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ABSTRACT
From deep optical images of three clusters selected by virtue of their X-ray luminosity and/or optical richness (1455+22; $z=0.26$, 0016+16; $z=0.55$ and 1603+43; $z=0.89$), we construct statistically-complete samples of faint field galaxies ($I\leq25$) suitable for probing the effects of gravitational lensing. By selecting clusters across a wide redshift range we separate the effects of the mean redshift distribution of the faint field population well beyond spectroscopic limits and the distribution of dark matter in the lensing clusters. A significant lensing signature is seen in the two lower redshift clusters whose X-ray properties are well-constrained. Based on these and dynamical data, it is straightforward to rule out field redshift distributions for $I\leq25$ which have a significant low redshift excess compared to the no evolution prediction, such as would be expected if the number counts at faint limits were dominated by low-$z$ dwarf systems. The degree to which we can constrain any high redshift tail to the no evolution redshift distribution depends on the distribution of dark matter in the most distant lensing cluster. In the second paper in this series, we use the lensing signal to reconstruct the full two-dimensional mass distribution in the clusters and, together with high resolution X-ray images, demonstrate that their structural properties are well-understood. The principal result is therefore the absence of a dominant low-$z$ dwarf population to $I\leq25$.

Key words: cosmology: observations – clusters: galaxies: evolution – galaxies: formation – galaxies: photometry – gravitational lensing.

1 INTRODUCTION
The surface density of faint galaxies is significantly in excess of predictions based on extrapolating to large redshift the known local properties of field galaxies, under the assumption of no evolution (Kron 1978, Peterson et al. 1979, Tyson & Jarvis 1979). The deepest optical counts are inconsistent with both modest or no evolution irrespective of the cosmological geometry (Tyson 1988, Yoshii & Takahara 1990, Metcalfe et al. 1990, Lilly et al. 1991) and reveal a gradual bluing with increasing apparent magnitude and no convincing turn-over to $B\sim28$ (Metcalfe et al. 1993).

The high surface density of blue light implies a star formation rate sufficient to yield a significant fraction of the metals in disk galaxies today (Cowie 1990). The nature of the population dominating the counts beyond $B\sim22$ is thus of considerable interest and depends critically on its redshift distribution. To the limits attainable with high throughput spectrographs on 4-m class telescopes ($B\sim24$), no significant departure from the predicted no evolution shape of the redshift distribution for $B$-selected samples has yet been seen (Broadhurst et al. 1988, Colless et al. 1990, 1993, Cowie et al. 1991, Glazebrook et al. 1993). The most rigorous statement on the redshift distribution can be made to $B\approx22.5$ (Colless et al. 1993a) where the incompleteness in a sizeable sample is less than 5%. At $B\approx24$ galaxies, where incompleteness remains $\approx15\%$ (Glazebrook et al. 1993), a proportion could be in a high redshift tail with $z\geq1$, but it is important to note that the incompleteness is negligible compared to the factor of $\times4-6$ by which the counts exceed the no evolution prediction. Notwithstanding the incompleteness, the bulk of the excess population of blue sources, if it exists as a separate entity, must lie within a volume consistent with the no-evolution prediction.

Determining the nature of the excess population is hard because of the difficulty of identifying representative ex-
amples for scrutiny. So long as the excess population is statistically-defined, physical properties such as luminosity functions or clustering scale-lengths cannot be reliably determined. Following Broadhurst et al. (1988)’s suggestion that the excess is associated with galaxies with intense [O II] spectral emission, Cole et al. (1993) found the excess population is co-spatial with its quiescent counterpart and Colless et al. (1993b) find such sources are often double systems. Such observations are consistent (but by no means prove) that star-formation induced by merging may simultaneously explain the counts and redshift data (c.f. Broadhurst et al. 1992). On the other hand, Efstathiou et al. (1991) and subsequent workers (e.g. Couch et al. 1993), present convincing evidence for a marked decrease in the angular clustering of $B \simeq 26$ galaxies which may support an alternative viewpoint that the blue light arises in recent star formation in a separate dwarf galaxy population whose present day counterparts cannot be found (c.f. Babul & Rees 1992). Some support for this model has come from limited spectroscopic surveys which have concluded that there is an excess of dwarf systems at the required redshifts (Cowie et al. 1991, Tresse et al. 1993).

The importance of the angular correlation function studies of faint field samples lies in the fact that the virtually all of the sources at $B=26$ represent the excess population i.e. difficulties in identifying the excess populations are largely removed. If it were possible to determine redshifts and luminosities for such a faint sample, even if only statistically, significant progress could be made. If the counts were dominated to the faintest limits by a recent era of dwarf galaxy formation, as proposed by Babul & Rees, conceivably the median redshift would hardly change for samples fainter than $B \simeq 24$. For a simple merger model, the median redshift closely tracks the non evolution prediction (c.f. Broadhurst et al. 1992), whereas if a significant fraction of $B=26$ galaxies originates in a primordial population there would be a rapid increase in the median redshift.

Unfortunately, conventional optical spectroscopy is rapidly approaching a hard faintness limit for two reasons. State of the art faint object spectrographs such as LDSS-2 (Allington-Smith et al. 1993) and MOSIS (Le Fèvre 1993) secure redshifts to $B=24$ in 4-6 hour exposures. Even with 10-m class telescopes, it will be painful to push the limits much beyond $B=25$. More importantly, Glazebrook et al. (1993) demonstrate convincingly how, as [O II] $3727$ Å is redshifted beyond $8000$ Å (for sources with $z > 1$) no useful diagnostic features can be seen in the optical region resulting in severe redshift incompleteness in any $z > 1$ tail. What is needed, therefore, is an independent method for determining the mean cosmological distance to a sample substantially fainter than $B=24$.

In this paper we describe how the gravitational lensing signal produced by rich clusters at different distances can constrain the redshift distribution of the faint galaxy population. The relevant samples are chosen to have $I \leq 25$ (corresponding approximately to $B \leq 27$). The technique is based on the weak distortion of background field galaxies first explored in a pioneering paper by Tyson et al. (1990). We have, however, extended the method, not only by imaging the field population to the same depth through several clusters at different distances, but also, significantly, by verifying the relative distribution of dark matter in the lensing clusters using a new inversion technique developed by Kaiser & Squires (1992). The latter result forms the basis of the second paper in this series (Paper II, Smail et al. 1994), which should ideally be read in conjunction with this paper.

A plan of this paper follows. In Section 2 we briefly review the lensing test proposed. This serves to explain in more detail the logic of this paper and its companion article. In Section 3 we discuss the new observations including target selection, data acquisition and reduction. Section 4 presents various statistical tests we have applied to the faint catalogues in the context of model redshift distributions. Our constraints are discussed in Section 5 and our conclusions are presented in Section 6.

2 THE LENSING METHOD AND PREVIOUS WORK

Our method to determine the mean distance to a $I \leq 25$ sample works as follows. The gravitational lensing of background galaxies by the cluster potential produces a coherent pattern of image distortions orthogonal to the cluster radius vector (Grossman & Narayan 1988). Although the weak signal is superimposed upon intrinsic ellipticities and orientations of the population, its coherent nature can be used to overcome the low signal to noise inherent in the statistics of faint image shapes. Of course, neither faint cluster members nor sources foreground to the cluster contribute to the lensing signal.

