Effective Control of the Error in a Direct Measurement of Core-Loss Power

F. Dong Tan, *Member, IEEE,* Jeff L. Vollin, *Member, IEEE,* Slobodan M. Ćuk

***Abstract*—Accuracy of a direct measurement of core-loss power is severely limited in many applications by the extra phase shift between the measured voltage and current. An analysis of the error is for the first time performed, which not only unfolds the hidden physical nature behind it, but also reveals a simple and effective method to control the sensitivity and hence the error. This method can also be used to satisfy a prescribed tolerance. Extensive measurements on a TDK PC40 core yield results which support the analysis.**

I. Introduction

In MAGNETIC designs, the core loss is one of the two essential parameters representing inherent characteristics of a magnetic material. Because of the nature of the magnetic field and the nonlinear mechanisms involved in the process of magnetization, the core-loss power cannot be expressed by accurate, yet simple closed-form formulas. Hence, approximate models, together with measurement techniques, are frequently used instead [1]–[4].

There are basically two categories of measurement techniques: indirect and direct methods. The indirect measurement method is typified by the calorimetric method. It enjoys high accuracy in measuring the dissipated power. However, it is very difficult to set up, and it cannot distinguish the copper loss (or the core loss) from the total loss.

Direct measurement methods measure directly the voltage, current, and the phase angle between them. Then the core-loss power is constructed mathematically. The appealing advantage is that it is easy to set up and to reproduce a measurement. Also, as reported in [5], a direct method generated results which were confirmed by an independent calorimetric method to be within $\pm 4\%$ in error.

The direct measurement method has been popular for simplicity and reasonable accuracy. (See, for example, [6]–[11]). However, in many applications, its accuracy is severely limited by the extra phase shift. Severe limitation in accuracy happens when the phase angle between the voltage and current approaches $90°$. It can be shown that a small percentage error in the measured phase angle can introduce an error in the measured core-loss power which can be larger than 100%. Researchers [8] and magnetic materials producers [1], [2] identified the problem long ago, but no solution has been proposed. Common practices have been limited to state simply the fact to caution the user [1], [2], to use awkward methods to control the extra phase shift [7], or to resort to exotic equipment [10], [11].

This paper presents for the first time an analysis of the error, which not only unfolds the hidden physical nature behind the error, but also reveals a simple and effective method to control it. This method is simple since it is easy to use and it employs conventional equipment. It is effective since it can reduce the error by an order of magnitude.

A. Outline of Discussion

Section II discusses an approximate model for the core-loss power. Section III describes details of a typical direct measurement setup. To facilitate the error analysis, an approximate equivalent circuit is derived for a toroidal two-winding transformer. Details on how to obtain data with dc bias are also included. Section IV presents an analysis for the error in a direct measurement of core-loss power, which reveals a simple and effective method to control it. The method can be used to satisfy a prescribed tolerance. Section V provides core-loss data at several hundred kilohertz for TDK PC40 and compares them to typical data provided by TDK. These two sets of data have good agreement, validating the analysis. Section VI summarizes key results.

II. An Approximate Model

In this section an approximate model is selected to illustrate the proposed method. Note that the proposed method is independent of the selection of models.

A. The Core-Loss Power

The core loss is the sum of three components: the hysteresis loss, the eddy current loss, and the residual loss. The hysteresis loss is due to the multivalued nature of the hysteresis loop. Suppose that a magnetic specimen is excited from zero to the maximum field and then back to zero field. At the end, the returned power is observed to

Manuscript received March 15, 1993; revised November 11, 1994.

F. Dong Tan was with the Power Electronics Group, 116-81 Caltech, Pasadena, CA 91125. He is now with QSC Audio Product, Costa Mesa, CA.

J. L. Vollin was with the Power Electronics Group, 116-81 Caltech, Pasadena, CA 91125. He is now with the Missile Systems Group, Hughes Aircraft Co., Tucson, AZ.

S. M. Ćuk is with the Power Electronics Group, 116-81 Caltech, Pasadena, CA 91125.

IEEE Log Number 9409001.

