

# Is the dependence of spectral index on luminosity real in optically selected AGN samples?

Su Min Tang,<sup>1,5★</sup> Shuang Nan Zhang<sup>1,2,3,4</sup> and Philip F. Hopkins<sup>5</sup>

<sup>1</sup>*Department of Physics and Centre for Astrophysics, Tsinghua University, Beijing 100084, China*

<sup>2</sup>*Department of Physics, University of Alabama in Huntsville, Optics Building 201C, Huntsville, AL 35899, USA*

<sup>3</sup>*Space Science Laboratory, NASA Marshall Space Flight Centre, SD50, Huntsville, AL 35812, USA*

<sup>4</sup>*Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918-3, Beijing 100039, China*

<sup>5</sup>*Harvard-Smithsonian Centre for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA*

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## ABSTRACT

We critically examine the dependence of spectral index on luminosity in optically selected AGN samples. An analysis of optically selected high- $z$  quasars showed an anticorrelation of  $\alpha_{\text{OX}}$ , the spectral index between the rest-frame 2500 Å and 2 keV, with optical luminosity. We examine this relationship by means of Monte Carlo simulations and conclude that a constant  $\alpha_{\text{OX}}$  independent of optical luminosity is still consistent with this high- $z$  sample. We further find that contributions of large dispersions and narrow range of optical luminosity are most important for the apparent, yet artificial,  $\alpha_{\text{OX}}-l_0$  correlation reported. We also examine another, but more complete, low- $z$  optical selected AGN sub-sample from Steffen et al., and our analysis shows that a constant  $\alpha_{\text{OX}}$  independent of optical luminosity is also consistent with the data. By comparing X-ray and optical luminosity functions, we find that a luminosity-independent  $\alpha_{\text{OX}}$  is in fact more preferred than the luminosity-dependent  $\alpha_{\text{OX}}$  model. We also discuss the selection effects caused by flux limits, which might systematically bias the  $l_X-l_0$  relation and cause discrepancy in optically selected and X-ray selected AGN samples. To correctly establish a dependence of  $\alpha_{\text{OX}}$  of AGNs on their luminosity, a larger and more complete sample is needed and consequences of luminosity dispersions and selection effects in flux-limited samples must be taken into account properly.

**Key words:** methods: statistical – galaxies: active – quasars: general – X-rays: galaxies.

## 1 INTRODUCTION

The dependence of the spectral index  $\alpha_{\text{OX}}$  of active galactic nucleus (AGN) on redshift and luminosity has important astrophysical implications on AGN evolution and thus has been studied for many years [e.g. Avni & Tananbaum 1982; Wilkes et al. 1994; Green et al. 1995; Bechtold et al. 2003; Vignali, Brandt & Schneider 2003a; Strateva et al. 2005; Steffen et al. 2006 (hereafter S06); Hopkins, Richards & Hernquist 2007; Kelly et al. 2007].  $\alpha_{\text{OX}}$  is defined as

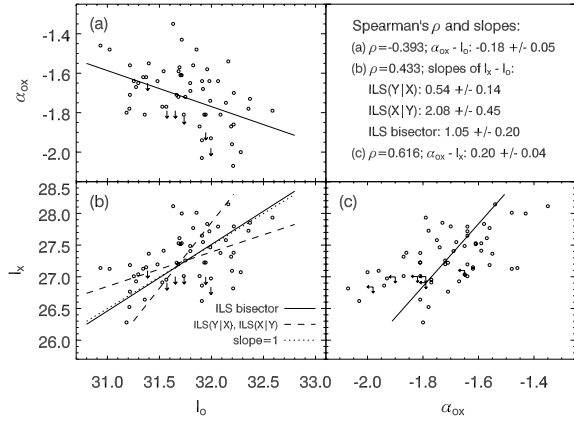
$$\alpha_{\text{OX}} = \frac{\log(f_{2\text{ keV}}/f_{2500\text{ Å}})}{\log(\nu_{2\text{ keV}}/\nu_{2500\text{ Å}})}, \quad (1)$$

where  $f_{2\text{ keV}}$  and  $f_{2500\text{ Å}}$  are the rest-frame flux densities at 2 keV and 2500 Å, respectively. Dependence of  $\alpha_{\text{OX}}$  on redshift means evolution of the accretion process in cosmic time. Most studies have concluded that there is no evidence for a dependence of  $\alpha_{\text{OX}}$  on redshift (e.g. Avni & Tananbaum 1982; Strateva et al. 2005), although some studies found that  $\alpha_{\text{OX}}$  is correlated with redshift (Bechtold et al. 2003; Kelly et al. 2007). Dependence of  $\alpha_{\text{OX}}$  on luminosity means a non-linear relationship between X-ray and optical lumi-

osity ( $L_X \propto L_0^e$ ,  $e \neq 1$ ), which provides insight into the radiation mechanism. An anticorrelation between  $\alpha_{\text{OX}}$  and the optical luminosity has been found by many authors in optically selected AGNs with follow-up X-ray measurements at some different epoch, which means that these AGNs span a larger range in optical luminosity than in X-ray luminosity (e.g. Vignali et al. 2003a; Strateva et al. 2005; Miyaji et al. 2006, hereafter M06; Strateva et al. 2006, hereafter S06). However, La Franca et al. (1995) found that a  $e = 1$  relationship is consistent with data when the photometric errors are taken into account, and Gaskell et al. (2004) suggested that the apparent dependence of  $\alpha_{\text{OX}}$  on luminosity is due to the dependence of the reddening on luminosity. Meanwhile, whether  $\alpha_{\text{OX}}$  depends on luminosity in X-ray selected AGN samples remains unknown (Hasinger 2005; Frank, Osmer & Mathur 2007).

As pointed out by Yuan, Siebert & Brinkmann (1998), one of the problems in such studies is that an apparent, yet artificial correlation between  $\alpha_{\text{OX}}$  and optical luminosity can be caused by dispersions in the optical luminosity. In Section 2, we present analysis of the relationship between  $\alpha_{\text{OX}}$  and optical/X-ray luminosity using data presented in M06, in which we find that dispersions in luminosity can be entirely responsible for the claimed dependence. We also discuss the determining factor for this behaviour.

★E-mail: stang@cfa.harvard.edu (SMT); zhangsn@tsinghua.edu.cn (SNZ)



**Figure 1.** 61 high- $z$  quasars in M06. Circles indicate X-ray detected quasars, while arrows indicate upper limits. (Panel a)  $\alpha_{\text{OX}}$  dependence on the rest-frame 2500 Å monochromatic luminosity. The solid line indicates linear regression results from the EM Algorithm in ASURV. (Panel b) Rest-frame 2 keV monochromatic luminosity against 2500 Å one. The solid line indicates the ILS bisector result, and dashed lines indicate ILS(Y|X) and ILS(X|Y) results from the EM Algorithm in ASURV, respectively. The  $e = 1$  relation is shown by dotted line for comparison. (Panel c)  $\alpha_{\text{OX}}$  dependence on the rest-frame 2 keV monochromatic luminosity. The solid line indicates linear regression result using X-ray detected quasars. Spearman correlation coefficients and slopes of fitting lines are indicated in the upper right panel.

Another problem is the degeneracy between redshift and luminosity in flux-limited samples, where redshift and luminosity are strongly correlated. In Section 3, we examine a sub-sample from S06 containing 187 AGNs, which more completely fills the redshift and optical luminosity plane and thus is less affected by such degeneracy. In Section 4, we compare the optical quasar luminosity function from Richards et al. (2006) with X-ray quasar luminosity function from Barger et al. (2005) in different  $\alpha_{\text{OX}}$  models. In Section 5, we discuss the selection effects in flux-limited samples and the consequent discrepancy in optically selected and X-ray selected samples. Discussion and conclusions are presented in Section 6.