The most elementary test measures the proportion of objects to a fixed apparent magnitude limit aligned tangentially to the radius vector to the lens centre. In the idealised case of a sample of identical lenses at different redshifts, $z_{lens,i} = 1...n$, the variation in the the fraction of aligned images with redshift delineates the shape of the field galaxy redshift distribution, $N(z)$. In practice, of course, clusters have a variety of lensing powers and the observed fraction is controlled not only by the combination of the field $N(z)$ and $z_{lens,i}$, but also by the amount and concentration of mass in the lens $M(r)$. To decouple these two factors, more complex analyses are required.

The first stage of complexity is to allow some freedom in the core radius, $r_c$ (kpc) and the depth of the gravitational potential well, parameterised by $\sigma_{cl}$ (km sec$^{-1}$) the velocity dispersion of the cluster. In a given cluster according to a simple isothermal model. These parameters can be constrained to some extent from X-ray imaging data and galaxy dynamics. By applying a joint likelihood technique across all 3 clusters, each of which has been imaged in the same conditions, we can test whether the lensing signals are consistent with a sequence of ‘test’ field redshift distributions, $N(z)$.

It may be, however, that the distribution of dark matter in a cluster bears little relation to that observed for the X-ray gas and cluster members. In such a situation, the test described above would give misleading results. The companion paper to this article shows how the lensing signal measured across the cluster can be inverted, using a new technique developed by Kaiser & Squires (1992), to define a projected 2-D map of the lensing mass at a resolution adequate to check its concentration. Although the technique does not yield an absolute estimate of the total cluster mass, the results show

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how closely the dark and baryonic matter are distributed. This important result is sufficient to remove the ambiguities essential to determining an accurate estimate of the median redshift for a $I \leq 25$ field sample.

Our method is qualitatively different to previous lensing probes of the faint galaxy distribution, and thus we briefly review those studies in the context of our paper. Tyson et al. (1990) presented a pioneering analysis of two X-ray luminous clusters: Abell 1689 ($z = 0.18$, $L_X = 1.7 \times 10^{45}$ ergs sec$^{-1}$, $\sigma_{cl} = 1800 \pm 200$ km sec$^{-1}$) and Cl1409+52 ($z = 0.46$, $L_X = 9.2 \times 10^{44}$ ergs sec$^{-1}$, $\sigma_{cl} \sim 3000$ km sec$^{-1}$). The small CCD format available at the time restricted imaging to radii $r \leq 380$ and 500 kpc respectively (we adopt $H_o=50$ km sec$^{-1}$ Mpc$^{-1}$, $q_o=0.5$ throughout). Samples were selected with $B \in [22, 26]$ (equivalent to $I \in [20, 24]$) but only the bluer galaxies showed alignment tangentially to the cluster centres. However, the excess aligned component is relatively small, amounting to $\simeq$30-40 galaxies in A1689 and only 12 in Cl1409+52. Using this signal, Tyson et al. claimed that at least 70% of the $B \in [22, 26]$ population has $z \geq 0.9$. Applying a statistic based on the alignment signal they also derived radial ‘mass’ profiles for the clusters but it transpires these profiles represent the surface potential (c.f. Kaiser & Squires 1992).

Our study extends the analysis of Tyson et al. (1990) by constructing a more extensive sample of faint galaxies with large format CCD detectors and applying a variety of new analytic techniques. There are also some important strategic differences. Firstly, by selecting in $B$ and restricting the cluster redshifts to $z \leq 0.5$, Tyson et al. would not be sensitive to a genuinely high redshift population whose Lyman limits would have shifted beyond the observing passband. By applying the Kaiser & Squires inversion technique. Simulations show this would not be possible at Tyson et al.’s brighter limit.

Very recently, Kneib et al. (1993) have utilised a rather different method to constrain the redshift distribution of $B \simeq 27$ galaxies. They select a single cluster, Abell 370 ($z=0.37$), whose mass profile is well-constrained from arcs of known redshift (Soucail et al. 1988). They visually identify a candidate list of likely arclets and attempt to ‘invert’ the lens equations to derive their cosmic distances individually. Whilst a promising technique, it is important to recognise that this approach cannot explore all regions of redshift space with uniform sensitivity. Thus whilst the individual arclet redshifts may be correct, by using a single cluster an unbiased $N(z)$ appropriate for a $B \leq 27$ sample cannot be simply constructed.

3 THE DATA

3.1 Observational Considerations

The lensing signal we seek is intrinsically very weak and could easily be affected by systematic errors. To account for such errors, we have simulated images of clusters taking into account all likely observational effects. These simulations, discussed below, are used to calibrate the statistics we apply to the real datasets. Foremost is the need to measure ellipticities of faint galaxies over a wide field. The typical scale length of a $I \simeq 25$ galaxy is 0.3-0.7 arcsec, thus sub-arcsecond seeing in the selection passband is critical. In addition, the pixel scale must sample the seeing disk appropriately to eliminate pixellation effects in the ellipticity measurements (pixels $\leq 0.3$ arcsec). With a large format EEV CCD, the f/4 TAURUS imaging Fabry-Perot system on the 4.2-m William Herschel Telescope (WHIT) has 0.27 arcsec sampling of a $5 \times 5$ arcmin field, making it ideally suited to this project.

The two passbands ($V$ and $I$) were selected to provide a colour baseline consistent with the sensitivity of the available large format EEV CCD. To reach a surface density of 40 arcmin$^{-2}$ in the $B$ band requires a completeness limit of $I=25$ (Lilly et al. 1991). Data in $V$ permits the discrimination of cluster members as well as exploration of a possible variation of the lensing signal with colour. Typically, $V-I \sim 1.5$ leading to a $V$ completeness limit of $V=26.5$.

Excellent seeing is only needed for the $I$ detections which are then used to provide statistical estimates of shapes and orientations on a galaxy-by-galaxy basis. Within the galaxy image, we estimated a minimum signal to noise per pixel of $\simeq 2.5$ corresponding to 50 sigma over a 1.5 arcsec FWHM. To achieve this signal to noise at the chosen completeness limit required on-source integrations of 20 ksec in $I$ and 10 ksec in $V$ per cluster.

Most published moderate and high-$z$ cluster catalogues were identified either from peaks in the projected surface density of optical galaxies (Abell et al. 1989, Gunn et al. 1989, Couch et al. 1991), or from deep X-ray observations (Henry et al. 1992). Although the negative evolution claimed for the number density of luminous X-ray clusters at relatively low redshifts (Edge et al. 1990) seems in conflict with the abundance of high redshift optical clusters (Couch et al. 1991), we conclude X-ray observations still provide us with a tracer of the most massive systems at a given epoch and should be less sensitive to projection effects. Accordingly, we chose 3 clusters (1455+22, $z = 0.26$; 0016+16, $z = 0.55$ and 1603+43, $z=0.89$) foremost on the basis of their X-ray luminosities. Further details of the clusters are provided in later sections (see also Table 1).