0018-9464/95$04.00

be less than the supplied. The lost power can basically be considered to have been used for the reorientations of the magnetic domains. This loss is proportional to the area encircled by the upper and the lower traces of the hysteresis loop. The area can be obtained by an integration and is proportional to B^2. Furthermore, if the shape of the loop remains the same for each successive recitation, the lost power is simply the product of the area and frequency. Hence, the hysteresis loss is directly proportional to frequency f and the square of peak flux density B^2 [3], [4].

If a time-varying magnetic field is applied to a specimen, eddy currents will be induced. These currents generate a certain amount of ohmic loss, which is normally called the eddy current loss. To reduce eddy current loss, materials with high resistivities such as ferrites are preferred. However, high resistivities are usually coupled with low permeability. Hence, a tradeoff exists between them for material designs. The mechanism for eddy current loss is the same as that in an conductor. Therefore, the eddy current loss is proportional to f^2 and B^2 [4].

The hysteresis loss and the eddy current loss account for a large portion of the total core loss. The rest of it is normally called the residual loss. Mechanisms behind these are complex. Experience indicates that the residual loss is proportional to frequency f and peak flux density B.

Since all three components are, to variable extend, related to f and B, the following empirical formula, which is frequently used in industry [12], is chosen as the model for the core-loss power,

$$P_{fe} = kV_e f^m B^n \tag{1}$$

where V_e is the volume of the core, f the operating frequency, and k, m, and n are real constants.

B. Curve Fitting

The mathematical model for the core-loss power requires the determination of the constants k, m and n. These constants can be determined by the technique of curve-fitting. The determined values for parameters k, m and n can then be regarded as inherent material constants. Since the technique of curve fitting is well known, details of it are not pursued further.

III. A Measurement Setup

Magnetic core manufacturers usually provide the loss data for symmetrical sinusoidal flux density, because of its convenience for measurements and specifications. Also, sinusoidal data can be used to estimate data for non-sinusoidal cases [12], [13]. (For example, it was found recently in [13] that the core loss corresponding to a bipolar square-wave voltage is related to that of a sinusoidal voltage by a multiplying factor $8/\pi^2$.) Hence, the practice is followed here.

Fig. 1 is a schematic of the actual setup of a popular scheme for direct measurement of the core loss. An

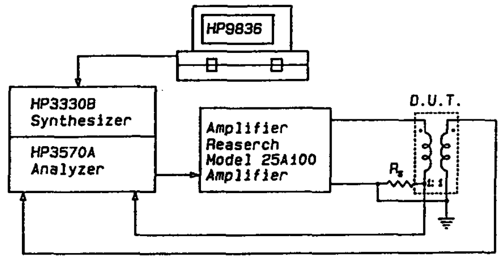

Fig. 1. Schematic for the measurement setup.

HP3330B synthesizer and an HP3570A network analyzer are used for the acquisition of measured data. A sinusoidal voltage from the synthesizer is amplified by an amplifier (Amplifier Research Model 25A100). This voltage is then applied across the primary of a toroidal transformer. The secondary winding is wound in a bifilar fashion with a turns-ratio 1 : 1. The measured voltage is the one across the open-circuited secondary terminals. The measured current is the sensed current from the current sensing resistor. This configuration can avoid the effect of the primary leakage impedance, the sum of the leakage inductance and the winding resistance, to the extent that the equivalent lumped circuit model is valid [14], [15].

The flux density is directly monitored by a scope, viewing the waveform of the measured voltage, since this voltage is proportional to the derivative of the flux density with respect to time. During all the measurements this waveform is closely monitored for any possible distortion in order to guarantee the desired sinusoidal flux density. For high-frequency applications (above a few hundred kilohertz), most designs are core loss limited. The flux density is kept well below saturation and the device works primarily on the linear part of the B–H loop. Therefore, the distortion can be assumed negligible.

Fig. 2 is a schematic diagram of the measurement fixture which is built with special attention to minimize parasitic capacitance and inductance. The sensing resistor is implemented with 10 carbon resistors connected in parallel. Its frequency response was measured to be flat from 1 kHz to 20 MHz. A 70-μH inductor is introduced to bypass the dc current in case a dc bias test is performed. A dc bias current I_{dc} is injected by an auxiliary circuit using an HP6130C digital voltage source controlled by an HP9836 computer (not shown in the figure). The fixture with a dc bypass inductor was checked for its frequency response. No additional phase shift is introduced in the frequency range of interest.