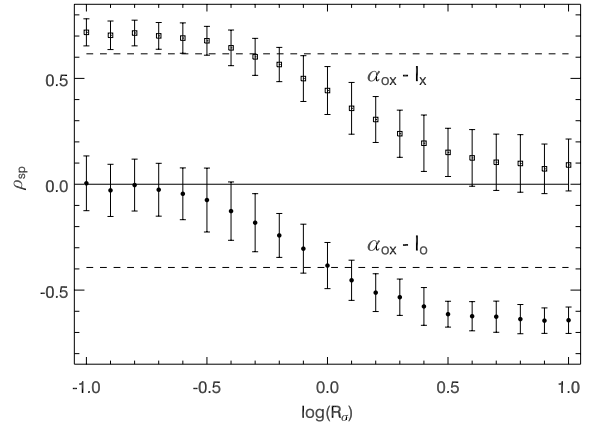
We mostly use the logarithms of luminosities and denote them as  $l_X = \log L_{2\text{keV}}$  and  $l_0 = \log L_{2500}$ , then  $\alpha_{\text{OX}} = 0.3838(l_X - l_0)$ , where  $L_{2\text{keV}}$  is the 2 keV monochromatic luminosity and  $L_{2500}$  is the 2500 Å monochromatic luminosity in units of  $\text{erg s}^{-1} \text{Hz}^{-1}$ . We adopt the currently favoured cosmology model with  $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$ ,  $\Omega_M = 0.3$  and  $\Omega_\Lambda = 0.7$  (e.g. Spergel et al. 2007).

## 2 ANALYSIS OF MIYAJI ET AL. (2006) SAMPLE

### 2.1 Data analysis

The sample we use in this section consisted of 61 high- $z$  ( $z > 2.9$ ) quasars in Fig. 2 of M06 (Vignali et al. 2003b, 2005), excluding the three quasars from archival *Chandra* data in table 3 in M06 which might be biased towards higher X-ray fluxes. Only six of them have no X-ray detection. M06 found a correlation of  $\alpha_{\text{OX}}$  with optical luminosity for this high- $z$  sample, while they kept the discussion on the  $l_0$ - $\alpha_{\text{OX}}$  relation open because of possible optical selection effects for variable AGNs which preferentially pick up the optically brighter phases.

To illustrate how dispersions produce artificial correlations in this sample, we carry out two independent analyses. The first one is linear regression of  $\alpha_{\text{OX}}$ ,  $l_0$  and  $l_X$  in observed data. Without further



**Figure 2.** Spearman rank correlation coefficients  $\rho_{\text{sp}}$  as a measure of the  $\alpha_{\text{OX}} - l_0$  and  $\alpha_{\text{OX}} - l_X$  correlation for Miyaji's sample and simulated samples. Open squares:  $\alpha_{\text{OX}} - l_X$  in simulated samples; solid circles:  $\alpha_{\text{OX}} - l_0$  in simulated samples; upper dashed line:  $\alpha_{\text{OX}} - l_X$  of true data; lower dashed line:  $\alpha_{\text{OX}} - l_0$  of true data; solid line indicates no correlation.

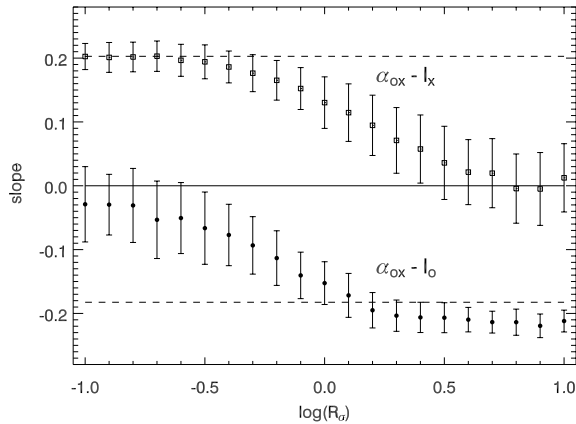
description, we perform linear regression using methods as follows throughout the paper.

(i) For the  $\alpha_{\text{OX}} - l_0$  correlation, we use the EM algorithm in ASURV (Isobe, Feigelson & Nelson 1986) to derive linear regression parameters, including X-ray undetected quasars.

(ii) The EM linear regression algorithm in ASURV is based on the traditional ordinary least-squares method which minimizes the residuals of the dependent variable (OLS(Y|X)). However, for  $l_X - l_0$  correlation, both variables are observed and a different result can be obtained if residuals of the independent variable are instead minimized (e.g. S06). Following S06, we perform linear regression with ASURV using EM algorithm, treating  $l_X$  as the dependent variables (ILS(Y|X)) and treating  $l_0$  as the dependent variables (ILS(X|Y)), then use the equations given by Isobe et al. (1990) to calculate the bisector of the two regression lines.

(iii) For the  $\alpha_{\text{OX}} - l_X$ , where both independent and dependent variables are upper limits, only Schmitt's binned method in ASURV is available which may suffer from several drawbacks (Sadler, Jenkins & Kotanyi 1989). Hence, we abandon upper limit points in  $\alpha_{\text{OX}} - l_X$  plane and only use X-ray detected quasars to derive linear regression parameters.

Spearman correlation coefficients are calculated using ASURV including X-ray undetected quasars. Observational data together with linear regression slopes and Spearman correlation coefficients for  $\alpha_{\text{OX}} - l_0$ ,  $l_X - l_0$  and  $\alpha_{\text{OX}} - l_X$  correlations are shown in Fig. 1. Conflicting correlations arise due to dispersions in luminosity:  $\alpha_{\text{OX}} = -0.18 l_0 + \text{const}$  as shown in solid line in Panel (a), but  $\alpha_{\text{OX}} = 0.20 l_X + \text{const}$  as shown in solid line in Panel (c). Therefore, the same data produce two totally different results:  $l_X = 0.53 l_0 + \text{const}$  or  $l_X = 2.08 l_0 + \text{const}$ , i.e.  $e < 1$  or  $e > 1$  if  $L_X \propto L_0^e$ . Therefore, depending upon how the regression is done, the conclusion on the relationship between the optical and X-ray luminosities can be significantly different. As shown in Panel (b), the slope of the  $l_X - l_0$  relation does depend on which luminosity is used as the dependent variable. When treating  $l_X$  as the dependent variable, the slope is  $0.54 \pm 0.14$  (the flatter dashed line), while it changes dramatically to  $2.08 \pm 0.45$  (the steeper dashed line) when treating  $l_0$  as the dependent variable. Using ILS bisector, we find the slope to be  $1.05 \pm 0.20$  (solid line), which is consistent with  $e = 1$  (dotted line).



**Figure 3.** Slopes of the  $\alpha_{\text{OX}}-l_o$  and  $\alpha_{\text{OX}}-l_X$  correlation for Miyaji's sample and simulated samples. Symbols and lines are as in Fig. 2.

The second part of the analysis is done with Monte Carlo simulations. Following Yuan et al. (1998), we assume intrinsic optical and X-ray luminosities  $\bar{l}_o$  and  $\bar{l}_X$  with a constant mean  $\bar{\alpha}_{\text{OX}}$ ,

$$\bar{l}_X = \bar{l}_o + \bar{\alpha}_{\text{OX}}/0.3838, \quad (2)$$

and  $\bar{\alpha}_{\text{OX}} = -1.728$  is given by the mean of observed  $0.3838(l_X - l_o)$  using the Kaplan–Meier estimator in ASURV, including the six quasars with upper limits. The above relationship is plotted as dotted line in Panel (b) of Fig. 1. The observed optical and X-ray luminosities are assumed to be the intrinsic luminosities modified by independent Gaussian dispersions

$$l_o = \bar{l}_o + \delta l_o, \quad l_X = \bar{l}_X + \delta l_X = \bar{l}_o + \bar{\alpha}_{\text{OX}}/0.3838 + \delta l_X, \quad (3)$$

where  $\delta l_o$  and  $\delta l_X$  are Gaussian-distributed dispersions with standard deviations  $\sigma_o$  and  $\sigma_X$ , respectively. Thus, the distribution of  $\alpha_{\text{OX}}$  is Gaussian with standard deviation

$$\sigma_{\alpha_{\text{OX}}} = 0.3838(\sigma_o^2 + \sigma_X^2)^{1/2}, \quad (4)$$