Observations were made in two runs in July 1990 and May 1991 on the WHIT: the journal is presented in Table 2. During the first run we encountered photometric conditions with very good seeing. However, during the second portion of the second run the seeing deteriorated beyond the limit we considered suitable for this study (1.0 arcsec FWHM). Accordingly the data taken during this period was only used for photometric work and was not used to analyse images for the lensing analysis.

The observations comprised multiple $\lesssim 1000$ sec exposures of two target clusters in a single passband each night. The exposures were ‘dithered’ on a rectangular grid with 10-15 arcsecond spacing and, by combining data for both clusters, a sky flatfield for the entire night was constructed and used to process the images. Numerous shorter exposures of standard stars were also taken at regular intervals through out the night to track variations in the transparency. Further
twilight flatfields were taken each night. The transparency was very stable with photometric zero point errors on the final frames below 0.01 mag.

The reduction of in-field dithered images is a more complex affair than normal image reduction, especially for a rich cluster containing many bright large galaxies. The reduction was performed using IRAF and consisted of the following steps. 1) After bias subtraction and trimming, large scale gradients across the images were removed using the twilight flats. 2) An object detection algorithm (FOCAS, Jarvis & Tyson 1981) was then used to replace all pixels contained within bright objects with random sky values drawn from surrounding regions. 3) The cleaned frames were then median-combined to give ‘superflats’ which contain both the pixel-to-pixel sensitivity variations and residual large scale variations arising from the mismatch of the twilight flatfield and the actual flatfield. 5) The pre-cleaned images were then flatfielded using the superflats to produce the final reduced images.

A comparison of offset frames of the same field after alignment revealed an uncorrected radial distortion in the positions of all sources. Investigations revealed this to be an inherent feature of the focal reducing optics. Uncorrected this would produce a serious reduction in the off-axis image quality. To remove this distortion, every image was geometrically remapped according to a system of fiducial references defined by a large number (≥100) of objects distributed over the frame. A spline surface was fit to the X and Y distortion vectors and each pixel was remapped according to linear interpolation within this surface. The remapping successfully removed the distortion to a level below 0.07 arcsec over the entire frame. The remaining residuals appear random and thus should add incoherently when the frames are stacked resulting in an uncorrelated rms error in the ellipticity of a typical faint object of ≤0.5%.

The processed frames were then combined using a broad median algorithm with an additive scaling offset to account for variations in sky brightness. This provided two final frames, one in each passband, for each of the three clusters. The typical 1σ surface brightness limit in the two bands are: $\mu_V = 28.9$ mag arcsec$^{-2}$ and $\mu_I = 27.8$ mag arcsec$^{-2}$. We now discuss the creation of reliable objects catalogues from these deep images.

### 3.2 Object Selection

The size and depth of the frames obtained are such that, when optimizing the analysis technique, it is most efficient to run tests on a small, but representative, 1 x 1 arcmin regions (Figure 1). The FOCAS object detection algorithm is described in detail by Jarvis & Tyson (1981) (see also Valdes 1982). The two main parameters which control the detection characteristics of the algorithm are the threshold cut (in units of the global sky sigma; $\sigma_{sky}$) and the minimum object area (in pixels). These parameters were optimised by visually checking the success rate of faint object detections against obvious spurious sources. The optimum combination was a threshold of 2.5 $\sigma_{sky}$ per pixel over an area of 10 pixels which corresponds roughly to a 25 $\sigma_{sky}$ detection within the seeing disk.

Object detection is performed on coadded $V + I$ images, combined such that in the final image a flat spectrum source ($f_\nu \approx$-const -- representative of the faintest objects) has equal flux contributions from each filter. Tests show that this approach provides both a fainter detection limit for images and improves detection of objects with very extreme colours. These frames are reproduced in Figures 2. After initial detection on the $V + I$ image, the object areas were evaluated and analyzed on the individual $V$ and $I$ frames. These catalogues were merged to define a final list of sources detected on both frames. Any objects whose isophotes touch the frame boundaries and thus have ill-defined shapes, are rejected at this point.

To determine the effective completeness limit for object detection in the individual bands we create a high signal to noise faint galaxy by median combining a large number of faint galaxy images from the data. This is then scaled and repeatedly added into a region of the cluster frame, the detection process is then rerun and the success of detection of the images as a function of magnitude gives the completeness limit for the catalogue for that band. As noted earlier we actually select from the combined $V + I$ image which will mean that the limits from the completeness simulations are in fact lower bounds on the actual completeness of our catalogues. We adopt a fixed $I$ magnitude limit in all three catalogues so that a well-defined redshift distribution can be compared across all clusters.

Standard aperture photometry in a 3 arcsec aperture is then performed on all the objects using seeing-matched images and aperture colours calculated. The resulting parameters for each object are its position, intensity-weighted second moments calculated from the better seeing $I$ image, isophotal $I$ magnitude and $V-I$ aperture colour. The colour magnitude data for each cluster is shown in Figures 3. The 80% and 50% completeness limits for the individual catalogues are marked on these figures. We now discuss the individual clusters and their associated faint object catalogues.

#### 3.3 The Lensing Clusters

##### 3.3.1 1455+22 (z=0.26)

This cluster (Figure 2(a)) was discovered as a serendipitous source in the EINSTEIN Medium Sensitivity Survey (Henry et al. 1992). On the basis of broad-band galaxy photometry, it was initially suspected to be a $z = 0.7$ cluster (Schild et al. 1980), but subsequent spectroscopy confirmed a lower redshift (Mason et al. 1981). Unfortunately, redshifts are only available for four members, including the dominant central galaxy ($z = 0.258$). Although the formal velocity dispersion is only $\sim 700$ km sec$^{-1}$, with only four velocities it would be possible for the observations to be consistent with an intrinsic dispersion of 1500 km sec$^{-1}$ 40% of the time. In contrast, it is one of the most X-ray luminous cluster known: $L_x \sim 1.598 \times 10^{45}$ ergs sec$^{-1}$ in the 0.3-3.5 keV band. The target has been imaged in deep pointed observations with ROSAT High Resolution Imager in parallel with our gravitational lensing study and this image is presented in the companion paper.

Examining the colour-magnitude diagram for this field (Figure 3(a)) it is straightforward to locate a well-defined cluster colour-magnitude relation, which extends over 6 magnitudes to $I=22.0$, at the appropriate colour for a pop-
ulation of early-type members at this redshift. (It should be noted that all 5 objects originally thought by Schild et al. to be at $z = 0.7$ lie on this relation). By comparing object densities on the colour-magnitude plane with those, suitably scaled, for the other clusters, no statistical excess representing the cluster was found either beyond $I = 22.0$ or bluer than the colour-magnitude relation. The tightness of the relation over this range ($\Delta(V-I) = 0.04$ mag) verifies the excellent photometric precision achieved. Using the test discussed in §3.2, the 80% photometric completeness limits were found to be $I = 25.3$ and $V = 26.5$. An $I = 25$ galaxy corresponds to a 17 sigma detection within the seeing disk.

