Transition in Incompressible Near-Wakes

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An experimental study of the near-wakes of a family of flat-based cylinders of varying chord-thickness ratio, L/d, shows that the phenomena of vortex shedding and transition are similar to those for a cylinder of circular section. The Reynolds number for appearance of turbulence in the wake is correlated with L/d.

ATTEMPTS to define a criterion for transition in a separated flow in terms of a Reynolds number based on distance x_s from the separation point cannot be successful because the instability of a free shear layer is practically independent of viscosity. The distance from separation to transition is simply proportional to the thickness δ_s at separation.[1] If the shear layer remains free, then the oscillations which begin immediately after separation are continuously amplified until transition occurs. But, if the shear layer reattaches onto a wall [Fig. 1(b)], or rejoins with an opposite shear layer [Fig. 1(a)], the amplification process may be terminated and transition averted or postponed. For example, it has been suggested[2] that, in the wake of a vortex shedding circular cylinder, transition to turbulence must occur (in the shear layers) before the vortices fully form and break away; otherwise it will not occur. The place where the vortices "roll up" and "break away" is, in fact, the region of rejoining of the shear layers {a rejoining point r can be defined for the steady mean flow [Fig. 1(c)] obtained by averaging over a time long compared to the shedding period}.

In previous work,[3] similar ideas were used to find a criterion for transition not to occur in the separated flow rettaching onto a step or splitter plate. It was based on the specification that the distance to transition be greater than the distance to reattachment, $x_t > x_r$. This then led to the relation $\delta_s/d >$ const (on the assumption that x_r was proportional to the base dimension d) and thus to the criterion in the form

$$(L/d)^2 > \text{const}\, R_L, \qquad (1)$$

where R_L is the Reynolds number based on the length L of a body belonging to the family of elongated shapes sketched in Fig. 1(a).

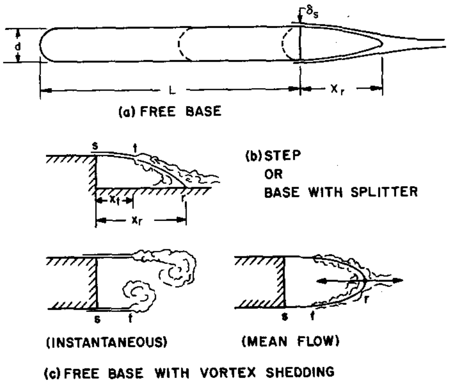

FIG. 1. Examples of near-wake and base flows.

The above may be interpreted as follows. The near-wake flow of this family of bodies (characterized by the fineness ratio d/L) is similar for similar values of δ_s/d. The implication is that, approaching transition, the near-wake flow is independent of viscosity—not only the free shear layer stability, but the near-wake dynamics as well. In fact, the reattachment or rejoining process should be little affected by viscosity, the dominant stresses being Reynolds stresses connected with the amplifying (not yet turbulent) nonstationary motion. It is quite possible to develop Reynolds stresses in this case which are as large or larger than in fully developed turbulent flow.[4]

An experimental determination of first appearance of transition in the near-wake of a free base was made for a family of elongated shapes like those sketched in Fig. 1(a). The models were constructed from $\frac{3}{32}$-in.-thick flat ground steel plates, on which elliptical leading edges (eccentricity 2 : 1) were machined. The three models used had lengths of $\frac{1}{2}$, $\frac{3}{4}$, and $1\frac{1}{2}$ inches, giving values of L/d of $5\frac{1}{3}$, 8, and 16, respectively. Each model spanned the 20-in.-square cross section of the open-circuit wind tunnel in which the experiments were performed.

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[1] H. Sato, J. Phys. Soc. Japan **11**, 702 (1956).
[2] A. Roshko, NACA Report 1191 (1954).
[3] A. Roshko and J. C. Lau, in *Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, California, 1965), p. 157.
[4] P. Bradshaw, J. Fluid Mech. **26**, 225 (1966).

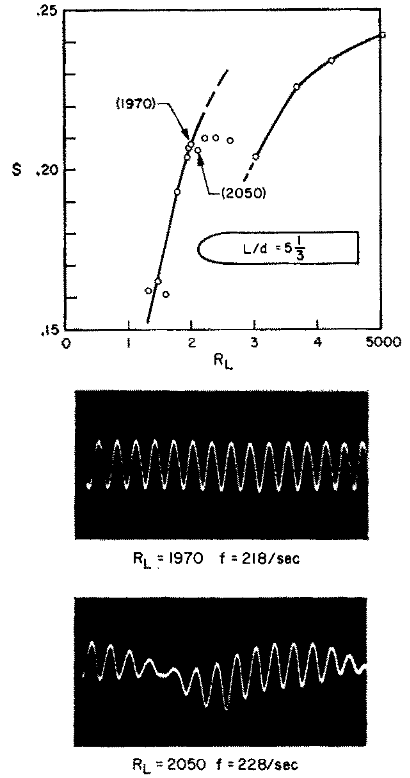

FIG. 2. Method of determining first appearance of turbulent disturbances in near wake. R_L = 1970, laminar periodic flow; R_L = 2050, disturbed flow.

