Large Amplitude Electrothermal Waves in a Nonequilibrium Plasma

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Steady, one-dimensional current streamers have been observed in a nonequilibrium plasma subjected to crossed E and B fields. Their half-width and amplitude agree with a nonlinear model of electrothermal waves.

Strong fluctuations in Hall potentials have been observed in conventional crossed-field configurations in magnetohydrodynamic devices for which nonequilibrium ionization was important.[1-3] These disturbances occurred when the Hall parameter, β, was greater than a value of the order of two. A possible explanation was offered by Kerrebrock,[1] who showed that this plasma was unstable to perturbations in the electron density. This instability is due to the strong coupling between the electric conductivity and heating of the electron gas, and is called the electrothermal instability.

The authors have been able to confirm the general dependence of the onset of the instability on the Hall parameter under different experimental conditions than those used by Kerrebrock and, in addition, have been able to obtain steady, one-dimensional waves, or current streamers, for direct study. The present note contains a brief description of these waves and a theoretical treatment of the steady, nonlinear wave problem. Similar work by Velikov[4] on unsteady disturbances has recently been brought to the authors' attention.

The experiments were carried out with the purpose of obtaining steady, one-dimensional waves, and the location of electrodes was picked to maximize the possibility of getting a steady disturbance rather than to duplicate any practical scheme of plasma acceleration or power generation.

Steady current streamers were obtained in a wide variety of test configurations, one of which is illustrated in Fig. 1. Here, three separate circuits discharge in parallel down the test duct to supply the axial component of current, and a perpendicular component is allowed to circulate through U-shaped tungsten wires connecting the top to the bottom of the duct. This current flow is illustrated in Fig. 2. For the center streamer, this arrangement duplicates, as nearly as possible, an infinite wave train. The gas in the duct was argon at atmospheric pressure and about 2000°K, seeded with 0.2% potassium. The velocity of the gas was approximately 60 m/sec.

Pictures in Fig. 3 were taken viewing parallel to the magnetic field through a quartz wall in the test section and with a 1/400-sec exposure time. In Fig. 3(a), the magnetic field is zero and the electron density and current, judging from the uniform radiation, are also uniform. The current streamer pattern that appeared when a magnetic field was applied is shown in Fig. 3(b). The half width of the central wave, measured from these pictures, is about $\frac{1}{4}$ cm, and the wave is remarkably one-dimensional over a 5-cm length. A measure of the current density variation across the streamers was obtained by comparing the light intensities in the current streamers with the light intensity emitted for various current density levels when no magnetic field was applied. These measurements indicate that current density, and thus electron density, variation has an amplitude, given as the difference between the maximum and minimum electron density, of $\Delta n_e = 1.5 n_{e0}$ where n_{e0} is the unperturbed value.

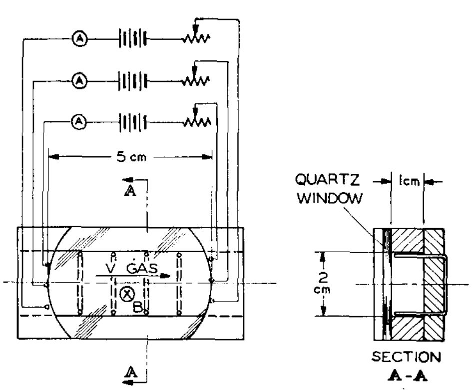

Fig. 1. Test section for three streamer configurations.

[1] J. Kerrebrock, AIAA J. **2**, 1072 (1964).
[2] J. Klepeis and R. J. Rosa, AIAA J. **3**, 1659 (1965).
[3] R. Dethlefson and J. L. Kerrebrock, in *Seventh Symposium on Engineering Aspects of Magnetohydrodynamics* (*Proceedings*), (Princeton University, Princeton, New Jersey, 1966), p. 117.
[4] A. A. Volkov and E. P. Velikov, *Electricity from MHD*, (International Atomic Energy Agency, Vienna, 1966), Vol. II, p. 395.

It was also possible to obtain two streamers in the 1×2-cm duct and one or two streamers in a 1×1-cm duct. In all these configurations, when one-dimensional, steady streamers were obtained, they had roughly the same half-width and amplitude. A large number of such tests indicate that these characteristics are roughly independent of the boundary conditions, number of electrodes, and the size of the duct. In addition, the amplitude of the waves decreases and waves die out as β decreases to a value of roughly 2. This behavior is in agreement with Kerrebrock's linear analysis.

A nonlinear differential equation for the electron density fluctuation across the electrothermal wave can be obtained by simplifying the complete set of equations, and it is integrated for conditions corresponding to the experimental situation of Fig. 3.