We define our ‘field’ sample to be those sources whose colours lie off the narrow colour-magnitude sequence of the cluster (marked as open circles on Figures 3). This is a fair approximation given the limits on a blue excess from the object densities on the colour-magnitude plane and the observation than even the strongest ‘Butcher-Oemler’ clusters the fraction of cluster galaxies lying outside this narrow colour-magnitude sequence is typically less than 30% (c.f. Oemler 1992). Two simple checks can be made of this procedure: in Figure 4 we compare the number counts derived with those of genuine ‘blank fields’ (Lilly et al. 1991) finding good agreement; we also examined the radial distribution of our ‘field’ objects which reveals no centrally clustered component.

The above procedure yields 180 early-type members brighter than $I = 22$ over the $5 \times 5 \text{arcmin}$ field. Only 17 galaxies in the inner 500 kpc lie within the range $[m_3, m_3 + 2]$, compared to 48 for the Coma cluster (Metcalfe 1983). Clearly, 1455+22 is only a third as optically rich as Coma, a result which is consistent with the poorly-determined velocity dispersion. On the other hand, the extremely high X-ray luminosity and the presence of a large cD galaxy both point to a deep and centrally concentrated cluster potential.

3.3.2 0016+16 ($z=0.55$)

This cluster was discovered by Richard Kron on a 4-m Mayall prime focus IIIa-F plate (Spinrad 1980). The redshift is $z = 0.545$ (Dressler & Gunn 1992) and the rest-frame velocity dispersion derived from 30 members is $\sigma_{cl} = 1324 \text{km sec}^{-1}$. The cluster has been the subject of several photometric studies because of Koo’s (1981) original claim that, whilst we have successfully removed the bulk of the cluster members, the foreground groups identified by Ellis et al. (1985) are apparent as an excess of objects brighter than $I = 22$ in the $I$ counts. Removal of this source of further contamination is not possible with the existing datasets.

Again, the scatter about the cluster colour-magnitude relation is surprisingly small ($\Delta(V-I) = 0.06$ mag). Since, at $z=0.55$, $V-I$ is equivalent to the restframe $U-V$ we can directly compare this value with the intrinsic dispersion seen in Coma ($\Delta(U-V) \leq 0.04 \text{mag}$) by Bower et al. (1992). At a look-back-time of 6 Gyr, there appears to be no evidence for a large increase in this dispersion. If 0016+16 is a representative cluster then within the framework developed by Bower et al. this yields a lower limit on the epoch of formation of cluster ellipticals of $z_{for} \gtrsim 3$ (Ellis 1993).

3.3.3 1603+43 ($z=0.89$)

A high redshift ($z \simeq 1$) cluster was considered essential in our survey in order to test the possibility of a truly high redshift ($z \geq 2$) component in the faint counts. Selecting such a cluster presented little difficulty since very few are known. At the time of its discovery, 1603+43 was the most distant optically-selected cluster known. It was discovered by Gunn et al. (1989) and the subsequent spectroscopic follow-up by Dressler & Gunn yielded redshifts for $\sim 5$ cluster members with a mean redshift of $z = 0.895$. The cluster was also included in the study of high redshift cluster populations by Aragón-Salamanca et al. (1993).

The very long exposure times on the WHT for this target produce faint 80% completeness limits of $I = 25.9$ and $V = 26.3$ (Figure 2(c)). The $I=25$ detection limit corresponds to a 21 $\sigma$ detection within the seeing disk. The colour-magnitude diagram (Figure 3(c)) shows that the cluster colour-magnitude relation is broader than in the lower redshift clusters and offset in colour compared to non-evolving ellipticals at $z=0.89$ (c.f. Aragón-Salamanca et al. 1993). An
excess of objects can be identified to at least $I=23.5$. To remove cluster members, we adopted a very broad colour criterion combined with a faint magnitude cutoff of $I=24.0$. In this way, we identified 70 cluster galaxies, with 33 in the range $m_I - 3 < m_3 + 2$ i.e. a richness comparable to Coma. The cluster centre is associated with a prominent ‘V’ of galaxies and the spatial distribution of members shows a bimodal structure with one peak over the ‘V’ of galaxies in a frame centre and the second peak lying to the west on the frame border.

This cluster was one of 4 high-redshift targets from the Gunn et al. sample imaged with the ROSAT PSPC (Cast
tander et al. 1993). The cluster was detected within a 2 × 2 arcmin aperture at a 6.5σ significance level in a total exposure time of 28 ksec, corresponding to $L_X \simeq 1.1 \pm 0.2 \times 10^{44}$ ergs sec$^{-1}$. Given the apparent optical richness of this cluster from the $K$-band imaging of Aragón-Salamanca et al., the low X-ray luminosity is somewhat surprising. Cast
tander et al. conjecture that the negative evolution in cluster X-ray properties claimed at low redshift (Edge et al. 1990) continues in more distant samples. Whilst these observations give no indication of the likely concentration of the mass in 1603+43, they do show that it has a much higher X-ray luminosity than many of the the lower redshift Couch et al. sample – some of which have gravitational arc candidates (Smail et al. 1991).

The optical richness of this cluster is consistent with the high spectroscopic identification rate for members and, when combined with the X-ray luminosity, provides good evidence that the cluster is massive. In the absence of the strong evolution observed the low redshift $\sigma_\text{cl}$ – $L_X$ relation would yield a rest-frame 1-D velocity dispersion of $\sigma_\text{cl} > 800$ km sec$^{-1}$. If the observed evolution arises from effects other than growth of the cluster potential wells, as has been proposed by Kaiser (1991), then this value is a lower limit to the dark matter’s velocity dispersion (the relevant quantity for the lensing studies).

3.4 Field Colour-Magnitude Distributions

Figure 4 shows the field colour-magnitude distributions constructed by the procedures discussed earlier for each of the 3 cluster areas. All show the well-known trend to bluer colours at fainter magnitudes. The median colour for the entire field sample brighter than $I = 25$ is $V - I = 1.55 \pm 0.10$. The lower envelope to the colour distribution is $V - I \sim 0.9$, similar to that of a flat spectrum source. Interestingly, the number of objects with flat spectrum colours increases rapidly beyond $I \sim 23$ ($B \sim 25$). Previous workers (Tyson 1988, Cowie et al. 1989) claimed a discontinuity in the photometric data at about this point. Certainly, the colour distributions brighter and fainter than $I = 23$ are highly inconsistent with being drawn from the overall parent population. However, when allowance is made for the bluing of the entire population, the shapes of the two distributions are very similar.

The deepest uniform $I$ sample consistent with the detection limits across the 3 clusters is $I_{\text{iso}} = 25.0$ giving >95% completeness limits in all 3 clusters. This limit corresponds to a minimum detection significance in the $I$ band of $\sim 17\sigma$ in the seeing disk. When combined with a similar detection requirement in the $V$ frame this creates a very robust sample with which to work.

3.5 Estimating Image Parameters of Faint Field Galaxies

Figure 1 illustrates a random 1 × 1 arcmin test area taken from the 1455+22 field. The frame contains 40 objects brighter than $I_{\text{iso}} = 25$ in the field and those with $I_{\text{iso}} \in 24 - 25$ are marked. The lensing technique relies upon our ability to estimate the ellipticities of these faint objects.