where  $\sigma_{\alpha_{\text{OX}}} = 0.154$  is the standard deviation around the linear relationship of equation (2) for this sample, using 55 X-ray detected AGNs. The ratio of the standard deviations of the optical to the X-ray luminosity dispersion is defined as

$$R_\sigma = \frac{\sigma_o}{\sigma_X}. \quad (5)$$

In the following, we make Monte Carlo simulations by considering 21 values of  $R_\sigma$  from 0.1 to 10, sampled evenly on a logarithmic scale. We use the observed optical luminosity as  $\bar{l}_o$  and keep redshift unchanged. Then  $l_o, l_X$  and fluxes can be determined using equations (3)–(5). The X-ray flux limit is determined as follows. Quasars in this sample are from different observations and thus not uniformly sampled. All quasars with  $z \leq 3.5$  are X-ray detected. In the range of  $z > 3.5$ , six quasars are not X-ray detected. Five of them, i.e. SDSS 1737+5828, PSS 1435+3057, SDSS 1532–0039, PSS 1506+5220 and PSS 2344+0342, were observed by *Chandra* with exposure times from 2.61 to 5.1 ks; SDSS 0338+0021 was observed by *XMM–Newton* with exposure time 5.49 ks. PSS 1506+5220 has one count in 0.5–2 keV, and all the other five have zero counts. All the other detected sources have counts larger than 1. The average rest-frame  $f_{2\text{keV}}$  flux for one count in 0.5–2 keV of the six quasars is  $0.8 \times 10^{-32} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$ . So we simply put a flux limit of  $f_{2\text{keV, upper}} = 1.5 \times 0.8 \times 10^{-32} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$ . We assume that simulated quasars with  $z > 3.5$  and  $f_{2\text{keV}}$  less than  $f_{2\text{keV, upper}}$  will not be detected: quasars with  $0 \leq f_{2\text{keV}} < 0.5f_{2\text{keV, upper}}$  will

be assigned zero count, and quasars with  $0.5f_{2\text{keV, upper}} \leq f_{2\text{keV}} < 1.5f_{2\text{keV, upper}}$  will be assigned 1 count. Then the upper limits of non-detected quasars are at the 95 per cent confidence level and will be calculated according to Kraft, Burrows & Nousek (1991), assuming one count corresponds to a flux of  $0.8 \times 10^{-32} \text{ erg cm}^{-2} \text{ s}^{-1}$ .

Then we simulate 100 samples for each of 21 different  $R_\sigma$  values. For each  $R_\sigma$ , we compute the average slopes and Spearman correlation coefficients  $\rho_{\text{sp}}$  from the 100 simulated samples and display the results in Figs 2 and 3. Parameters for simulated samples are calculated in the same way as for Panels (a) and (c) in Fig. 1. As discussed in Strateva et al. (2005), the measurement errors and variability effects bring  $\sigma_X \sim 0.23$  and  $\sigma_o \sim 0.17$ , and  $\sqrt{\sigma_X^2 + \sigma_o^2} \sim 0.29$ . The observed dispersion is 0.4 for the M06 sample and around 0.35–0.4 for other samples. The extra dispersions could be assigned to either  $\sigma_X$  or  $\sigma_o$  as unknown dispersions, so the possible range of  $R_\sigma$  should be  $0.5 \sim 1.4$ , i.e.  $\log(R_\sigma) \sim -0.3$ – $0.15$ . In the range of  $\log(R_\sigma) \sim -0.2$ – $0$ ,  $\rho_{\text{sp}}$  and slopes in simulated samples for both  $\alpha_{\text{OX}}-l_X$  and  $\alpha_{\text{OX}}-l_o$  correlations are consistent with observed values within  $1.5\sigma$ , as shown in Figs 2 and 3.

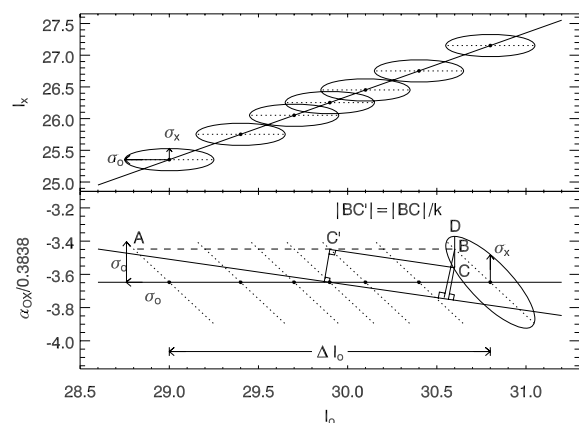
From the above two analyses, we conclude that a constant  $\alpha_{\text{OX}}$  which does not depend on optical luminosity is still consistent with data in this high- $z$  sample.

## 2.2 Determining factor for an artificial correlation caused by luminosity dispersions

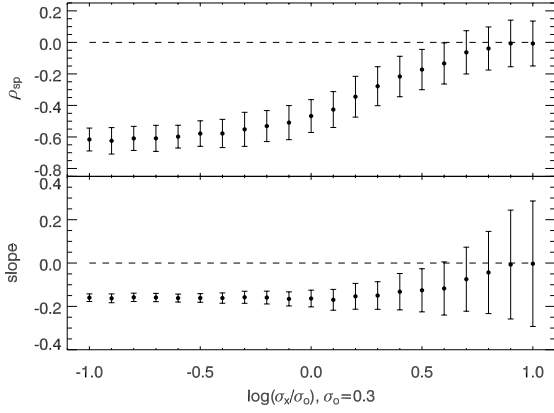
Yuan et al. (1998) pointed out that a dispersion larger for the optical luminosity than for the X-ray luminosity tends to result in apparent, yet artificial correlation of  $\alpha_{\text{OX}}-l_o$ . To quantitatively examine the effect of dispersions on the artificial correlation, we make a simple analytic calculation as follows.

As shown in Fig. 4, assuming  $l_X = l_o + \text{const}$ , dispersions in  $l_X$  and  $l_o$  are  $\sigma_X$  and  $\sigma_o$ , respectively, and the span range of  $l_o$  is  $\Delta l_o$ . Then locations of AGNs in the  $\alpha_{\text{OX}}-l_o$  plane with average  $l_X$  and  $l_o$  range of  $l_o$  are indicated by the dotted lines in the lower panel, where an apparent correlation appears. We only consider the simplest situation:

- (i) AGNs are evenly distributed along  $l_o$  (A concentration around a central  $l_o$  is equivalent to a smaller  $\Delta l_o$ ).
- (ii) AGNs are only distributed  $0.3838f\sigma_o$  away from  $\bar{\alpha}_{\text{OX}}$ , i.e.  $\alpha_{\text{OX}} = \bar{\alpha}_{\text{OX}} \pm 0.3838f\sigma_o$ , where  $f$  is an unknown positive coefficient to be determined. We will discuss the value of  $f$  later.



**Figure 4.** Schematic sketches for the  $l_X-l_o$  and  $\alpha_{\text{OX}}-l_o$  relationship. Here,  $l_X = l_o + \bar{\alpha}_{\text{OX}}/0.3838$  and  $\bar{\alpha}_{\text{OX}} = -1.4$  (see Section 2.2 for details).



**Figure 5.** Spearman rank correlation coefficients  $\rho_{\text{sp}}$  and best-fitting slopes of the  $\alpha_{\text{OX}}-l_0$  correlation for simulated samples for various  $\sigma_X$ , with constant  $\sigma_o = 0.3$  and  $\bar{\alpha}_{\text{OX}} = -1.729$ .  $l_0$  and redshift distributions are from Miyaji et al. (2006).