The governing equations for the behavior of the electrons in a dense, slightly ionized plasma in the presence of electric and magnetic fields are the continuity equation, Ohm's law, an ionization rate expression, and an energy equation of the form:

$$\frac{\partial(\epsilon_e + \epsilon_i)n_e}{\partial t} - \frac{5}{3}\frac{\mathbf{J}\cdot\nabla\epsilon_e}{e} = \mathbf{J}\cdot\mathbf{E} - \dot{R} - \dot{\Omega} + \nabla\cdot\kappa_e\mu\nabla\epsilon_e. \quad (1)$$

A typical formulation of these equations and a definition of the notation is given in Ref. 1.

For the one-dimensional problem with variation in the x direction only (illustrated in Fig. 4), E_y and J_x are constants which are called E_0 and J_0, respectively. Ohm's law gives the other two components, J_y and E_x, and the Joule heating term $(\mathbf{J}\cdot\mathbf{E})$ can then be expressed as

$$\mathbf{J}\cdot\mathbf{E} = \sigma E_0^2 + 2\beta J_0 E_0 + \frac{1+\beta^2}{\sigma} J_0^2 - \frac{2}{3}\frac{\epsilon_e}{e n_e} J_0 \frac{\partial n_e}{dx}. \quad (2)$$

Local thermodynamic equilibrium is assumed to hold, and consequently Saha's equation can be used to relate the electron density n_e, to the electron energy, ϵ_e. From the Saha equation, it is apparent that a change of 20% in ϵ_e produces an order of magnitude change in n_e. Therefore, as in Ref. 5,

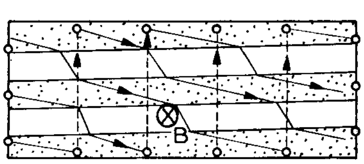

FIG. 2. Schematic current pattern which produced current streamers in Fig. 3.

[5] T. A. Cool and E. E. Zukoski, Phys. Fluids 9, 780 (1966).

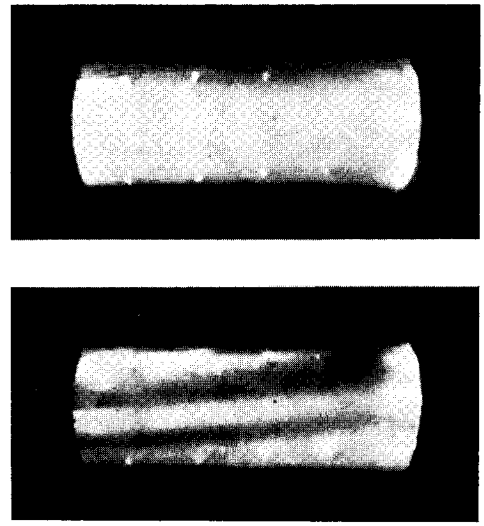

FIG. 3. Pictures taken through the window in the test section. (a) Uniform luminosity produced when only an electric field was applied; (b) the current streamer pattern that appeared when a magnetic field was added.

the terms $\partial\epsilon_e/\partial t$ and $\nabla\epsilon_e/\epsilon_e$ are dropped in comparison to $\partial n_e/\partial t$ and $\nabla n_e/n_e$.

The diffusion approximation was made to the radiation term

$$\dot{R} \approx \kappa_R \frac{\partial^2 n_e}{\partial x^2} + R_L n_e.$$

Although this is not a precise representation for this term, it is felt that it gives the correct local behavior. In this approximation, radiation with a mean free path less than $\lambda/2\pi$, where λ is the wavelength of the disturbance, was included in the diffusion term, and radiation with a mean free path greater than the characteristic length of the duct was included in the escape term $R_L n_e$. The remaining radiation with mean free path of the order of the wavelength was not taken into account. Calcula-

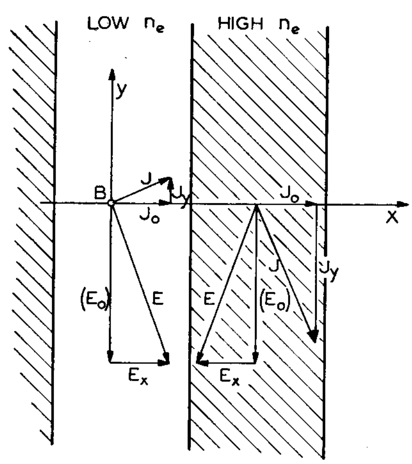

FIG. 4 Coordinate system for one-dimensional problem.