The problem of measuring reliable ellipticities for faint objects remains an area of active research. Intensity-weighted second moments (as used in FOCAS) can yield reliable ellipticity and orientation estimates for bright sources but, for the faintest objects under consideration here, the outer isophotes are heavily influenced by noise. For this reason, the intensity-weighted and unweighted moments give very similar results.

To circumvent this, we developed an alternative approach. Instead of using the detection isophote to define pixel membership for an object, we select a circular aperture and, to reduce the noise from the outer regions, a radial weighting function is applied when calculating the second moments in this aperture. The optimal weighting function for a particular object then has the same profile as the object. To simplify matters, we adopted a generic circular Gaussian with a variable width as a weighting function – this simplification has been shown to be reasonable (Bernstein, priv. comm.). The width was determined from the intensity-weighted radius of the object broadened by convolving with the seeing psf. We refer to moments measured using this algorithm as ‘optimally weighted’.

Two separate tests were undertaken to estimate the reliability of the ellipticity measurements for the faintest objects in our sample. The first test involved estimating the ellipticity errors of $I = 25$ galaxies using simulations. The second test measured the scatter in an individual measurement from two independent observations of the same field. The simulations consisted of a large number of artificial frames populated by objects with known ellipticities. For the comparison test, individual exposures comprising the final 1455+22 $I$ frame were combined to create two independent frames each with a total exposure time of 9.5 ksec. These were then analysed and the resulting catalogues matched to allow comparison of the measured image parameters (Figure 6). Both tests have some drawbacks – the simulation results are dependent upon the form of the galaxy profile used, while the real observations are of necessity shallower than the final image.

In the simulations, the intensity-weighted FOCAS moments provide an unbiased and reasonably accurate estimate of the input object ellipticity ($\langle \Delta \epsilon \rangle = 0.16$). The optimally-weighted moments are systematically rounder by about 0.1 than the input (Figure 6). However, the comparison test showed that the optimally weighted moments have a roughly four-fold reduction in the scatter in the ellipticities measured for an object from both frames ($\langle \Delta \epsilon \rangle = 0.04$ versus $\langle \Delta \epsilon \rangle = 0.16$ for the objects with $I_{\text{iso}} \in 24 - 25$). A similar reduction in scatter and introduction of a systematic offset has also been reported by Bernstein (priv. comm.).

In our analysis we use the more efficient optimally-weighted moments for tests where the systematic bias introduced could be modelled (such as the mass mapping pre-
sent in Paper II), otherwise the intensity weighted FOCAS moments were used.

4 STATISTICAL ANALYSES OF GRAVITATIONAL LENSING

4.1 Model Redshift Distributions

Our primary goal is to use gravitational lensing as a tool to constrain the redshift distribution of faint field galaxies well beyond the spectroscopic limits of the largest current telescopes. As we are mainly concerned with establishing a statistical result for the mean distance to the faint population at $I < 25$, we have tested our lensing signals against three model redshift distributions, $N(z)$, which encompass the various physical models discussed in §1.

The three model distributions adopted for the $I < 25$ samples are: (i) the no evolution (‘N.E.’) prediction which maintains a reasonable fit to the deepest spectroscopic observations thus far and might be considered an appropriate model for the merger-induced star formation picture (Broadhurst et al. 1992); (ii) the shallow prediction which maintains the form of the distribution observed at $I=21$ irrespective of the limiting magnitude. Fainter than $I=21$, galaxies simply pile up in the same redshift range as might be expected if there was a well-defined era of recent dwarf formation (Babul & Rees 1992). (iii) Finally we have a deep prediction which includes a significant proportion of galaxies with $z > 1$ as originally claimed by Tyson et al. (1990). We adopted the distributions of White & Frenk (1991) which are based on a hierarchical model for galaxy formation and transformed roughly from $B$ to $I$ using a fixed colour term.

The 3 model redshift distributions are summarised in Figure 7. The potential of our clusters to distinguish between these models can be examined by considering the proportion of $I < 25$ galaxies lying beyond our clusters. For the 3 model $N(z)$ the fractions behind $1455+22$, $0016+16$ and $1603+43$ are, respectively, Shallow (63%, 1%, 0%), NE (96%, 69%, 20%) and Deep (97%, 83%, 65%). Whilst a continuum of intermediate possibilities are physically plausible, particularly between the no evolution and deep cases, the 3 models are, we believe, sufficient for this exploratory study.

4.2 The Lensing Tests

While image parameters have been determined for statistically-complete catalogues of field galaxies in the 3 cluster areas, we still have to develop algorithms for estimating the coherent lensing signal. In recent years a number of statistical methods have been developed to analyse the weak lensing of faint galaxies by rich clusters (Kochanek 1990, Miralda-Escudé 1991a, 1991b, Kaiser & Squires 1992). In general, these methods aim to derive the mass profile of the lensing cluster, rather than the properties of the faint galaxy population – which are assumed to be known.

The analyses fall into two main classes: parametric likelihood tests which assume some functional form for the relative mass distribution in the lens and then attempt to determine the most likely values of the model parameters (Kochanek 1990, Miralda-Escudé 1991a, 1991b) and non-parametric tests which directly infer the 2-D projected mass distribution (Kaiser & Squires 1992). The former methods are capable of testing the faint galaxy properties, whereas the latter methods are better suited for investigating the relative distribution of mass in the lensing cluster.

Throughout this paper, we will assume our lenses can be modelled by a spherically-symmetric non-singular isothermal sphere parameterized by a core radius, $r_c$, and a rest-frame one dimensional velocity dispersion, $\sigma_v$. We chose this simplification initially to make progress in the absence of any other information. As we explained in §1, however, the companion article (Paper II) presents the non-parametric analyses using the Kaiser & Squires statistic and those results allow us to test directly the parametric methods adopted in this paper. The uncertainties in assuming the clusters can be parameterised by simple isothermal models are reviewed in that paper.

4.3 Parametric Methods

The parametric tests compare the observed distributions of image parameters with those calculated for a family of lensing clusters for each of the various $N(z)$. These ‘model’ distributions were first calculated using an analytic prescription of the lensing effect of a given cluster, assuming the data has very simple noise properties. However, in tests, we found that although this method is sensitive to the redshift distributions, it yields cluster parameters which are systematically offset from their true values. This is presumably because the real data contains systematic effects not represented in the analytical treatment. To correct for this degradation, we undertook more realistic simulations which attempt to include all the likely sources of observational noise and estimate, as accurately as possible, to calibrate the offset in the cluster parameters.

To determine the suitability of a given $N(z)$ for a given set of cluster parameters, we adopted a simple maximum likelihood technique. Consider two redshift distributions, a test hypothesis (say, the deep case) and a null hypothesis (the no evolution case). For both we estimate the probability that the observed dataset for a given cluster can be reproduced according to a family of lens models. Applying the maximum likelihood method to each hypothesis will yield two estimates of $\sigma_v$, denoted $\hat{\sigma}_0$ and $\hat{\sigma}_1$, two estimates of $r_c$ ($\hat{r}_0$ and $\hat{r}_1$) and two probabilities $p_0$, $p_1$. These probabilities are determined by comparing the observed image orientations, ellipticities and radial positions from the lens centre with those predicted by the models (c.f. Smail et al. 1991).