Then, we fit the observed AGNs in the  $\alpha_{\text{OX}} - l_0$  plane, assuming a least chi-squared fitting procedure with same weight in  $l_0$  and  $\sigma_o$  (i.e.  $\alpha_{\text{OX}}/0.3838$ ) as

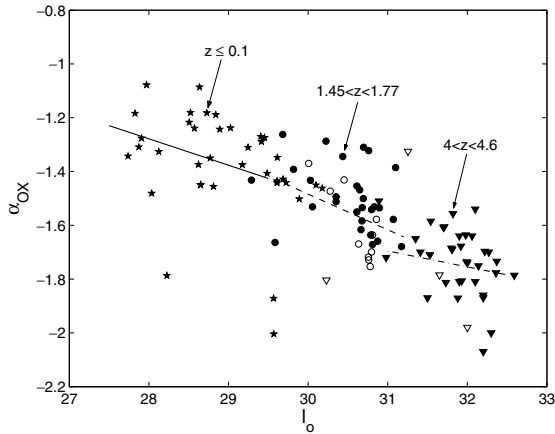
$$\chi^2 = \left\{ \sum_i [l_0(i) - \hat{l}_0(i)]^2 + [\alpha_{\text{OX}}(i) - \hat{\alpha}_{\text{OX}}(i)]^2 / 0.3838^2 \right\} / \sigma_c^2, \quad (6)$$

where the  $l_0(i)$  and  $\alpha_{\text{OX}}(i)$  are observed values,  $\hat{l}_0(i)$  and  $\hat{\alpha}_{\text{OX}}(i)$  are the values in the fitting line (indicated by a long solid line with a negative slope in the lower panel of Fig. 4) with the least  $[l_0(i) - \hat{l}_0(i)]^2 + (\alpha_{\text{OX}}(i) - \hat{\alpha}_{\text{OX}}(i))^2 / 0.3838^2$ , and  $\sigma_c$  is the typical constant error of  $l_0$ . Then,

$$\chi^2 = \frac{k^2}{3(1+k^2)} \left[ \frac{\Delta l_0^3}{4} + 3\Delta l_0 (f\sigma_o)^2 \left(1 + \frac{1}{k}\right)^2 \right] / \sigma_c^2, \quad (7)$$

where  $k$  is the slope of the fitting line. The best-fitting slope  $k$  can be derived by solving

$$\frac{d\chi^2}{dk} = 0, \quad \frac{d^2\chi^2}{dk^2} > 0. \quad (8)$$



**Figure 6.**  $\alpha_{\text{OX}}$  dependence on the rest-frame 2500 Å monochromatic luminosity for AGNs in three redshift bins in S06 sample:  $z \leq 0.1$  (stars),  $1.45 < z < 1.77$  (circles) and  $4 < z < 4.6$  (downward-pointing triangles). X-ray detected AGNs are represented using filled symbols while upper limits are represented using open symbols. The solid line, dashed line and dash-dotted lines indicate linear regression results from the EM Algorithm in ASURV for  $z \leq 0.1$ ,  $1.45 < z < 1.77$  and  $4 < z < 4.6$  samples, respectively.

The solution is

$$k_f = \frac{\Delta l_0^2}{24f^2\sigma_o^2} - \sqrt{\left(\frac{\Delta l_0^2}{24f^2\sigma_o^2}\right)^2 + 1} \quad (9)$$

$$\simeq -12 \left(\frac{f\sigma_o}{\Delta l_0}\right)^2. \quad (10)$$

The deviation of the approximation in equation (10) from equation (9) is less than 20 per cent when  $\frac{24f^2\sigma_o^2}{\Delta l_0^2} < 1$ . Considering two sub-samples with the same weight and different slopes  $k_1$  and  $k_2$ , respectively, the slope of combined sample including all points in each sub-samples would be  $k_c = \tan((\arctan(k_1) + \arctan(k_2))/2) \simeq (k_1 + k_2)/2$ , where the last  $\simeq$  is valid only if  $k_1 \ll 1$  and  $k_2 \ll 1$ . Therefore, we can derive the slope of the combined sample considering different  $f$  values with different weights by an integration

$$k = \int_0^\infty k_f p(f) df, \quad (11)$$

where  $p(f)$  is the probability of  $f$ .

Then we take the distribution of  $f$  into account. If  $\sigma_X = 0$ , and  $\sigma_o$  follows Gaussian distribution, the probability of  $f$  would be

$$p(f) = \sqrt{\frac{2}{\pi}} e^{-\frac{f^2}{2}}, \quad f \geq 0. \quad (12)$$

Then

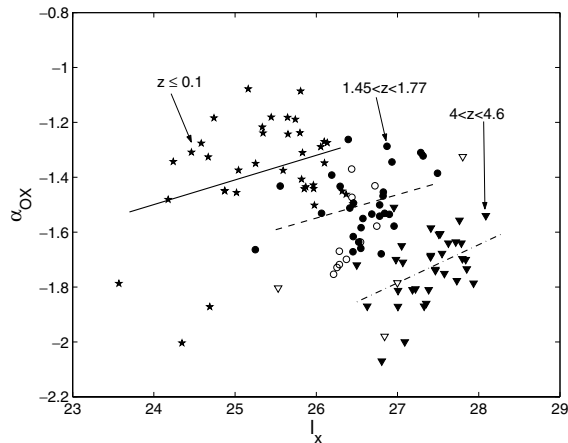
$$k \simeq \int_0^\infty -12 \left(\frac{f\sigma_o}{\Delta l_0}\right)^2 \sqrt{\frac{2}{\pi}} e^{-\frac{f^2}{2}} df = -6 \left(\frac{\sigma_o}{\Delta l_0}\right)^2 \quad (13)$$

$$\simeq \frac{\Delta l_0^2}{12\sigma_o^2} - \sqrt{\left(\frac{\Delta l_0^2}{12\sigma_o^2}\right)^2 + 1}. \quad (14)$$

Equation (13) shows that the slope of the artificial  $\alpha_{\text{OX}}-l_0$  correlation is directly proportional to  $\sigma_o^2/\Delta l_0^2$ . The significance of the correlation, which could be measured by Spearman correlation coefficient, is always positively correlated to the slope value in the artificial  $\alpha_{\text{OX}}-l_0$  correlation, and as shown in Figs 2–3. Therefore, the result in equation (13) also means the significance of the artificial  $\alpha_{\text{OX}}-l_0$  correlation is proportional to  $\sigma_o^2/\Delta l_0^2$ .

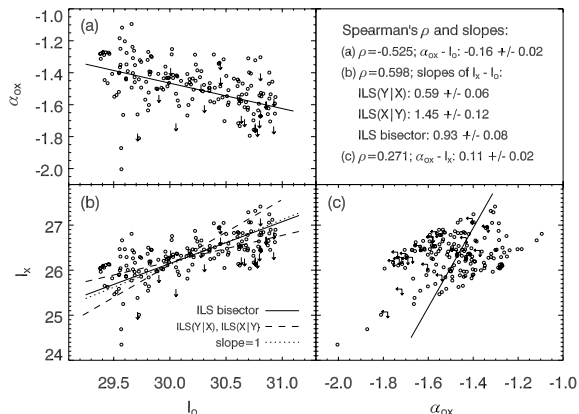
For the M06 sample,  $\Delta l_0 \sim 1.7$  and  $\sigma_o \sim 0.40$  when  $\sigma_X = 0$ , then  $k \sim -0.3$ . Such estimation of an artificial slope using equations (13) or (14) is qualitatively consistent with the slope in the  $e = 1$  simulations, where the value is  $k \sim -0.18$  as shown in Fig. 3. A reason for the discrepancy is that the absolute slope value is not  $\ll 1$ , therefore approximations used in our estimation are deviated from true values. Another possible reason, i.e. different fitting procedure used in simulations and our estimation, would more or less contribute to the lower absolute slope values in simulations. In simulations, a linear regression method, which only takes residuals of the dependent variable into account, is used, thus always leading to a lower absolute slope value than methods considering residuals in both variables as used in equation (7), and as shown in Panel (b) in Fig. 1.

We now discuss consequences of non-zero  $\sigma_X$ . When  $\sigma_X > 0$ , the distribution of  $f$  will be extended. As shown in Fig. 4, Point B becomes a distribution in the range of CD within  $1\sigma$ . When calculating the  $\chi^2$  of a linear regression with slope  $-k$ , the contribution of the broadening in B is equivalent to an extension along the  $l_0$  axis. For example, as shown in Fig. 4, the contribution of Point C is equivalent to Point C' which has the same  $\alpha_{\text{OX}}$  as B but smaller



**Figure 7.**  $\alpha_{\text{OX}}$  dependence on the rest-frame 2 keV monochromatic luminosity for AGNs in three redshift bins in S06 sample:  $z \leq 0.1$  (stars),  $1.45 < z < 1.77$  (circles) and  $4 < z < 4.6$  (downward-pointing triangles). X-ray detected AGNs are represented using filled symbols while upper limits are represented using open symbols. The solid line, dashed line and dash-dotted lines indicate linear regression results using X-ray detected sources for  $z \leq 0.1$ ,  $1.45 < z < 1.77$  and  $4 < z < 4.6$  samples, respectively.

