example, been discussed by: R. Capps, Phys. Rev. Letters **10**, 312 (1963); R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963); E. Abers, F. Zachariasen, and C. Zemach, *ibid.* **132**, 1831 (1963); M. Baker and S. Glashow *ibid.* **128**, 2462 (1962), Sec. V [remark on the possibility of obtaining parity conservation from a restriction or vector function].

bers, as calculated by dynamics, must equal the input numbers. This principle produces a series of self-consistency equations, one solution of which, corresponding to the real universe, is presumed to exist. If alternative solutions corresponding to other universes also exist, at least their number will be severely limited.

It is important to keep carefully in mind the distinction between the bootstrap philosophy itself and the exceedingly crude methods of performaing dynamical calculations which are presently used to implement and test it. The techniques available to us now do the same disservice to any theory, whether it be a bootstrap theory or a Lagrangian field theory with elementary particles. What this means is simply that testing the bootstrap hypothesis to see if it agrees with nature is as difficult as testing individual Lagrangian field theories of strong interactions has always been. The bootstrap hypothesis is not tied to some particular method, that is, to some particular way of calculating the dynamics of strongly interacting particles, and is not automatically discredited by the failure of the approximation method.

(iii) When the self-consistent solution contains, as nature does, a multiplicity of particles of like spin, their masses and interaction strengths may well be equal because the determining equations possess symmetry with regard to them. This is the dynamical origin of internal symmetries.[1]

Parity conservation fits naturally into this program. The effect of parity conservation is simply that a system with a certain orbital angular momentum may not couple to a system with a certain different orbital angular momentum, even though the total angular momentum is the same in both cases. But since couplings are determined dynamically, the fact that certain classes of couplings vanish must also emerge dynamically. The problem is to understand why this can happen.

The remaining space-time symmetries are charge conjugation and time reversal, related to P by CPT. The general arguments on the origin of P apply equally well to C and T.

In the next section, a simple bootstrap model of pion-nucleon interactions is treated by the N/D method. The system of spin 0 plus spin $\frac{1}{2}$ is a natural one to study in a first exploration of parity conservation. States of total spin j may have orbital angular momentum $l=j+\frac{1}{2}$ and $l=j-\frac{1}{2}$ which are coupled or not depending on parity conservation. The only simpler system is spin zero plus spin zero which has one state for each j and does not need a subclassification by parity. A model based on a world containing no particles with spin higher than zero would seem to have little to say about parity, since in such a world there is, without the assumption of parity conservation, no feature by which the bootstrap mechanism could distinguish between scalar and pseudoscalar particles.

The model cannot *prove* that parity conservation in the strong interactions follows from our mechanism. Its purpose is to convince the reader that such an explanation is plausible and, in fact, attractive. Moreover, it emphasizes that the ultimate question of parity conservation, or invariance under any internal symmetry, depends on values of kinematical factors or integrals which cannot be predicted in advance. There do emerge, however, certain quantitative features related to the signs of the forces and the range of an exchange interaction which are not especially model dependent. These may be of significance in some future rigorous calculation. Finally, the model makes it clear that the parity-conserving situation does *not* arise in the bootstrap as a sort of continuous limit of a parity-violating one. Thus, one would not necessarily expect a small change in the parameters of the parity-conserving solution to generate a small amount of parity violation.

The general $\pi\bar{N}N$ interaction is taken as[2]

$$(4\pi)^{1/2}\bar{N}(a+b\gamma_5)N\pi. \tag{1.1}$$

The nucleon should appear as a bound state in both the $S_{1/2}$ and $P_{1/2}$ waves of $\pi+N$. The constants a, b are determined by the bootstrap principle. Parity conservation, if valid, will be expressed by

$$ab=0.$$

We consider two cases: (a) π and N are simple particles (no isospin); and (b) π and N are isovector and isospinor, respectively. The difference between the two cases lies solely in the sign of the force. In the first case, a parity-conserving self-consistent scheme can be found if the force is strong enough, but a parity-violating scheme does not exist. The second case has the feature that if the force is sufficiently strong, there are two bootstrap solutions, one conserving parity and one violating parity.

Our model does not test C since (1.1) is automatically C invariant when a and b are real, as they must be if the interaction is to be Hermitian. A model which could is the bootstrap of a spin $\frac{1}{2}$ nucleon as a bound state of itself and a spin-1 meson. Such a model would be similar to the one we are discussing here. An example of a model which would test invariance under C, P, and isospin separately is the bootstrap of charged and neutral pions as composites of 3π and 4π states (and 2π states also, if you like).[3]

We have no comments about parity conservation in electromagnetism.