We compare the hypotheses by constructing the likelihood ratio $\Lambda = p_0/p_1$. If the ratio is large ($\Lambda \gg 1$), the alternate hypothesis is rejected in favour of the null hypothesis. This approach can obviously be extended to test the relative likelihoods for our three model $N(z)$. In addition to selecting the most likely $N(z)$, the maximum likelihood method returns preferred values for the lens parameters for each cluster.

4.3.1 Analytical Solutions

The analytical test works as follows. For each $N(z)$, a combination of core radius and velocity dispersion for the lens are chosen from a grid of values. Galaxies are then drawn randomly from the hypothesised redshift distribution. The
model galaxies are distributed uniformly across the source plane with ellipticities drawn from the observed blank field distribution and random orientations. The image distortion arising from the lens is calculated using the formalism of Miralda-Escudé (1991a) which yields the radial position ($r$), the orientation relative to the lens centre ($\theta$), and the ellipticity ($\epsilon$) of the model image. The procedure is repeated until there are sufficient objects to allow a fair comparison of the model distributions with the observations. A linear Kolmogorov-Smirnov test compares the observed and predicted distributions and the final likelihood that the model could create the observations is determined by combining the probabilities ($\log \hat{p} = \log(p_{\text{th}} p_{\text{fr}})$). The test is extremely powerful when applied to strong lensing systems. However, as the lensing signal diminishes so does its distinguishing power.

Figure 8 shows how accurately our lensing test can determine the correct input cluster parameters for two different kinds of simulations. Both simulations adopt the observational parameters (cluster redshift, frame size and field galaxy magnitude limit) for the 1455+22 dataset and assume, as input, that the cluster is a spherical system with $r_{\text{cl}}$=1400 km sec$^{-1}$ and $r_c = 100$ kpc and the galaxies are drawn from the no evolution redshift distribution. The logarithmically-spaced probability contours show the derived cluster parameters assuming the observed galaxies are drawn from either the no evolution ($H_0$) or shallow ($H_1$) redshift distributions. The filled circle denotes the correct input value in the $r-r_c$ plane. In the top panels, the catalogue was constructed using the analytical formalism of Miralda-Escudé (described above), whereas in the bottom panels the simulations attempt to allow for as many of the observational selection effects as possible by constructing a realistic frame of the simulated cluster (see §4.3.2 for a detailed description).

In the case of the analytical models, the test readily returns the correct redshift distribution: the probability ratio is $\log \hat{p}/p_0 \equiv \log \Lambda < -10$. In addition the input lens parameters are correctly recovered. The shape of the likelihood contours can be understood in terms of a trade-off between an increase in $r_{\text{cl}}$ – which strengthens the lens – and an increase in $r_c$ – which weakens it. The shifts between the contours for the two redshift distributions arises because more distant galaxies are more easily distorted. Examining the individual distributions, we find greatest power comes from the orientations which constrain the solution to lie somewhere along a slanted locus. The ellipticities and radial positions then confine the solution to a point on this locus. Whilst the overall likelihood is derived assuming that the probabilities from the three K-S tests for $r$, $\theta$ and $\epsilon$ are independent, this is not a critical assumption given the dominant power of the orientation distribution.

For the catalogue from the simulated frame, the test still correctly distinguishes between the two possible redshift distributions with $\log \Lambda = -1.8$. However, a systematic offset in the best fit lens parameters appears. The calibration and correction for this offset is the motivation for creating simulated frames and its source is discussed below. The ability of the test to determine the correct lens parameters is very sensitive to the strength of the observed lensing signal. For weak signals the likelihood peak flattens and while the test can still determine the correct redshift distribution, the lens parameters become less meaningful.

The analytical catalogue (created in the same manner as the analytic models) obviously disregard a number of complications. The effects of noise on the image measurements and the degradation of the induced distortion by seeing are ignored. Both these effects will introduce a systematic error in the measured ellipticity. However, their effect on the image orientations will be random. By concentrating on the image orientation and radial distributions it is hoped that the effects of these systematic errors will be minimized. Furthermore, the fixed source magnitude limit results in a paucity of objects in the lens centre on the image plane. This is because amplification bias has been neglected, this acts to populate this region by magnifying galaxies fainter than the observation’s magnitude limit into the sample. The combined effect of all these processes is to reduce the observed lensing signal in the simulated frames (Figure 8). By using simulated observations this degradation can be calibrated and the observations corrected for it.

4.3.2 Simulated Frames

To calibrate the statistical tests applied to the real observations we simulate a set of frames which are analysed in the same manner as the real data using FOCAS. This approach was chosen to cater for most of the biases that are likely to occur in the data which cannot be handled analytically. Each of these may degrade the strength of the lensing signal causing systematic errors in the derived lens parameters. The most obvious effect is atmospheric seeing, but sky noise and undetected merged images also contribute. Underlying correlations in the data due to redshift dependency of certain image characteristics are also a concern unless properly modelled. The lensing is performed using the same technique as in the analytic simulations with a number of additional features:

- The effects of seeing, pixellation and sky noise on the measurements are included in the simulated image to match those appropriate for a particular observation.
- Instead of a cumulative $N(z)$ to a given magnitude limit, a differential distribution is used which allows empirical control of changes to the form of the $N(z)$ as a function of magnitude allowing us to easily model the effects of amplification bias in our observations. Furthermore, as the distortion at a particular radius on the source plane increases with redshift more distant sources will tend to be more distorted. However, as the signal to noise in a given ellipticity measurement decreases for fainter objects it is important that the faintest, possibly most distant and hence most strongly lensed objects are most effectively degraded. Ideally, we would also like to include any correlation of source size or ellipticity with redshift but there is currently no observational data on either of these correlations. Images to $I=23$ unaffected by atmospheric seeing were kindly supplied by the HST Medium Deep Survey (MDS) Team and scale lengths for sources selected randomly from their first deep image (Griffiths et al. 1992) measured for this purpose.
- The effects of crowding on image detection and the distortion of image isophotes by undetected faint images are also be included. The latter effect is incorporated by lensing galaxies fainter than the adopted magnitude limit (to
5 RESULTS AND DISCUSSION

In discussing the results, we will first present the basic evidence for gravitational lensing in our sample, and examine its dependence on the colour of the faint field population. Here we will, essentially, follow the original method used by Tyson et al. (1990) and utilise the orientations of faint galaxies perpendicular to the cluster radius vector. Some constraints on the mean redshift of the $I \leq 25$ sample is possible from this analysis, particularly if we introduce mass estimates for 1455+22 from X-ray studies. We then apply the more rigorous maximum likelihood analysis to examine the datasets for all 3 clusters in the context of the 3 model $N(z)$ discussed in §4. The strength with which we can rule out various redshift distributions depends on how much freedom we are willing to assign to the cluster parameters. The reader is again referred to the companion article (Paper II) for further information.