$l_0$ . Therefore, a non-zero  $\sigma_X$  tends to smooth the distribution of  $l_0$  and extend its range. When  $\sigma_X/k$  is comparable with  $l_0$ , it will extend the range of  $l_0$  significantly. To show this effect, we do another simulation. Based on the M06 sample with a given  $\sigma_0 = 0.3$ , we calculate the Spearman correlation coefficients and slope for  $\alpha_{\text{OX}}-l_0$  correlation with different  $\sigma_X$ , while other conditions are set to be the same as simulations in Section 2.1. As shown in Fig. 5, when  $\log(\sigma_X/\sigma_0) > 0$ , i.e.  $\sigma_X > 0.3$  and  $\sigma_X/k > 1.7 \sim \Delta l_0$ , both the Spearman correlation coefficient and the slope tend to move toward zero when  $\sigma_X$  increases. When  $\log(\sigma_X/\sigma_0) > 0.5$ , i.e.  $\sigma_X > 0.9$  and  $\sigma_X/k > 5.3 \sim 3\Delta l_0$ , there is no correlation in  $\alpha_{\text{OX}}-l_0$  within  $1\sigma$ . However, as discussed in Section 2.1, even if all extra dispersion in



**Figure 8.** 187 low- $z$  AGNs from S06. Circles indicate X-ray detected AGNs, while arrows indicate upper limits. (Panel a)  $\alpha_{\text{OX}}$  dependence on the rest-frame 2500 Å monochromatic luminosity. The solid line indicates linear regression results from the EM Algorithm in ASURV. (Panel b) Rest-frame 2 keV monochromatic luminosity against 2500 Å one. The solid line indicates the ILS bisector result, and dashed lines indicate ILS(Y|X) and ILS(X|Y) results from the EM Algorithm in ASURV, respectively. The  $e = 1$  relation is shown by dotted line for comparison. (Panel c)  $\alpha_{\text{OX}}$  dependence on the rest-frame 2 keV monochromatic luminosity. The solid line indicates linear regression result using X-ray detected quasars. Spearman correlation coefficients and slopes of fitting lines are indicated in the upper right panel.

$\alpha_{\text{OX}}$  comes from  $l_X$ ,  $\sigma_X$  is unlikely to exceed 0.4 and  $\log(\sigma_X/\sigma_0)$  is unlikely to exceed 0.3.

The effects of  $\sigma_0$  on the relationship of  $\alpha_{\text{OX}}-l_X$  is similar with the effects of  $\sigma_X$  on the relationship of  $\alpha_{\text{OX}}-l_0$ . Thus, we do not need to repeat the above analysis. Moreover, a similar effect as presented here for the  $\alpha_{\text{OX}}-l_0$  correlation would also affect any correlation with a dependent variable  $B$ , which is not directly observed but derived from  $B \propto A^{-1}$ , where  $A$  is the independent variable, such as the Baldwin effect, which has also been pointed out by Yuan et al. (1998).

In summary, the significance of artificial correlation in  $\alpha_{\text{OX}}-l_0$  is approximately proportional to  $\sigma_0^2/\Delta l_0^2$ , and decreases when  $\sigma_X$  increases and becomes comparable with  $k\Delta l_0$ , where  $k$  is the absolute value of the artificial slope and  $\Delta l_0$  is the range  $l_0$  span.

### 3 DATA ANALYSIS OF A SUB-SAMPLE OF AGNS FROM STEFFEN ET AL. (2006)

Another problem in the study of  $\alpha_{\text{OX}}-l_0$  relationship is the degeneracy between redshift and luminosity in flux-limited samples. S06 used a much larger sample than M06 with  $\Delta l_0 \sim 5$ , which suppresses the false slope artefacts discussed in Section 2. The observed change in  $\alpha_{\text{OX}}$  across this larger baseline  $\Delta l_0$  in their sample is sufficiently large that  $\alpha_{\text{OX}}$  must depend on luminosity, or redshift, or both. To distinguish between luminosity dependence and redshift dependence, first, S06 performed partial correlation analysis using Kendall's generalized partial  $\tau$  to quantitatively show the correlation significance of  $\alpha_{\text{OX}}-l_0$  and  $\alpha_{\text{OX}}-z$ . They found a  $13.6\sigma$  correlation of  $\alpha_{\text{OX}}-l_0$  when controlling  $z$ , and a  $1.3\sigma$  correlation of  $\alpha_{\text{OX}}-z$  when controlling  $l_0$ . However, Kelly et al. (2007) have shown that interpretation of Kendall's  $\tau$  is problematic, and Kendall's  $\tau$  for the  $\alpha_{\text{OX}}-z$  correlation is not necessarily expected to be non-zero when  $\alpha_{\text{OX}}$  is correlated with  $z$ . Based on simulations, Kelly et al. (2007) pointed out that the lack of evidence for a significant correlation between  $\alpha_{\text{OX}}$  and  $z$  based on Kendall's  $\tau$  in S06 may be the result of an incorrect assumption about the distribution of  $\tau$  under the null hypothesis. In spite of most previous studies, Kelly et al. (2007) found that  $\alpha_{\text{OX}}$  is correlated with both  $l_0$  and  $z$ . Moreover, in the partial correlation analysis of  $\alpha_{\text{OX}}-l_0$ , consequences of luminosity dispersions, as discussed in Section 2, were not taken into account. To show this effect, we select AGNs in three redshift bins, with each bin containing 38 sources, to control the redshift. Then we examine the  $\alpha_{\text{OX}}-l_0$  and  $\alpha_{\text{OX}}-l_X$  relations in each bin, as shown in Figs 6 and 7. Similar to Fig. 1, in each redshift bin,  $\alpha_{\text{OX}}$  is anticorrelated with  $l_0$ , but positively correlated with  $l_X$ , which is caused by luminosity dispersions. Because dispersions always strengthen the anticorrelation of  $\alpha_{\text{OX}}-l_0$ , dependence of  $\alpha_{\text{OX}}$  on  $l_0$  might be biased toward higher significance by luminosity dispersions, whereas the dependence on  $z$  does not suffer such bias.

Second, S06 compared  $\alpha_{\text{OX}}$  residuals as a linear function of  $l_0$  and  $z$ . As shown in their Fig. 8, there are systematic residuals of  $\alpha_{\text{OX}}-\alpha_{\text{OX}}(z)$ , which indicate that  $\alpha_{\text{OX}}$  cannot be linearly dependent on redshift alone. However, spectral index might depend on redshift in a non-linear form, as shown in fig. 12 of Strateva et al. (2005). Moreover, it is also possible that  $\alpha_{\text{OX}}$  depends on both  $l_0$  and  $z$ , as shown in Kelly et al. (2007). Using different parametric models for the redshift and optical luminosity dependencies, Kelly et al. (2007) found the model that is best supported by their data has a linear dependence of  $\alpha_{\text{OX}}$  on cosmic time, and a quadratic dependence of  $\alpha_{\text{OX}}$  on  $l_0$  [the definition of  $\alpha_{\text{OX}}$  in Kelly et al. (2007) is different from our definition with an opposite sign]. Since  $l_0$  and  $z$  are coupled together in flux-limited samples, different parametric models will

**Table 1.** Slopes of  $l_X$ - $l_o$  relation in M06 and low- $z$  sub-sample derived from different regression methods.