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[2] Our γ_5 is anti-Hermitian. We follow the notation of Ref. 5. Note that a and b in Eq. (1.1) must be real in order that the interaction be Hermitian.

[3] It is not sufficient to couple π to 2π and 3π states alone, for if P conservation eliminates the π to 2π coupling, then G parity is conserved and C will follow from isospin conservation.

II. MODEL

Although our primary purpose in setting up a model for the study of parity conservation is just to show that it can be done, we may get some additional insight if our choice of parameters is guided by actual physical considerations. Accordingly, we shall simulate the problem of building a composite nucleon out of $\pi+N$ with the force given by the nucleon exchange diagram. We already know[4,5] that a parity-conserving bootstrap is possible with isospinor nucleons and isovector pions if both N and $N^*(1238)$ exchanges are used; the N exchange force is too weak to do the job by itself. For the sake of simplicity, we will ignore the extra force, but the kinematical constants may be allowed to vary in such a way as to increase the effective strength of the N exchange force.

The actual magnitude of the force at threshold energy is indicated by Eq. (A13) and Eq. (A14) of Appendix A, for values of the mass ratio in the interval $0<\mu/m\leq 1$. In no case would we consider $\mu>2m$ because the pion would be unstable.

The range of possibilities can be approximated by the one-pole formula

$$t^{\text{Born}}=\frac{R}{s-s_p}\begin{pmatrix} a^2-\bar{b}^2 & a\bar{b}(q/q_0) \\ a\bar{b}(q/q_0) & -(q/q_0)^2(a^2+y\bar{b}^2) \end{pmatrix}, \qquad (2.1)$$

where b^2 has been replaced by $\bar{b}^2=\mu b^2/(2m)$. (We shall not worry about using the variable $\sqrt{s}=W$ rather than s for this system.) For the output, we take

$$t^{\text{out}}=-\frac{C}{s-m^2}\begin{pmatrix} a^2 & a\bar{b}(q/q_0) \\ a\bar{b}(q/q_0) & \bar{b}^2(q/q_0)^2 \end{pmatrix}. \qquad (2.2)$$

The value of y ranges from about $\frac{1}{2}$ for $m\gg\mu$ to about $1/23$ for $m\approx\mu$. q_0 is roughly $(\mu m)^{1/2}$. For flexibility, we suppose R, C, may take on any positive values.

First we consider the case of no isospin. The relevant mathematics is worked out in Appendices A and B. A parity-conserving bootstrap with $a=0$, $\bar{b}\neq 0$ is impossible because the force is repulsive in the $P_{1/2}$ state. By (B.27), a parity-conserving bootstrap with $a\neq 0$, $\bar{b}=0$ does work if

$$1<R/C<\infty. \qquad (2.3)$$

Is a parity-violating solution possible? Applying the prescription of Appendix B with $r_x=R$, $z=1$, $R_{11}=R(a^2-\bar{b}^2)$, $R_{22}=R(-a^2-y\bar{b}^2)$, and noting (B.15) and (B.16), we infer

$$(Ra^2-R_{11})/(R\bar{b}^2-R_{22})=J_2/J_1, \qquad (2.4)$$

which leads to

$$(a^2/\bar{b}^2)=[J_1-(1+y)J_2]/J_2. \qquad (2.5)$$

But we now observe that $J_2>J_1$. It follows that either a^2 or $\bar{b}^2$ is predicted to be negative. That is, the parity-nonconserving bootstrap necessarily *fails*.

The reason why $J_2>J_1$ can be expressed in general terms. The J_2 integral contains the factor q/q_0 where q_0^{-1} represents the range of the force. The force in question is of the exchange type (namely, a pole in the u channel) and its range, as already indicated, is $q_0^{-1}\sim(\mu m)^{-1/2}$. This is in contrast to the range of a direct force (a pole in the t channel) which would be $q_0^{-1}\sim m^{-1}$ in this case. But the integrands of J_1, J_2 do not cut off until $q\approx m$. Therefore, the factor q/q_0 makes $J_2>J_1$. Now *any* bootstrap in which one particle is considered to be a bound state of itself and another will have an exchange-type force contributing to the interaction. If this force dominates, it is tempting to speculate that this fact plays a role in guaranteeing parity conservation. The argument is admittedly fragile.