The whole measurement setup is then calibrated with terminals 1, 2, and 3 all shorted together, which accounts for all the remaining parasitics associated with coax cables, probes, etc.

Finally, the device under test is demagnetized before a measurement by reducing the magnitude of the ac excitation voltage gradually to zero (dynamic demagnetization).

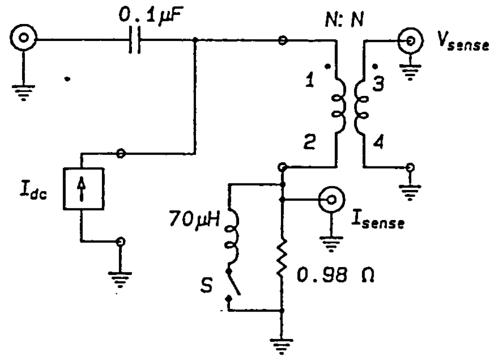

Fig. 2. Schematic for the measurement fixture.

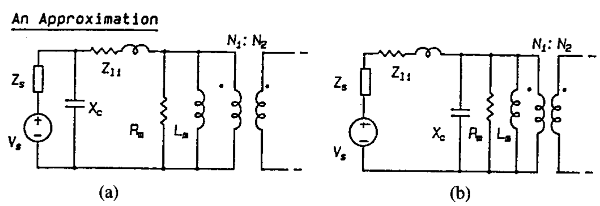

Fig. 3. Equivalent circuits for a two-winding transformer.

A. An Equivalent Circuit

Fig. 3(a) is a general equivalent circuit model for a transformer. It was used successfully for very wideband RF transformer designs in the frequency range from 100 kHz to 30 MHz [15]. Z_s is the output impedance of the source. X_c is the reactance presented by the shunt parasitic capacitance of an inductive winding. Note that it also includes the input capacitance of a voltage probe used to measure the voltage. Z_{l1} is the series combination of the leakage inductance and the winding dc and ac resistances. L_m is the magnetizing inductance associated with the main flux of the transformer. R_m is a series resistance representing the core loss.

An approximate circuit is shown in Fig. 3(b). The condition for a good approximation can be established as

$$Z_s + Z_{l1} \ll X_c. \tag{2}$$

This condition is frequently well satisfied since the parasitic capacitance is on the order of ten picofarads.

IV. The Error Analysis

The error associated with a direct measurement of core-loss power is first analyzed. Then an error introduced by parasitic capacitance is identified.

A. The Error Due to the Extra Phase Shift

The core-loss power P of the device under test can be constructed from the measured voltage V, current I, and the phase angle ϕ between them by the following well-known expression:

$$P = VI \cos \phi. \tag{3}$$

The potential error associated with the measured power is given by the corresponding total increment of P

$$\Delta P = \frac{\partial P}{\partial V} \Delta V + \frac{\partial P}{\partial I} \Delta I + \frac{\partial P}{\partial \phi} \Delta \phi. \tag{4}$$

Since the voltage and the current can usually be measured with sufficient accuracy, ΔV and ΔI can be assumed to be vanishingly small. Hence, without loss of generality, the total increment of the power can be simplified as

$$\Delta P = \frac{\partial P}{\partial \phi} \Delta \phi \tag{5}$$

where $\Delta\phi$ is the extra phase shift associated with the measurement setup, $\partial P/\partial \phi$ is the partial derivative, and ΔP the power error due to $\Delta\phi$.

A more useful expression related to the power error ΔP can be obtained if the relative value of the power error is used. Straightforward algebra leads to

$$\left| \frac{\Delta P}{P} \right| = |-\tan \phi \Delta \phi| = |\tan \phi| \, |\Delta \phi|. \tag{6}$$

Equation (6) contains rich information about the physical nature of the relative power error. Table I tabulates several values of the error for different values of ϕ. It is seen that the error increases drastically as ϕ approaches 90°. For example, at $\phi = 88°$, $\Delta P/P$ is 150%.

More importantly, the value of the error is highly sensitive to the value of ϕ. For example, if $\phi = 85°$ and the extra phase shift is 3°, then the relative error is (149.9 − 59.8)% = 90%. That is, a 3/85 = 3.5% error in the phase measurement introduces 90% additional error in the measured core-loss power. Even worse, this high sensitivity of the error only gets much larger when the phase angle approaches 90° further.