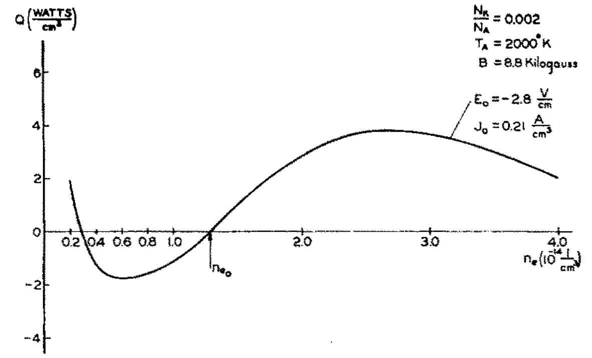

FIG. 5. Electronic heating term, $Q(n_e)$.

tions for a sine wave disturbance in n_e indicate that the omitted term would increase the effective diffusion coefficient by less than a factor of two.

For the conditions of the experiments, the electronic heat conduction was much less than that due to radiation, and this term, $\nabla \cdot \kappa_e \mathbf{u} \nabla \epsilon_e$, was dropped. The heat loss due to elastic collsions, $\dot{\Omega}$, was calculated by the technique discussed in Ref. 5.

With these approximations, the energy equation becomes

$$\frac{\partial n_e}{\partial t} + \frac{a}{n_e}\frac{\partial n_e}{\partial x} - b\,\frac{\partial^2 n_e}{\partial x^2} = \frac{Q(n_e)}{(\epsilon_e + \epsilon_i)}\,, \qquad (3)$$

where

$$a = \frac{2}{3}\,\frac{J_0 \epsilon_e}{e(\epsilon_e + \epsilon_i)}\,, \qquad b = \frac{\kappa_R}{\epsilon_e + \epsilon_i}\,,$$

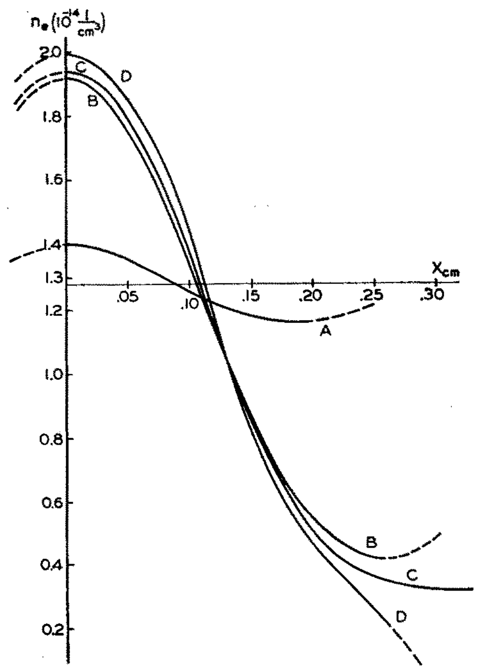

FIG. 6. Wave shapes at various amplitudes for $Q(n_e)$ in Fig. 5.

and

$$Q(n_e) = \left(\sigma E_0^2 + 2\beta J_0 E_0 + \frac{1+\beta^2}{\sigma}\,J_0^2 - \dot{\Omega} - R_L n_e\right).$$

$Q(n_e)$ is shown in Fig. 5 for a typical set of conditions. The solution for uniform n_e and for specified J_0, E_0, and B, is n_{e0}. However, since dQ/dn_e at n_{e0} is positive, this is an unstable point, and perturbing n_e about n_{e0} gives a growth rate of 10^5 sec and a velocity of 10 m/sec for linear disturbances. These results are similar to those obtained by Kerrebrock from a linear analysis of Eq. (1).

For cases of interest, the speed a/n_e is small. Therefore, in the first approximation to the steady problem, the first two terms of Eq. (3) are negligible, and the equation reduces to

$$-\kappa_R\,\frac{d^2 n_e}{dx^2} = Q(n_e). \qquad (4)$$

That is, the electronic heating is balanced only by radiative diffusion.[6] Note that this equation is similar to that describing a nonlinear spring except that the independent variable is x rather than t. Integration of Eq. (4) gives

$$x = \int \left\{\int \frac{2Q}{\kappa_R}\,dn_e + C_1\right\}^{-\frac{1}{2}} dn_e.$$

For the conditions of the experiment, given in Fig. 5, the expression was integrated numerically. This gave the family of curves for $n_e(x)$ for various amplitudes, C_1, shown in Fig. 6. The curves show that for a critical value of C_1 a solitary wave, labeled C, is obtained. For larger values, the electron density decreases toward zero with increasing x, and consequently these solutions are not of interest in regions away from the boundary. The solutions for values of C_1, less than critical, are periodic and hence resemble the waves found in the laboratory. The half-width of the waves varies from 0.18 cm at a very low amplitude to 0.24 cm for the solitary wave. The maximum variation of electron density for a periodic solution is that for the solitary wave and has a value of $\Delta n_e/n_{e0} = 1.3$.