It is worth emphasizing that the conserving and nonconserving solutions come from completely different conditions, (B.13) and (B.14). If we make a slight perturbation in the dynamics, J_1 and J_2 will be slightly changed, but it does *not* follow that the system acquires a slight amount of parity breaking.

Now we go to the isospin case. The formulation is the same except that t^{Born} changes sign. There is no bootstrap with $a\neq 0$, $\bar{b}=0$ because the force is repulsive in the $S_{1/2}$ channel. There will be a bootstrap with $a=0$, $\bar{b}\neq 0$ if

$$1<Ry/C<2. \qquad (2.6)$$

Again we ask if there can be a nonconserving bootstrap. The reasoning which led to (2.5) now yields

$$(a^2/\bar{b}^2)=[J_1+(1-y)J_2]/(2J_1+J_2). \qquad (2.7)$$

The ratio is now positive. If one pursues the rest of the problem out to its end, one obtains R/C as the ratio of two polynomials in α, where α is defined in (B.22). This equation will have a solution for α, with $1<\alpha<\infty$, and hence there will be a bootstrap solution, provided

$$1\lesssim R/C<\infty. \qquad (2.8)$$

For the isospin case, then, we have two solutions of the bootstrap: one with parity conservation and one without.

Finally, then, what may we conclude from these models? Presumably two things: First, parity conservation in the strong interactions may indeed be a consequence of the bootstrap hypothesis; second, however, whether or not this is the case depends on what world is being considered, and specifically on details of that world such as how many particles there are, what value various forces and kinematical factors have, and so on.

APPENDIX A

We review the partial-wave analysis for πN scattering, not assuming parity conservation, and ignoring isospin.

[4] G. F. Chew, Phys. Rev. Letters, 9, 233 (1962).
[5] E. Abers and C. Zemach, Phys. Rev. 131, 2305 (1963).

Let the S matrix be written

$$S=1+\frac{im}{2\pi E_\pi E_N}T\delta^4(P)\,, \tag{A.1}$$

with T defined by Feynman's rules and normalizations of $(2\pi)^{-3}$ particles per unit volume. Then T will have the form

$$T=\omega(p_4)[A+\tfrac{1}{2}B\gamma(p_1+p_3)+C\gamma_5]\omega(p_2)\,, \tag{A.2}$$

where we follow the customary notation.[5] The reduction to two-component spinors is

$$T=u_4^*[f_1+(\sigma\cdot\bar{n})(\sigma\cdot n)f_2+i(\sigma\cdot n-\sigma\cdot\bar{n})f_3]u_2\,, \tag{A.3}$$

where

$$f_1=((E+m)/2W)(A+(W-m)B)\,, \tag{A.4}$$

$$f_2=((E-m)2W)(-A+(W+m)B)\,, \tag{A.5}$$

$$f_3=(q/2W)C\,. \tag{A.6}$$

If the t matrix is mormalized to $e^{i\delta}\sin\delta/q$, we have, for spin j and channels labeled by orbital angular momentum,

$$t(j\pm\tfrac{1}{2}\to j\pm\tfrac{1}{2}) =\tfrac{1}{2}\int dx[f_1P_{j\pm1/2}(x)+f_2P_{j\mp1/2}(x)]\,, \tag{A.7}$$

$$t(j\pm\tfrac{1}{2}\to j\mp\tfrac{1}{2}) =\tfrac{1}{2}\int dx\, f_3[P_{j-1/2}(x)-P_{j+1/2}(x)]\,. \tag{A.8}$$

For the case at hand, the input t matrix is given by the nucleon exchange diagram. From this we find, with the couplings defined as in Eq. (1.1),

$$\begin{aligned} t_{11}\equiv t(S_{1/2}\to S_{1/2}) &=\frac{E+m}{4Wq^2}Q_0(x)[(W-3m)a^2+(W-m)b^2] \\ &\quad+\frac{E-m}{4Wq^2}Q_1(x)[(W+3m)a^2+(W+m)b^2]\,, \end{aligned} \tag{A.9}$$

$$t_{22}\equiv t(P_{1/2}\to P_{1/2})=t_{11} \text{ with } Q_0, Q_1 \text{ interchanged}\,, \tag{A.10}$$

$$t_{12}=t_{21}\equiv t(P_{1/2}\to S_{1/2}) =(m/2Wq)[Q_0(x)-Q_1(x)]ab\,. \tag{A.11}$$

Here, $x=1-(s-m^2-2\mu^2)/2q^2$. The output t matrix is given by the pole diagram. We are interested in the matrix elements only in the neighborhood of the nucleon pole.