The method of determining transition was similar to that used in Ref. 2, namely observation of the signal from a hot-wire anemometer, placed a few diameters downstream of a shoulder of the model. The characteristic change from a regular, periodic signal is shown in Fig. 2. When this irregularity occurs, there is also a characteristic break in the Strouhal number–Reynolds number curve, as shown in the figure. This is similar to what occurs for a circular cylinder at a Reynolds number of about 150 (up to 200 has been observed[5]). Points corresponding to this first appearance of a turbulent disturbance are plotted (crosses) in Fig. 3. The line $R_L = 70(L/d)^2$ drawn through them defines the constant in Eq. (1). The constant is only a little smaller than that previously found[3] for flows reattaching onto a step or splitter plate, corresponding to the line $R_L = 100(L/d)^2$. The constant in the latter case could possibly be somewhat less than 100—the first appearance of turbulent disturbances was not as precisely determined as in the present experiments.

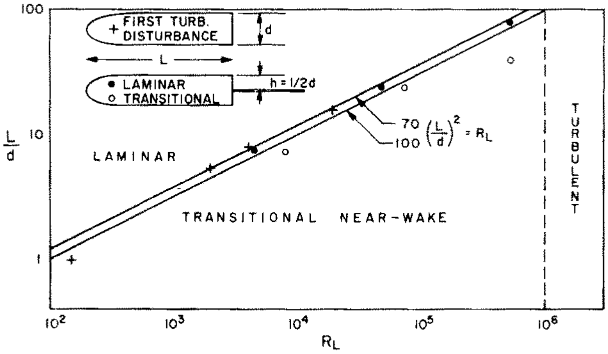

FIG. 3. Regimes of near-wake flow.

It seems remarkable that there should be so little difference in the criteria for the two cases, namely, a step flow with reattachment onto a wall and a free wake in which periodic vortex shedding is a prominent feature. It indicates, perhaps, that the mean flow in the latter case is not greatly different from the step flow. It should, however, be noted that the free base flows do not appear to follow the criterion to as high values of L/d and R_L as do the reattaching flows. (Some results of experiments on bodies with $L/d = 32$ and 40 are not included here because of somewhat ambiguous indications of the first appearance of a turbulent disturbance.)

If $R_L = 10^6$ (broken line in Fig. 3) is taken as a nominal value for transition to occur in the boundary layer on the body *before separation* (in which case the wake will be completely turbulent), then the regimes delineated in Fig. 3 may be defined. The near-wake is *transitional* (with transition point somewhere between separation and closure) in the region between the lines $R_L = \text{const}\,(L/d)^2$ and $R_L = 10^6$. From this diagram it is seen that the range of transitional flow in the near wake is small for bodies of large L/d, but can extend over several decades of Reynolds number for small L/d. Thus the near-wake of a circular cylinder is transitional from $R \doteq 150$, at which turbulence first appears in the vortex-formation region, up to $R \doteq 3 \times 10^6$, at which transition finally establishes itself upstream of separation.[6]

This diagram is highly idealized, with effects of geometry (nose shape, etc.) accounted in only a crude way. Departures from the correlation can be expected at low L/d, where geometric effects are important and where, due to the low R_L, the assumptions about the negligible role of viscosity may be bad. We would also expect departures at

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[5] M. S. Bloor, J. Fluid Mech. **19**, 290 (1964).

[6] A. Roshko, J. Fluid Mech. **10**, 345 (1961).

high $R_L(\sim 10^5)$, where development of instability may begin in the boundary layer before separation. We expect, however, that in a more exact treatment the transitional regime will roughly conform to that indicated in Fig. 3.

Some interesting problems are implicit in this class of transitional, separated flows: (1) There is a strong interaction between the unstable, oscillatory flow (before transition) and the flow field. The large Reynolds stresses that can be developed[4] can modify the flow field, this can affect the development of the instability, etc. One aspect of this, the restabilization, or shifting to another mode, due to rejoining of shear layers, is, in fact, the basis for our correlation. (2) There is a possibility for the following kind of instability. The Reynolds stresses developed in the amplifying disturbance tend to shorten the distance to rejoining, this decreases the distance available for amplification, this reduces the stresses, which increases the distance to rejoining, etc. The irregular, low frequency disturbances which characterize the first appearance of turbulence[5] (Fig. 2) might be associated with such an instability. (3) Berger[7] has shown that, by oscillating a circular cylinder normal to the flow direction, at appropriate frequencies, the range of laminar, pure-periodic vortex shedding can be increased to a cylinder Reynolds number of 350. One suggestion for this effect is that the forced oscillation "organizes" the flow in the spacewise direction and delays the development of three-dimensional disturbances. [The irregular disturbances described in (2) are attributed to three-dimensional effects.] Another possibility is that augmented, laminar velocity fluctuations induced by the cylinder oscillation increase the Reynolds stresses and shorten the distance to rejoining sufficiently to delay transition, according to the ideas developed here.

ACKNOWLEDGMENTS

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[7] E. Berger, in Jahrbuch der Wissenschaftlichen Gesellschaft für Luft- und Raumfahrt, 1964; p. 164.