5.1 Orientation Histograms

Tyson et al. (1990) introduced the simple test of measuring the fraction of galaxies aligned tangentially to the cluster radius vector. We start by analysing such histograms for each of the clusters in turn. Making the gross simplification that our clusters are identical objects and ignoring amplification bias, we directly infer the fraction of galaxies behind each cluster and hence $N(z)$.

We select galaxies in elliptical annuli aligned with those defined by the cluster members (see Paper II) in order to remove the effect of the lens ellipticity on the orientation histogram. The centres used are the optically defined centres of the clusters. Paper II demonstrates that these centres are consistent with those defined by both the mass and X-ray gas distributions in our cluster sample. Figure 9 shows the resulting orientation histograms for the $I \leq 25$ field samples for each of the three clusters. Both of the lower redshift clusters show an obvious excess of tangentially-aligned images. In 1455+22 the excess is approximately 190 in a total of 810 objects (23±2%) corresponding to a surface density of 7.0 galaxies arcmin$^{-2}$, whereas for 0016+16 it is ≥80 out of 356 galaxies (21±2%) or 4.5 arcmin$^{-2}$. 1603+43 shows no alignment excess.

Under the assumption of identical clusters, we can compare these orientation histograms with our three hypothetical $N(z)$ distributions. Although considerably idealised, this is illustrative in determining which cluster is the most critical in estimating $N(z)$. Using a K-S test to compare the real and analytical orientation distributions, we rule out the Shallow distribution at the 99.7% level using the combined result from all three clusters. As expected, 0016+16 provides by far the strongest rejection since, according to the Shallow $N(z)$, only 2% of the $I \leq 25$ population should be beyond $z=0.55$. Were 1603+43 to be as massive and concentrated as the other clusters, the data would also rule out the Deep $N(z)$.

The signal in the two lower redshift clusters is sufficiently strong that we can sub-divide the sample to check Tyson et al.’s claim that the the excess is predominantly seen in the bluest or faintest galaxies. We note, however, that by virtue of using the EEV CCD we only have a $V-I$ baseline compared to Tyson et al.’s $B-K$. We chose to split the samples at the mean sample colour of $V-I = 1.5$ and $I_{250} = 23$ where the colour distribution rapidly begins to shift to the blue (Figure 5(d)).

The orientation histograms for the four sub-classes are shown in Figure 10(a) for 1455+22 and Figure 10(b) for 0016+16. Clearly all four sub-classes for both clusters show similar alignments and so we cannot improve the signal contrast by applying photometric selections. The photometric distribution of galaxies in the aligned bins ($\theta > 60$ deg-
degrees) are completely consistent with those in the unaligned bins. Indeed, for 1455+22, the two most obvious arclets have colours which fall either side of that of the cluster members, and the radial distribution of the aligned component shows no variation of colour with radius.

We can place a strict limit on the maximum possible colour difference for the excess aligned population if we assume that they are drawn from a population with a colour distribution similar in shape to that observed in the unaligned bins but shifted to the blue. The upper limits then refer to the maximum colour shift allowed. For 1455+22 the 90% confidence limit is \( \Delta(V-I) = -0.2 \) and for 0016+16 it is also \( \Delta(V-I) = -0.2 \). In other words, the aligned component cannot be drawn from a population which is much bluer in \( V-I \) than the main population. Alternatively, the strong shift to the blue seen in the field colours beyond \( I \geq 23 \) is not primarily due to the existence of a distant blue \( z \geq 1 \) galaxy population. This constraint argues against generic Deep models containing a primordial population of distant star forming galaxies unless they exhibit a wide spread in \( V-I \) colours.

5.2 Constraints from Current Spectroscopic Surveys

The relatively strong alignment signal in the samples brighter than \( I \geq 23 \) prompts us to consider a ‘boot-strap’ method for determining the redshift of the faint galaxy population. If we can measure the lens parameters for a bright sample for which the field redshift distribution is already secure from conventional spectroscopy, we can then apply the lens model to derive the median redshift of the \( I \leq 25 \) sample.

At the current time, the deepest \( I \)-selected field survey are those of Lilly (1993) and Tremes et al. (1993) limited at \( I \leq 22 \). To undertake this analysis, we selected galaxies in 1455+22 and 0016+16 satisfying \( I \in [20, 22] \) and \( e_{\text{opt}} \geq 0.05 \). To test the method, we chose two input redshift distributions: the one observed by Lilly (hypothesis: \( H_0 \)) and the Shallow distribution (hypothesis: \( H_1 \)). We applied a combined maximum likelihood estimator on the radial distribution and orientations and, as expected, 0016+16 is the better discriminator. Taking both clusters, the Shallow redshift distribution was rejected at the 95% level to this apparent magnitude limit. The maximum likelihood parameters for the 1455+22 lens were \( \sigma_c = 1300 \) km sec\(^{-1} \), \( r_c = 400 \) kpc with \( \log \rho_0 = -0.1 \). For 0016+16, \( \sigma_c = 1800 \) km sec\(^{-1} \) and \( r_c = 70 \) kpc with \( \log \rho_0 = -1.9 \). As these estimates are derived from analysis of a sample of bright galaxies they are less affected by the systematic biases detailed in §4.3.

The uncertain cluster parameters inferred from the small samples that overlap in apparent magnitude with the current spectroscopic redshift surveys indicate that this ‘boot strap’ technique is probably too hazardous a method for estimating \( N(z) \) to \( I \leq 25 \). We will see later, however, that the derived cluster parameters are not that erroneous. The prospect of deeper surveys (and thus higher galaxy surface densities) from 10-m class telescopes will make the ‘boot strap’ method a viable approach in future.

5.3 Maximum Likelihood Analyses

We now apply the likelihood analysis described in §4.3 to the datasets constructed from the three clusters. We use the complete \( I \in [20, 25] \) samples restricting the FOCUS ellipticities to \( \epsilon \in [0.1, 0.8] \). The likelihood test compares the observed \( r, \theta, \epsilon \) distributions with those from 10 combined realisations of each lens model calculated for each of the three model \( N(z) \). The parameter space searched is: \( \sigma_c \in [400, 2000] \) km sec\(^{-1} \) and \( r_c \in [0, 2] \) arcmin. The upper limit on the core radius translates into metric radii of 0.6, 0.9 and 1.0 Mpc for the 3 clusters. The rather small lower limit for the cluster velocity dispersion protects against the probability peak moving outside our searched parameter range due to the degradation illustrated in Figure 8.

For 1455+22 the likelihood distributions (Figure 11(a)) have very similar shapes to those seen in the simulations presented earlier (Figure 8). The maximum probabilities for the three redshift distributions are: \( \hat{\rho}_{\text{shallow}} = -1.1 \), \( \hat{\rho}_{\text{NE}} = -0.7 \) and \( \hat{\rho}_{\text{Deep}} = -0.8 \). Thus, the no evolution \( N(z) \) is marginally preferred. The lens parameters for this fit are: \( \sigma_c = 630 \pm 150 \) km sec\(^{-1} \) and \( r_c = 210 \pm 100 \) kpc. The best fit velocity dispersion is strictly a lower limit to the actual value due to the systematic effects illustrated in the simulations above. The errors quoted are 90% confidence limits taking the errors in \( \sigma_c \) and \( r_c \) to be orthogonal.