$$t^{\text{out}}=-\frac{1}{W-m}\begin{pmatrix} a^2 & abq/2m \\ abq/2m & b^2q^2/4m^2 \end{pmatrix}. \tag{A.12}$$

We may estimate the magnitudes of the nucleon exchange matrix elements by evaluating them at threshold, $W\to m+\mu$. If we also set $\mu/m\to\infty$, this gives

$$t^{\text{Born}}\approx\frac{1}{\mu}\begin{pmatrix} a^2-(\mu/2m)b^2 & abq/(2m) \\ abq/(2m) & -(q^2/\mu m)[\tfrac{1}{3}a^2+(\mu/12m)b^2] \end{pmatrix}. \tag{A.13}$$

Alternatively, to get the other extreme, so to speak, we may put $\mu=m$; then,

$$t^{\text{Born}}=\frac{1}{\mu}\begin{pmatrix} \tfrac{1}{2}(a^2-b^2) & abq/(2\mu) \\ abq/(2\mu) & (-q^2/24\mu^2)(23a^2+b^2) \end{pmatrix}. \tag{A.14}$$

Equations (A.13) and (A.14) give us an idea of the signs of the forces in the different channels due to nucleon exchange and the limits of variation of their magnitudes for various values of μ/m.

APPENDIX B

We consider the dynamics of a coupled channel bootstrap in the single pole approximation to the N/D method. This is the most simplified version of bootstrap dynamics that may still have a qualitative similarity to a correct calculation. We begin with the Born t matrix:

$$t^{\text{Born}}(s)=\frac{1}{s-s_p}\begin{pmatrix} R_{11} & R_x(q/q_0) \\ R_x(q/q_0) & R_{22}(q/q_0)^2 \end{pmatrix} \tag{B.1}$$

with momentum factors characteristic of S-wave and P-wave channels. The R's are constants, and will depend on the coupling constants a, b. The full t matrix satisfies unitarity:

$$\text{Im}t_{ij}=t_{ik}t_{kj}^*q_k\theta(s-s_t)\,,\quad q=(s-s_t)^{1/2}\,, \tag{B.2}$$

where s_t is the threshold energy. We seek a bound state at $s=m^2$; $s_p<m^2<s_t$. This approach will have some approximate validity if m^2 is far from s_p; it is meaningless if $m^2\leq s_p$. At $s=m^2$, t must have the form of the output matrix

$$t^{\text{out}}=\frac{-C}{s-m^2}\begin{pmatrix} a^2 & zab(q/q_0) \\ zab(q/q_0) & z^2b^2(q/q_0)^2 \end{pmatrix}, \tag{B.3}$$

where C, z, q_0 are kinematic "constants."

First, divide out the momentum factors in the usual way, forming $\bar{t}^{\text{Born}}$ and $\bar{t}$, where

$$\bar{t}_{ij}^{\text{Born}}(s)=R_{ij}/(s-s_p)\,, \tag{B.4}$$

with $R_{12}=R_{21}=R_x$, and

$$\text{Im}\,\bar{t}_i=t_{ik}t_{kj}^*q_kC_k\theta(s-s_t)\,, \tag{B.5}$$

with $C_1=1$, $C_2=(q/q_0)^2$. Then $\bar{t}=ND^{-1}$ where, by well-known procedures,

$$N_{ij}=\bar{t}_{ij}^{\mathrm{Born}}, \tag{B.6}$$

$$D=\begin{pmatrix} 1-R_{11}J_1 & -R_xJ_1 \\ -R_xJ_2 & 1-R_{22}J_2 \end{pmatrix}, \tag{B.7}$$

$$\begin{aligned} J_1&=\frac{s-s_p}{\pi}\int_{s_t}^{\infty}\frac{q'ds'}{(s'-s)(s'-s_p)^2}, \\ J_2&=\frac{s-s_p}{\pi}\int_{s_t}^{\infty}\frac{q'(q'/q_0)^2ds'}{(s'-s)(s'-s_p)^2}. \end{aligned} \tag{B.8}$$