Analytically, (6) says that as ϕ approaches 90°, $|\tan \phi|$ approaches infinity. Hence, $\Delta\phi$ has to go to zero in order for $\Delta P/P$ to have a small but finite value. This is a severe limitation inherently associated with the accuracy of a direct measurement of core-loss power. It has hindered wider application of the direct measurement method. A simle and effective method to control the error has long been desired.

Previous efforts in this regard have resulted in limited success [7], [10], [11]. The reason behind the limited success can be explained readily by (6). If ϕ approaches 90°, any effort to control $\Delta\phi$ is doomed ineffective, since $|\tan \phi|$ is very large and $\Delta\phi$ always has a finite value ($\Delta\phi$ can be small but cannot be eliminated in any actual measurement setup).

Furthermore, (6) suggests a simple and effective alternative. Indeed, (6) shows that the relative error $|\Delta P/P|$ is simply $|\Delta\phi|$ scaled by $|\tan \phi|$. Hence, reduction of the scaling factor will reduce the error for a given $\Delta\phi$. By the nature of the tan function, the sensitivity of the error can also be reduced effectively by moving ϕ away from 90°. For example, if $|\phi|$ is around 45°, $|\tan \phi|$ will be of unity. Then the relative power error is primarily determined by $\Delta\phi$, which can be minimized by engineering effort.

An example will drive home the idea. Assume the de-

TABLE I
SENSITIVITY OF THE ERROR IN MEASURED CORE-LOSS POWER

ϕ	60°	70°	80°	85°	87°	88°
$\Delta P/P(\%)$	9.1	14.4	29.7	59.8	99.8	149.9

sired relative power error is 10%. From (6), straightforward algebra yields,

$$-\frac{1}{10|\Delta\phi|} \leq \tan\phi \leq \frac{1}{10|\Delta\phi|}. \tag{7}$$

It gives a range of values for ϕ within which the relative power error will be at most $\pm 10\%$ away from the actual value. This range of values can be considered as a feasible region for ϕ corresponding to given values of $|\Delta\phi|$ and $|\Delta P/P|$. If the extra phase shift is limited to be $|\Delta\phi| = 3°$, a typical practical value, the corresponding feasible region of the phase angle is given by $|\phi| \leq 63°$. Then, according to (6), the relative power error is less than $\pm 10\%$.

B. Desired Phase Angle

In an actual measurement, the phase angle may not be in the feasible region frequently. In many applications of magnetic devices, low loss and high permeability are frequently required of a magnetic material. For a finite value of inductance, very low loss (a vanishingly small value for the resistance) directly translates into a phase angle close to 90°, where error sensitivity is very high as discussed previously.

To cope with this difficulty, one can use a low loss capacitor across the secondary (or the primary) to bring the phase angle to the desired value. Fig. 4 illustrates the underlying principle, which is a simple "detuning" process.

It is pointed out that this technique was mentioned in [16], [17], but its importance was not at all fully realized.

Note also that introduction of an additional capacitor will degrade the accuracy, as one may observe. There are two things one needs to look out for. Additional capacitance in the primary may upset the condition for the equivalent circuit in Fig. 3(b). Additional capacitance in the secondary may worsen the loading effect on the secondary, which exists due to the small input capacitance of the probe.

On the other hand, the degradation can be compensated for by tightening up the prescribed tolerance from, say, from $\pm 10\%$ to $\pm 5\%$. Equation (6) then shows that the feasible region is reduced from $\pm 63°$ to $\pm 45°$. Note that this self-adaptive feature of the approach, although advantageous, cannot eliminate the error due to the added capacitance. However, when compared to the error for the case with no added capacitance, the error for the case with added capacitance is not only greatly reduced (an order of magnitude reduction), but also well controlled.

C. An Error Due to the Parasitic Capacitance

The parasitic capacitance associated with the toroidal transformer can also introduce large error in determining

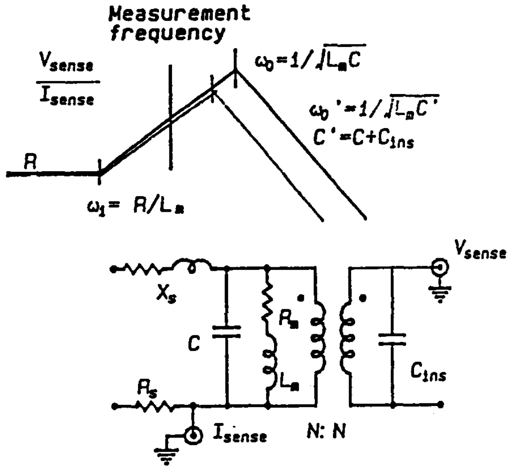

Fig. 4. Bring the phase angle to the desired value.