Regression method	M06	Low- $z$ sub-sample
ILS( $\alpha_{OX}   l_o$ )	$0.53 \pm 0.13$	$0.58 \pm 0.05$
ILS( $l_X   l_o$ )	$0.54 \pm 0.14$	$0.59 \pm 0.06$
ILS( $\alpha_{OX}   l_X$ )	$2.08 \pm 0.48$	$1.40 \pm 0.10$
ILS( $l_o   l_X$ )	$2.08 \pm 0.45$	$1.45 \pm 0.12$

lead to different results and their best model results depend on the form of the models. In summary, dependence of  $\alpha_{OX}$  on  $z$ , though with lower significance in partial correlation analysis, could not be excluded in S06 sample.

To avoid possible bias from  $\alpha_{OX}$ - $z$  correlation, here we examine a sub-sample from S06 containing 187 AGNs, as shown in the dotted-line box in fig. 3 in S06 (Kelly et al. 2005; Shemmer et al. 2005; Strateva et al. 2005; Vignali et al. 2005; S06), which more completely fills the redshift and optical luminosity plane and thus is less affected by such bias. We refer this sample as the ‘low- $z$  sub-sample’. We perform linear regressions in the same procedure as described in Section 2.1. Fig. 8 presents our results for this sample. Similar to the M06 sample, the low- $z$  sub-sample shows conflicting correlations due to dispersions in luminosity:  $\alpha_{OX} = -0.16l_o + \text{const}$  as shown in solid line in Panel (a), but  $\alpha_{OX} = 0.11l_X + \text{const}$  as shown in solid line in Panel (c). Therefore, the same data produce two totally different results:  $l_X = 0.58l_o + \text{const}$  or  $l_X = 1.40l_o + \text{const}$ , i.e.  $e < 1$  or  $e > 1$  if  $L_X \propto L_o^e$ . As shown in Panel (b), the slope of the  $l_X$ - $l_o$  relation also depends on which luminosity is used as the dependent variable. When treating  $l_X$  as the dependent variable, the slope is  $0.59 \pm 0.06$  (the flatter dashed line), while it changes dramatically to  $1.45 \pm 0.12$  (the steeper dashed line) when treating  $l_o$  as the dependent variable. Using ILS bisector, we find the slope to be  $0.93 \pm 0.08$  (solid line), which is consistent with  $e = 1$  (dotted line). In Panel (c), the fit looks different from the trend by eye (bisector fit). As discussed in Section 2, when using traditional ordinary least-squared method which minimizes the residuals of the dependent variable, the fit tends to be flatter than the bisector one where residuals of both dependent and independent variables are taken into account. When data points are concentrated at the centre, as in Fig. 8(c), inconsistency of the two fits becomes large.

Interestingly, but not surprisingly, results of ILS( $\alpha_{OX} | l_o$ ) are consistent with results of ILS( $l_X | l_o$ ), and results of ILS( $\alpha_{OX} | l_X$ ) are consistent with results of ILS( $l_o | l_X$ ), as shown in Table 1 and Figs 1 and 8. The reason is that artificial correlations seen in  $\alpha_{OX}$ - $l_o$  relation, i.e.  $0.3838(l_X - l_o) - l_o$  relation, corresponds to least-squares method which minimizes the residuals of  $l_X$ , which is the same as in  $l_X$ - $l_o$  relation, and thus leads to a slope less than the true one where residuals of both variables are considered. Such results indicate that for  $l_X$ - $l_o$  relation, regression results based on the traditional ordinary least-squares method which minimizes residuals of the dependent variable suffer from effects caused by luminosity dispersion and are thus not reliable. Instead, weighting both  $l_o$  and  $l_X$  in the regression, i.e. ILS bisector, would be a more robust methodology. Moreover, the degree to which the slopes of ILS( $\alpha_{OX} | l_X$ ) and ILS( $\alpha_{OX} | l_o$ ) are inconsistent indicates the degree of artificial correlation discussed in Section 2. When the slopes converge to the same value, the artificial correlation would be suppressed.

We conclude for this sample that a constant  $\alpha_{OX}$  which does not depend on luminosity is also consistent with data.

#### 4 COMPARISON OF OPTICAL AND X-RAY LUMINOSITY FUNCTIONS

For a complete sample of broad line AGN including optical and X-ray observations (detections or upper limits), slopes in optical and X-ray quasar luminosity functions (LFs) should be the same after correct transformations. When converting optical luminosity to X-ray luminosity, different  $\alpha_{OX}$  models would lead to different X-ray LF shapes in the optical frame. Thus, we can test whether a particular  $\alpha_{OX}$  model is correct by comparing the two LFs. We use the optical quasar LF from Richards et al. (2006) and AGN hard X-ray LF from Barger et al. (2005).

We investigate the following two  $\alpha_{OX}$  models:

- (i) a constant  $\alpha_{OX}$ ,  $l_X = l_o - 3.92$ ;
- (ii)  $\alpha_{OX}$  from S06,  $l_X = 0.72l_o + 4.53$ .

We follow Hopkins, Richards & Hernquist (2007) to calculate the binned LFs. For each  $\alpha_{OX}$  model, an overall normalization factor is applied in the X-ray LF to get the minimum  $\chi^2$ , which means that we are comparing just the slopes of LFs. Results are presented in Fig. 9. A constant  $\alpha_{OX}$  which does not depend on luminosity (left-hand panels) is consistent with data, and is more preferred than the  $\alpha_{OX}$  models given by S06 (right-hand panels).

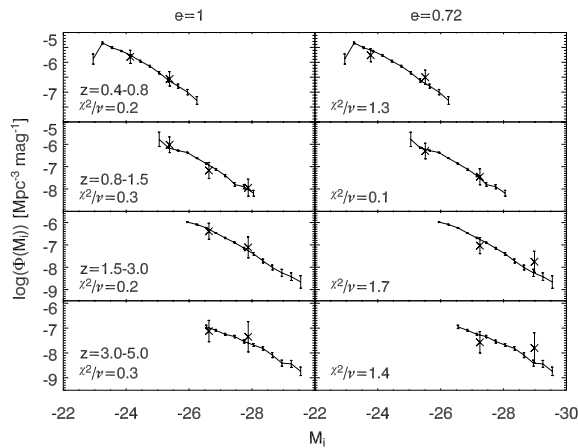
However, from this comparison we cannot reach a strong conclusion that luminosity-dependent  $\alpha_{OX}$  is excluded completely, for three reasons as follows. First, there are very few data points here in the X-ray LF. Second, as pointed out by Richards et al. (2005), such comparison is not strictly quantitative since X-ray selected samples and optically selected samples are not identical. Moreover, the bright-end slopes from different X-ray samples are different (Ueda et al. 2003; Barger et al. 2005; Hasinger, Miyaji & Schmidt 2005). Hopkins, Richards & Hernquist (2007) combined a large set of LF measurements and took obscuration and scattering into account, and in their analysis the luminosity functions can be reconciled reasonably well with the  $\alpha_{OX}$  model in S06. However, since the constraints on the present bright-end X-ray LFs are poor, the fact that the LFs could be reconciled reasonably well with S06 in their work probably just reflects the large X-ray error bars.

#### 5 SELECTION EFFECTS IN FLUX-LIMITED SAMPLES: OPTICALLY SELECTED SAMPLES VERSUS X-RAY SELECTED SAMPLES

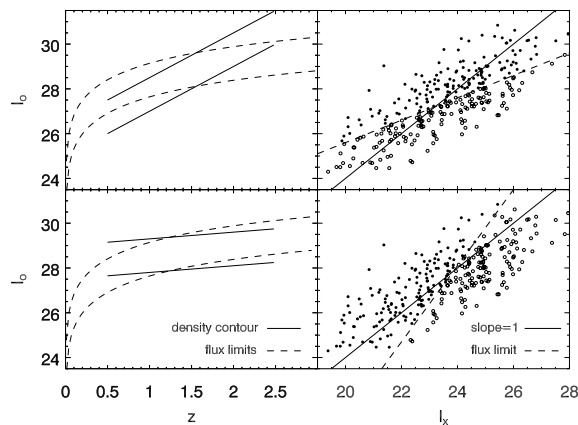
In this section, we discuss the selection effects in flux-limited samples. For a given AGN luminosity function, assuming the observed optical and X-ray luminosities are the intrinsic values modified by dispersions which might be caused by variabilities or observational errors, there are three possibilities in a flux-limited sample:

- (a) lower fraction of more luminous AGNs are missed.
- (b) higher fraction of more luminous AGNs are missed.
- (c) same fractions of more luminous and fainter AGNs are missed, so the relationship between  $l_X$  and  $l_o$  remains unchanged.