The most interesting features of these solutions are that the nonlinearity in $Q(n_e)$ has given an estimate of maximum amplitude to which a periodic wave can grow and that the amplitude and half width of the associated wave are the same as those measured in the laboratory.

The present work is incomplete in that no analysis

[6] Reference 4 treats the unsteady problem where the first term, $\partial n_e/\partial t$, balances $Q(n_e)$ in Eq. (3).

has been made of the stability of the large amplitude waves; however, the experimental evidence indicates that they are stable.

In conclusion, steady current streamers can exist, and their amplitude ($\Delta n_e/n_{e0}$) and half-widths have been measured to be 1.5 and $\frac{1}{4}$ cm, respectively. These characteristics are in agreement with an extension of the theory of electrothermal waves to the case of steady, one-dimensional, large amplitude disturbances. In this limit, the driving force for the electrothermal waves, unstable electron heating, is balanced by radiative transport of heat. Work is continuing on the theoretical and experimental aspects of this problem.

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Wave Propagaton in a Partly Ionized Gas. II

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Simple dispersion relations for longitudinal waves are derived for plasmas satisfying the hydrodynamic equations subject to the conditions: $10^{10} < N_{n0} < 10^{16}$ cm^{-3}; $10^9 < N_{e0} < 10^{14}$ cm^{-3}; $T_e \gg T_{i,n}$; $T_{i,n} < 10^4$ °K $< T_e$. In addition, the exact solutions to the dispersion equation are obtained (for arbitrary number densities and temperatures) in the low-frequency limit, and the damping factor for the low-frequency acoustic mode is calculated.

1. INTRODUCTION

The dispersion relations for plane waves in a homogeneous plasma are required not only for simple wave propagation problems, but also for a wide class of radiation, initial value, and boundary value problems. Therefore this question has received wide attention in the literature both from the kinetic theory[1] and hydrodynamic[2] models for a plasma. One aspect of the problem which has received relatively little attention, however, is the propagation of longitudinal waves in a partly ionized gas, where, for sufficiently low frequencies, the motion of ions, electrons, *and* neutrals are all of importance. This has been treated to some extent for the case when the species of particles all have equal ambient temperatures,[3,4] when the degree of ionization is very low (and it is also assumed that $T_e \gg T_i, T_n$),[5,6] and for low frequencies.[7]

Each of these treatments, however, is limited by factors such as the range of frequencies where the results can be applied, the exact limitations of the approximations used, or the complexity of the solutions. Hence, in this paper we wish to describe a new solution to this problem which leads to relatively simple dispersion relations for a broad range of conditions that are satisfied in many plasma experiments. Although this solution, too, has its limitations, we feel that, because of its simplicity, this solution may be useful in many plasma physics problems.[8]

2. THE DISPERSION EQUATION

The dispersion equation for longitudinal waves in a partly ionized gas with equal ambient electron and ion number densities, no magnetic field, adiabatic equations of state for each species,[9] and momentum transfer terms proportional to the drift velocity difference between species is given in Ref. 3 as

$$(C_1^2/m) - C_3C_5 = 0, \tag{2.1}$$

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[1] See for example, T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill Book Company, Inc., New York, 1962), p. 107.

[2] See for example, W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (The Massachusetts Institute of Technology Press, Cambridge, Massachusetts, 1963), p. 1.

[3] B. S. Tanenbaum and D. Mintzer, Phys. Fluids **5**, 1226 (1962).

[4] D. Kahn, Phys. Fluids **8**, 399 (1965).

[5] G. M. Sessler, Phys. Fluids **7**, 90 (1964).

[6] U. Ingaard and K. W. Gentle, Phys. Fluids **8**, 1396 (1965).

[7] L. W. Parker, Phys. Fluids **9**, 274 (1966).

[8] The results have already been used, for example, to calculate power radiated by a current source in a plasma [D. J. Connolly and B. S. Tanenbaum, J. Appl. Phys. **38**, 2557 (1967)].

[9] Note that our omission of energy transfer effects due to collisions between particles of different species excludes the wave amplification mechanism described in Ref. 6 as well as some additional damping effects described in Ref. 4.