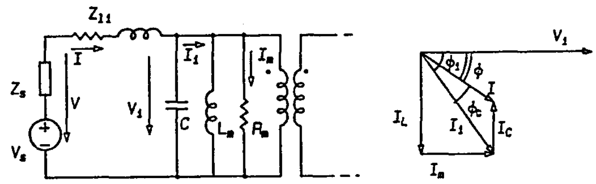

Fig. 5. A correction for measured current.

the permeability. Fig. 5 shows a derived approximate equivalent circuit together with the corresponding phasor diagram of various currents. V_1 is the measured voltage, I is the measured current, and I_1 is the current needed for determination of the permeability. L_m' and R_m' are the corresponding parallel impedance equivalent to the series impedance consisted of L_m and R_m. From the principle of the conservation of energy, the following is obtained:

$$I_1 = \frac{\cos\phi}{\cos\phi_1} I \tag{8}$$

where $\phi_1 = \phi + \phi_c$. The angle ϕ_c is due to the capacitive current I_c, and its value can be determined from an independent measurement of the frequency response of the impedance defined by V_1/I.

It can be seen that the closer the measuring frequency is to the self-resonance frequency of the inductive device under test, the greater the error is due to I_c. This error needs to be compensated if the determination of permeability is pursued.

V. EXPERIMENTAL VERIFICATIONS

A toroidal transformer was wound on a TDK T16-28-13 core with six turns for the bifilar winding. Data were obtained according to the proposed method at 50, 100, 200, 300, 400, and 500 kHz, respectively. With each frequency, the flux density was swept. Flux densities were maintained in the same range, 50–320 mT, for all measurements. Table II is a summary of the values for k, m, and n obtained from the curve fitting.

TABLE II
DETERMINED PARAMETERS FOR CORE LOSSES FOR TDK PC40

Freq.(kHz)	k^*	m^*	n^*	F^*
50	2.10	1.43	2.41	0.119
100	2.09	1.42	2.41	0.065
200	2.02	1.42	2.38	0.038
300	2.09	1.43	2.40	0.040
400	2.10	1.43	2.43	0.062
500	2.10	1.44	2.42	0.074
Average	2.08	1.43	2.41	0.066

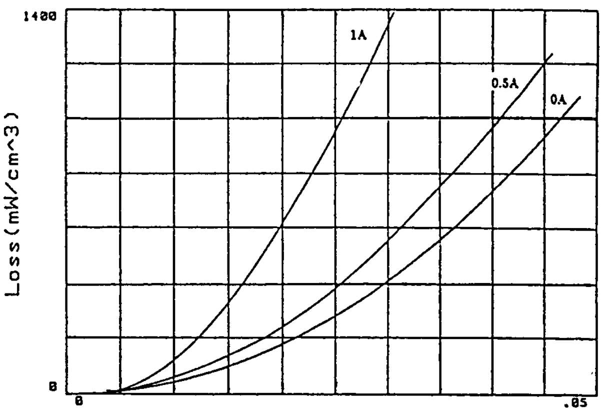

Fig. 6. Core losses for PC40 with dc biases.

The core loss for TDK PC40 material can be represented by

$$P_{fe} = 2.08 \times 10^{-6} f^{1.43} B^{2.41} \quad (9)$$

where P_{fe} is in watts per cubic centimeters, f in hertz, and B peak flux density in tesla. On the other hand, the data provided by TDK gives

$$P_{fe} = 2.00 \times 10^{-6} f^{1.46} B^{2.57}. \quad (10)$$

The small difference between the two may be attributed to two facts: 1) the sample-to-sample variation of a core material, and 2) the anisotropy compensation temperature for PC40 is placed around 80° according to TDK data. Our measurements were, however, done at room temperature. Hence the loss is expected to be slightly larger. Therefore, the agreement of the two sets of data is good, validating the proposed approach.