Assuming the slope of  $l_X$ - $l_o$  relation (without further description, slope= $e$  in  $l_X = e \times l_o + \text{const}$  throughout this section) is unity, and at a certain redshift the number density of AGN decreases with luminosity, Fig. 10 shows schematic sketches for the first two cases in optically selected AGN samples. The upper panels are for Case (i), where the density contour lines in luminosity functions in the  $l_o - z$  plane (each line corresponds to a given constant AGN number density as a function of redshift) are steeper than the flux limits, hence lower fraction of more optical luminous AGNs are missed,



**Figure 9.** Comparison of the SDSS DR3 optical quasar luminosity function from Richards et al. (2006) (lines with error bars) with AGN hard X-ray luminosity function from Barger et al. (2005) (crosses with error bars). Left-hand panels show the LFs with  $e = 1$  (luminosity-independent spectral index). Right-hand panels show the results with  $e = 0.72$  (spectral index is anticorrelated with luminosity). Different rows show results at different redshift range. For each panel, an overall normalization factor is applied in X-ray LF to get the minimum  $\chi^2$ .



**Figure 10.** Schematic sketches for selection effects in optically selected AGN samples. The upper panels are for Case (i) and the bottom panels are for Case (ii). The left-hand panels show flux limits (dashed lines) in  $l_0 - z$  plane, compared with the density contour lines in luminosity functions (solid lines). The right-hand panels show observed AGNs (solid circles) which are above the flux limits (dashed lines), and missed AGNs (open circles) which are below the flux limits, in  $l_0 - l_X$  plane. Slope is defined as in  $l_X = \text{slope} \times l_0 + \text{const}$ . Slope=1 are indicated by solid lines. Note that in the upper right panel slope  $> 1$  for detected AGNs, and in the lower right panel slope  $< 1$  for detected AGNs, since  $l_X$  is the x-axis and  $l_0$  is the y-axis. These sketches, much simplified and only qualitatively correct, are shown for illustration only.

and then the slope is biased toward more than unity. The bottom panels are for Case (ii), where the density contour lines in luminosity functions in the  $l_0 - z$  plane are flatter than the flux limits, hence higher fraction of more optical luminous AGNs are missed, and then the slope is biased lower less than unity. The left-hand panels show flux limits (dashed lines) in  $l_0 - z$  plane, compared with the density contour lines in luminosity functions (solid lines). The left-hand panels show observed AGNs (solid circles) which are above the flux limits (dashed lines), and missed AGNs (open circles) which

are below the flux limits, in  $l_0 - l_X$  plane. Slope=1 are indicated by solid lines.

The density contour lines in real luminosity functions are much more complicated than shown in Fig. 10, and the real slopes of density contour lines depend on redshifts and luminosities. Moreover, since the fraction of missed AGNs depends on the dispersions of luminosities around the linear relationship of  $l_X - l_0$ , the biases also depend on the dispersions. To test whether the slope of  $l_X - l_0$  relation could be biased by flux limits in realistic optically selected AGN samples, we carry out Monte Carlo simulations. We use optical analytical luminosity function from Richards et al. (2005) for  $z < 3$  AGNs, and Richards et al. (2006) for  $z > 3$  AGNs. We simulate three optically selected sub-samples, in order to mimic the SDSS, COMBO-17 and high- $z$  samples in S06:

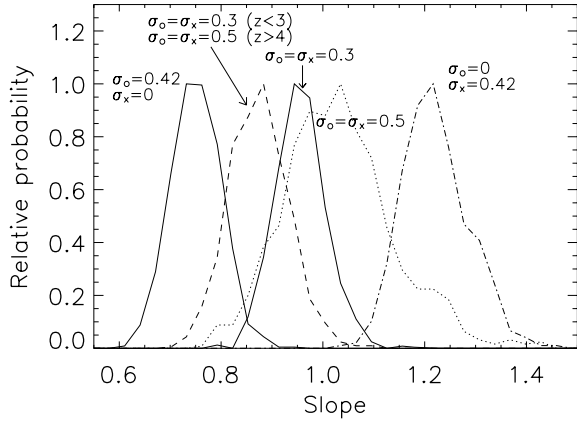
- (1) a shallow  $z < 3$  sub-sample with  $m_g < 19$  containing about 155 AGNs, which is similar to the SDSS sample in S06.
- (2) a deeper  $z < 3$  sub-sample with  $m_g < 21$  containing about 52 AGNs, which is similar to the COMBO-17 sample in S06.
- (3) a  $4 < z < 6$  sub-sample with  $m_g < 20$  containing about 55 AGNs, which is similar to the high- $z$  sample in S06.

We do not try to mimic the nearby Seyfert 1 and BQS samples in S06 which are located in  $z < 0.4$ , since the LF in this redshift range has larger errors due to smaller volume. For the two  $z < 3$  sub-samples, a detection efficiency factor is taken from fig. 6 in Richards et al. (2006). For the  $z > 4$  sub-sample, a constant detection efficiency is used according to Richards et al. (2006). To show the effects of luminosity dispersions in optically selected flux-limited samples, 1000 simulations, each containing the above three sub-samples, are carried out for each of the following five dispersion models:

- (i)  $\sigma_o = 0.42, \sigma_X = 0$ ;
- (ii)  $\sigma_o = \sigma_X = 0.3$  for  $z < 3$  AGNs,  $\sigma_o = \sigma_X = 0.5$  for  $z > 4$  AGNs;
- (iii)  $\sigma_o = \sigma_X = 0.3$ ;
- (iv)  $\sigma_o = \sigma_X = 0.5$ ;
- (v)  $\sigma_o = 0, \sigma_X = 0.42$ ;

where  $\sigma_o$  and  $\sigma_X$  are defined as in Section 2.1. Models (i) and (v) are extreme cases, in order to show the bias direction when  $\sigma_o$  or  $\sigma_X$  is dominating. Models (ii)–(iv) can show the effects of dispersion and dispersion evolution in cosmic time.

The Monte Carlo analysis was performed by generating a combined sample containing the above three sub-samples for each of the five dispersion models as follows. First, the redshift and luminosity ranges are divided into grids with  $\delta z = 0.1$  and  $\delta M_g = 0.3$ , then the detection probability in a given grid is proportional to  $dV(z) \times \Phi(z, M_g) \times \eta$ , where  $dV(z)$  is the volume element in comoving space,  $\Phi(z, M_g)$  is the luminosity function and  $\eta$  is the detection efficiency. In a given grid, AGNs are randomly produced following an uniform distribution, and the total number of AGNs in the grid is determined by a Poisson process with expectation  $\bar{N}(z, M_g) = dV(z) \times \Phi(z, M_g) \times \eta \times C$ , where  $C$  is a constant for a given sub-sample with given luminosity dispersions, adjusted to make the average number of detected AGNs to be 155, 52 and 55 for the three sub-samples, respectively. The conversion from  $M_g$  to the intrinsic optical luminosity  $\bar{l}_o$  follows Richards et al. (2005), and the intrinsic X-ray luminosity  $\bar{l}_X = \bar{l}_o - 4$ . Second, luminosity dispersions are applied, where the observed  $l_0$  and  $l_X$  are drawn from the Gaussian distribution around  $\bar{l}_o$  and  $\bar{l}_X$  with given dispersions  $\sigma_o$  and  $\sigma_X$ , respectively. Third, flux limits are applied and AGNs with  $m_g > \text{flux limit}$  are detected. The above procedure selects about 262



**Figure 11.** The distributions of slopes in simulated optically selected flux-limited samples, where  $l_X = \text{slope} \times l_o + \text{const}$ . The relative probabilities are normalized with peak values equal to unity. From left- to right-hand side are the five dispersion Models (i) to (v) listed in the text, respectively.

AGNs for each dispersion model, where about 155 in sub-sample (1), 52 in sub-sample (2) and 55 in sub-sample (3). Then, the above procedure is repeated 1000 times to get 1000 independent samples.