Let $\Delta(s)=\det D(s)$. The bootstrap requirement is that t reduce to t^{out} at $s=m^2$. This leads to four (mutually consistent) equations for the three unknowns m^2, a, b, namely,

$$\Delta(m^2)=1-R_{11}J_1-R_{22}J_2+J_1J_2\det R=0, \tag{B.9}$$

$$a^2=(R_{11}-J_2\det R)/[-C\Delta'(m^2-s_p)], \tag{B.10}$$

$$z^2b^2=(R_{22}-J_1\det R)/[-C\Delta'(m^2-s_p,)], \tag{B.11}$$

$$zab=R_x/[-C\Delta'(m^2-s_p)]. \tag{B.12}$$

Here, Δ' means the derivative of $\Delta(s)$ with respect to s evaluated at $s=m^2$, and the J's are all evaluated at $s=m^2$. In any model, R_x must be of the form $R_x=abr_x$ where r_x is not singular for $ab=0$; this is clear because the two channels are not coupled unless $ab\neq 0$. Then (B.12) might be satisfied in two quite different ways:

$$ab=0 \tag{B.13}$$

and

$$r_x=-zC\Delta'(m^2-s_p). \tag{B.14}$$

We temporarily put aside possibility (B.13), which corresponds to parity conservation, and follow up the implications of (B.14). In our model, r_x is independent of a, b, so that (B.9) and (B.10) can be simplified:

$$a^2=(z/r_x)(R_{11}-J_2\det R), \tag{B.15}$$

$$z^2b^2=(z/r_x)(R_{22}-J_1\det R). \tag{B.16}$$

By (B.9) and then (B.10) and (B.11),

$$\begin{aligned} \Delta'&=J_1'(-R_{11}+J_2\det R)+J_2'(-R_{22}+J_1\det R) \\ &=C\Delta'(m^2-s_p)[a^2J_1'+b^2z^2J_2'] \end{aligned} \tag{B.17}$$

from which the Δ' may be canceled.

Now the J's can be integrated explicity. We have

$$J_1=\frac{1}{(s_t-s_p)^{1/2}+(s_t-m^2)^{1/2}}-\frac{1}{2(s_t-s_p)^{1/2}}. \tag{B.18}$$

More important are the ratios

$$J_2/J_1=(s_t-s_p)/(q_0)^2(1+2/\alpha), \tag{B.19}$$

$$J_1'/J_1=\alpha/(m^2-s_p), \tag{B.20}$$

$$J_2'/J_1=\frac{s_t-s_p}{q_0^2}\frac{(2+1/\alpha)}{m^2-s_p}, \tag{B.21}$$

where

$$\alpha=\left(\frac{s_t-s_p}{s_t-m^2}\right)^{1/2}. \tag{B.22}$$

Then (B.17) becomes

$$\frac{1}{C}=\alpha J_1a^2+\frac{2\alpha+1}{\alpha+2}J_2b^2z^2. \tag{B.23}$$

The procedure for solving the bootstrap can now be simply stated: First, determine the kinematical constants r_x, C, z, q_0, s_t, s_p, and the dependence of R_{11}, R_{22} on a^2, b^2 from some theory or approximation. Second, solve (B.15) and (B.16) for a^2, b^2 in terms of J_1, J_2. The solutions will be ratios of polynomials in J_1, J_2. Third, substitute these solutions into (B.23) and use (B.19) to reduce the result to an equation for α involving only the kinematical constants. This will be a polynomial equation in α. Fourth, solve for α and obtain the self-consistent mass from (B.22). Finally, the values of a^2, b^2, already known in terms of J_1, J_2, can be calculated from (B.15), (B.16).

The permissible values for α corresponding to $s_p<m^2<s_t$ are $1<\alpha<\infty$. If the calculated value of α falls outside this range, or if one of a^2, b^2 is negative, the bootstrap fails. Otherwise, it succeeds and gives a self-consistent set of parameters violating parity conservation.

Finally, we return to the alternative (B.13). If $a\neq 0$, $b=0$, we have a single-channel problem with

$$t^{\mathrm{Born}}=R_{11}/(s-s_p)\equiv a^2R/(s-s_p) \tag{B.24}$$

and

$$t^{\mathrm{out}}=-Ca^2/(s-m^2). \tag{B.25}$$

Then α is determined to be

$$\alpha=R/C, \tag{B.26}$$

giving a successful bootstrap if

$$1<R/C<\infty. \tag{B.27}$$

If we take $a=0$, $b\neq 0$, $t^{\mathrm{Born}}=b^2R/(s-s_p)$, and the condition is found to be

$$\frac{2\alpha+1}{\alpha+2}=\frac{R}{Cz^2}, \tag{B.28}$$

which gives a bootstrap if

$$1<R/Cz^2<2. \tag{B.29}$$

If $R/Cz^2>2$, the force is so strong that $m^2<s_p$ and the single-pole model breaks down. A proper calculation might still yield a bootstrap.