Fig. 6 presents core losses for PC40 at different dc bias current values. It can be seen that the core loss increases drastically with the increase of dc bias current. The implication of this result is significant. A magnetic device with a dc bias flux can have core loss several times larger than what is expected from using manufacturer's data with no dc bias. It is pointed out that the resources and timing didn't allow an investigation into the physical explanation of the large increase of core loss to proceed at the time this work was performed.

VI. CONCLUSIONS

An analysis of the error in a direct measurement of core-loss power is performed for the first time. The nature of the error is similar to that of a *tan*-function.

A method is then proposed to control the error. This method is simple since it is easy to use and it employs conventional equipment. It is also effective since it can reduce the error by an order of magnitude.

A TDK PC40 sample core is measured extensively at 50–500 kHz. Results are compared with typical data provided by TDK. Good agreement is observed, validating the analytical results.

This approach can be used to determine accurately the core-loss power for a given magnetic material at different frequencies and flux densities, in the event that the data are not available or the sample-to-sample variation has to be considered. Applications of the approach are independent of the selection of models for the core loss power. This approach is equally applicable to cases where dc bias is present.

REFERENCES

[1] "Soft Ferrites: A User's Guide," Magnetic Materials Producers Ass., Chicago, IL, pp. 27–29, 1992.
[2] T. Mitsui and G. V. Schaick, "Ferrite power material for high-frequency applications," *Powertechnics Mag.*, pp. 15–19, Feb. 1991.
[3] M. R. Bozorth, *Ferromagnetism.* New York: Van Nostrand, 1951.
[4] E. C. Snelling, *Soft Ferrites: Properties and Applications.* second edition, England: Butterworths, 1988.
[5] D. K. Conroy, G. F. Pierce, and P. R. Troyk, "Measurement techniques for the design of high frequency SMPS transformers," in *IEEE APEC Proc.*, pp. 341–351, 1988.
[6] B. Carsten, "Fast, accurate measurement of core loss at high frequencies," in *PCIM Proc.*, 1986, pp. 14–24.
[7] T. Sato, Y. Sakaki, "100 kHz-10 MHz iron loss measuring system," *IEEE Trans. Magn.*, vol. 23, no. 5, pp. 2593–2595, Sept. 1987.
[8] V. J. Thottuvelil, T. G. Wilson, and H. A. Owen, "High-frequency measurement techniques for magnetic cores," *IEEE Trans. Power Electron.*, vol. 5, no. 1, pp. 41–53, Jan. 1990.
[9] A. F. Goldberg, "High field properties of nickel-zinc ferrites at 1–10 MHz," in *IEEE APEC Proc.*, pp. 311–318, 1988.
[10] P. M. Gradzki and F. C. Lee, "High frequency core loss characterization technique based on impedance measurement," in *HFPC Proc.*, pp. 108–115, 1991.
[11] J. A. Ferreira and J. van Wyk, "Experimental evaluation of losses in magnetic components for power converters," *IEEE Trans. Ind. Applic.*, vol. 27, no. 2, pp. 335–339, 1991.
[12] R. Severns, "HF core losses for non-sinusoidal waveforms," in *HFPC Proc.*, pp. 140–148, 1991.
[13] W. Roshen, "Ferrite core loss for power components design," *IEEE Trans. Magn.*, vol. 27, no. 6, pp. 4407–4415, Nov. 1991.
[14] H. P. J. Wijn and J. J. Went, "The magnetization process in ferrites," *Physica*, vol. XVII, no. 11-12, pp. 976–992, 1951.
[15] D. Maurice and R. H. Minns, "Very-wide band radio-frequency transformers," *Wireless Eng.*, pp. 168–177, June 1947, and pp. 209–216, July 1947.
[16] D. Y. Chen, "Comparisons of high frequency magnetic core losses under two different driving conditions: A sinusoidal voltage and a square-wave voltage," in *IEEE PESC Rec.*, pp. 237–241, 1978.
[17] D. Y. Chen, "High-frequency core loss characteristics of amorphous magnetic alloy," in *Proc. IEEE*, vol. 69, no. 7, pp. 853–855, July 1981.

F. Dong Tan biography not available at time of publication.

Jeff L. Vollin biography not available at time of publication.

S. M. Ćuk biography not available at time of publication.