The slope in each simulated sample is calculated using FITEXY (Press et al. 1992) assuming the same error in  $l_o$  and  $l_X$ . The distributions of slopes in simulated optically selected flux-limited samples are shown in Fig. 11. The relative probabilities are normalized with peak values equal to unity. From left- to right-hand side are the five dispersion models (i) to (v), respectively. The mean values  $\pm$  standard deviations of slopes in simulations of the five models are:  $0.76 \pm 0.05$ ,  $0.89 \pm 0.06$ ,  $0.97 \pm 0.05$ ,  $1.04 \pm 0.11$ , and  $1.23 \pm 0.06$ , respectively. In Models (iii) and (iv),  $\sigma_o = \sigma_x = 0.3$  and  $0.5$ , flux-limited simulations are consistent with the assumed slope=1, i.e. the slopes are not biased. However, when  $\sigma_o \neq \sigma_x$  or  $\sigma_o, \sigma_x$  change in cosmic time, slopes in flux-limited simulations might be biased, as shown in Models (i), (ii) and (v). Slopes in Model (i), i.e.  $0.76 \pm 0.05$ , is consistent with the slope in S06, i.e.  $0.72 \pm 0.01$ . However, since  $\sigma_o = 0.42$  and  $\sigma_x = 0$  are an unrealistic extreme case, this does not mean that the non-unity slope of  $l_X-l_o$  relation is totally caused by such selection effects. Note that slope values in Fig. 11 depend on the selection method, i.e. flux limits and number of AGNs in each sub-sample, hence another different optical samples will have different results.

We do not simulate X-ray selected flux-limited samples for two reasons. First, if the slope of  $l_o-l_X$  equals unity, X-ray AGNs are identical to optical AGNs, and the X-ray LF will be the same as optical LF with  $l_X = l_o + \text{const}$ , as suggested in Section 4. Therefore, the results here can be applied to X-ray selected sample with similar flux limits after switching  $\sigma_o$  and  $\sigma_x$ . Second, X-ray LFs have larger errors than optical LFs due to smaller samples. Therefore, it is our purpose to just point out the fact that the slope will be biased in flux-limited X-ray samples, rather than focusing on a particular X-ray selected sample.

While a number of previous studies of optical selected AGNs have reported that  $\alpha_{OX}$  is anticorrelated with luminosity (Strateva et al. 2005; S06), whether  $\alpha_{OX}$  depends on luminosity in X-ray selected AGN samples remains unknown. Hasinger (2005) found no  $\alpha_{OX}$  dependence on either luminosity or redshift in soft X-ray selected samples. Frank et al. (2007) found in their *Chandra* Deep Field-North sample  $l_X = (0.808 \pm 0.047)l_o + \text{const}$ . They also found the slope decreases when only brighter sources are included, and

the slope increases when only fainter sources are included. It is possible that the slope might be biased in this flux-limited sample and the magnitude of biases is different when using different flux limit. As shown in Fig. 11, if  $\sigma_o > \sigma_x$ , an optically selected sample like in S06 will be biased toward a flatter slope. Moreover, if the X-ray LF is similar to optical LF and the X-ray sample consisted of AGNs with similar flux limits, the X-ray sample will be biased toward a steeper slope. Therefore, even if optically selected AGNs and X-ray selected AGNs are identical, the slope in the optically selected sample will be flatter than the slope in the X-ray selected sample, which can properly explain the observed discrepancy.

In summary, selection effects in flux-limited samples might bias the  $l_o-l_X$  relation and cause discrepancy in the  $l_o-l_X$  relation in optically selected samples and X-ray selected samples, especially when  $\sigma_o \neq \sigma_x$  or  $\sigma_o, \sigma_x$  change in cosmic time. The magnitude of the bias and discrepancy depend on the luminosity function, flux limits of the sample and dispersions in optical and X-ray luminosities. Note that even if such selection effects do bias the slope of the  $l_o-l_X$  relation toward the observed discrepancy between optical and X-ray samples, it is not necessarily the only reason. It is possible that optically selected samples and X-ray selected samples consisted of different AGNs, so slopes of the  $l_o-l_X$  relation in optically selected samples will be different from slopes in X-ray selected ones. As discussed in Brusa et al. (2007), about 40 per cent of the X-ray selected AGNs in their COSMOS sample would have not been easily selected as AGN candidates on the basis of purely optical criteria, either because similar colours to dwarf stars or field galaxies, or because they are not point-like sources in morphological classification. Moreover, optically selected AGNs and X-ray selected AGNs might be typically in different evolution stages and thus are not identical (Shen et al. 2007).

## 6 DISCUSSION AND CONCLUSIONS

Recently Shen et al. (2006) have studied a sample of 46420 SDSS quasars and shown that the X-ray to optical spectral index is strongly and similarly dependent on UV luminosity at all redshifts and at all luminosities at  $z < 3$ . Because all quasars in this sample are detected in the ROSAT X-ray band either individually, or by stacking quasars of similar UV luminosity and redshift, their sample of X-ray detected quasars is much larger than any previously available one, due to the power of the stacking technique. However their result is not in conflict with what we have presented in this paper, because (1) the SDSS sample is still optical flux limited and most X-ray detection is near the flux limit for each group of sources stacked together, so that the effects of flux limit should still exist, and (2) our results are not sensitive to the sizes of our quasar samples used.

In summary, we have investigated the correlation between the spectral index  $\alpha_{OX}$  and optical/X-ray luminosities in AGNs by means of linear regressions, Monte Carlo simulations, simplified analytic estimations and comparison of X-ray and optical luminosities functions. We have reached to following five conclusions.

(1) The dependence of  $\alpha_{OX}$  on optical luminosity found in M06 may not be an underlying physical property. It remains unknown whether  $e < 1$  or  $e > 1$  if  $L_X \propto L_o^e$  in this high- $z$  sample.

(2) The luminosity dependence can be artificially generated very easily by luminosity dispersions. The significance of artificial correlation in  $\alpha_{OX}-l_o$  is approximately proportional to  $\sigma_o^2/\Delta l_o^2$ , where  $\sigma_o$  is the optical luminosity dispersion and  $\Delta l_o$  is the range that  $l_o$  spans, and decreases when  $\sigma_x$  increases and becomes comparable with  $k\Delta l_o$ , where  $k$  is the absolute value of the artificial slope. This



effect also affects the Baldwin effect. Instead of regressions only weighting one variable, weighting both  $l_o$  and  $l_x$ , i.e. ILS bisector, in the regression would be a more robust methodology to avoid such bias.

(3) In a more complete low- $z$  sub-sample from S06,  $\alpha_{OX}$  must depend on luminosity, or redshift, or both. However, a luminosity-independent  $\alpha_{OX}$  is still consistent with data. Redshift dependencies cannot be ruled out and may be large, but somewhat hidden because of luminosity dispersions, which generate artificial luminosity correlations in each redshift bin.

(4) In the comparison of X-ray (Barger et al. 2005) and optical quasar (Richards et al. 2006) LFs, a luminosity-independent  $\alpha_{OX}$  is consistent with data, and more preferred than the luminosity-dependent  $\alpha_{OX}$  model given by S06.

(5) Selection effects in flux-limited samples might bias the  $l_o - l_x$  relation and cause discrepancy in the  $l_o - l_x$  relation in optically selected sample and X-ray selected sample, especially when  $\sigma_o \neq \sigma_x$  or  $\sigma_o, \sigma_x$  change in cosmic time. The magnitude of the bias depends on the luminosity function, flux limits of the sample and dispersions in optical and X-ray luminosities.

It therefore remains inconclusive whether the anticorrelation between AGN spectral index and optical luminosity is true. Even if  $\alpha_{OX}$  does depend on optical luminosity, the currently adopted slope value might be biased and deviate from the intrinsic value. To correctly establish a dependence of  $\alpha_{OX}$  of AGNs on their luminosity, a larger and more complete sample, such as from multiwavelength surveys, is needed and consequences of luminosity dispersions and selection effects in flux-limited samples must be taken into account properly.

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