As before, 0016+16 (Figure 11(b)) provides the most significant constraint. Here we obtain \( \hat{\rho}_{\text{shallow}} = -6.2 \), \( \hat{\rho}_{\text{NE}} = -2.8 \) and \( \hat{\rho}_{\text{Deep}} = -3.3 \) ruling out the Shallow model and again preferring the no evolution \( N(z) \). The lens parameters for the N.E. fit are: \( \sigma_c = 860 \pm 250 \) km sec\(^{-1} \) and \( r_c = 210 \pm 250 \) kpc, where again \( \sigma_c \) represents a lower limit to the actual value.

For 1603+43, the three models have maximum probabilities of: \( \hat{\rho}_{\text{shallow}} = -1.2 \), \( \hat{\rho}_{\text{NE}} = -1.3 \) and \( \hat{\rho}_{\text{Deep}} = -1.4 \). Given the uncertain cluster mass a very wide parameter space was searched. For the N.E. model a large parameter space is compatible with the observations (Figure 11(c)). For both the N.E. and Deep models we obtain \( \sigma_c \geq 400 \) km sec\(^{-1} \) and \( r_c \approx 900 \) kpc for the maximum likelihood solutions. However, it is apparent from Figure 11(c) that if a lower limit is placed on the velocity dispersion of this cluster or an upper limit on its core radius, we would strongly reject the Deep redshift distribution.

Combining all three clusters we can reject the Shallow redshift distribution at the 99.98% level (3.8\( \sigma \)) but without further assumptions about the cluster properties we can only claim a marginal preference for the N.E. model over the Deep distribution at the level of 23% (1.2\( \sigma \)).

We can refine the derived cluster parameters for the lower redshift clusters from simulated frames (§4.3.2). The relatively strong signal in these systems gives a well-determined transformation. From the likelihood fits for the no evolution \( N(z) \) we already have lower bounds of \( \sigma_c \approx 600 \) km sec\(^{-1} \) and 850 km sec\(^{-1} \) for 1455+22 and 0016+16 respectively. We simulated images for a family of clusters with different lens parameters \( (\sigma_c, r_c) \) at the two cluster redshifts \( (z = 0.26, z = 0.55) \) using the N.E. redshift distribution. The likelihood analysis was then run on FOCUS catalogues created from each of these images and the input lens parameters of the simulation whose measured parameters are...
Lensing Studies of Clusters: I – Faint Galaxy \(N(z)\).

Our primary conclusion is the rejection of a redshift distribution to \(I \leq 25\) significantly shallower than the no evolution prediction. Such a model would be expected if the ultra-faint counts were dominated by a population of low redshift \((z < 0.5)\) dwarfs. The detailed dynamical and/or X-ray data on our two lower redshift clusters, endorsed by the lensing maps presented in Paper II, makes this a very robust conclusion. The lensing signal seen in 0016+16 alone is consistent with the bulk (70-80%) of the field galaxies to \(I \leq 25\) lying beyond the cluster \((z \lesssim 0.6)\).

To constrain redshift distributions deeper than the no evolution form, we have to consider our most distant cluster, 1603+43. While less information is available about this cluster than for the two lower redshift systems, precision data is not so important as any lensing signal would indicate the presence of a distant population \((z \gtrsim 2)\) to \(I \leq 25\). The absence of any signal suggests that a substantial \((>20\%)\) tail beyond \(z \sim 2\) (such as implied in the White & Frenk model) is unlikely. To formally reject a high redshift component at any level, however, a lower limit is needed on the velocity dispersion of 1603+43. At present, this can only be derived from X-ray observations, which, interpreted in terms of low redshift correlations, predict \(\sigma_{\text{cl}} > 800 \text{ km sec}^{-1}\) for an assumed \(r_c < 250 \text{ kpc}\). Adopting the nominal offset to the model velocity dispersions discussed in §4.3.2, these parameters for 1603+43 would allow us to reject the 20\% \(z > 2\) tail at a confidence limit of \(>95\%\).

Support for our conclusion of an absence of a high-\(z\) component to the \(I \leq 25\) \((B \leq 27)\) redshift distribution is provided by the redshift distribution recently derived by Kneib et al. (1993) for \(\sim 40\) arclets seen through the rich cluster Abell 370. The median redshift they obtain for their sample is \(z \sim 0.9\), very close to that of the no evolution model (Figure 7). Only 15\% of their ‘best’ sample have \(z \gtrsim 1.5\).

We can compare the derived line of sight velocity dispersions for the two lower redshift clusters with the predicted distribution from the CDM simulations of Frenk et al. (1990) rescaled to \(b = 1\) in line with the COBE observations. The high inferred velocity dispersions of the clusters, especially for 0016+16 (in both \(N(z)\) cases) given its redshift, are at the extremes of the standard cold dark matter predictions. This is particularly interesting as the derived dispersions are free of the projection effects commonly invoked to force agreement between the predicted cluster velocity dispersion distribution and that observed locally with spectroscopic samples. This discussion is extended in Paper II where we compare the 2-D distributions of the baryonic and total mass in our clusters. However, we conclude here that selection of clusters according to X-ray luminosity does provide massive systems. Although, the absence of strongly lensed arcs in these two high dispersion clusters implies that X-ray selection does not necessarily guarantee a potential compact enough to produce a giant arc.

Although our results rely to some extent on the nature of the clusters used as lenses, we demonstrate in Paper II that the mass distribution assumed for the two lower redshift systems can be directly checked from the lensing data, with satisfactory conclusions. The prospects with 8-10 metre telescopes for improving our understanding of the dynamical state of 1603+43 via \(\sim 30-50\) redshifts (such as are available for 0016+16) and for enlarging the overlap between lensing-based and spectroscopically-based field redshift distributions (§5.2) are excellent. We can thus look forward to tighter constraints on the statistical distances to faint galaxies in the near future.

6 CONCLUSIONS

We have developed new and powerful lensing techniques that are able, with some limitations, to simultaneously constrain the statistical distances \(N(z)\) to the field population at limits well beyond reach of current spectrographs and the distribution of dark matter in a non-parametric manner. The latter conclusions are discussed in more detail in the companion article, Paper II. We describe detailed tests of these techniques which, we believe, ensure that systematic biases inherent in our observational datasets are well-understood.

Our conclusions at this stage can be summarised as follows:

- The strongest constraint we can provide on the redshift distribution of a sample of \(I \leq 25\) field galaxies is the absence of a significant population of faint low-\(z\) dwarfs such as might be expected if either the faint end of the local galaxy luminosity function is seriously underestimated (c.f. Koo et al. 1993) or if there is strong evolution of the faint end slope at low redshifts. We reject a model \(N(z)\) with 98\% of the \(I \leq 25\) population at \(z \leq 0.55\) at the 99.98\% level. The very deep surface brightness limit of our imaging data makes this a particularly effective constraint.

- Our constraints on models with large fractions of high redshift galaxies are weaker due to the lack of detailed information about the dynamical state of our most distant cluster, 1603+43. Further study of this system and the properties of other high \(z\) clusters is necessary to make progress. At present we are unable to formally reject models with as high as 20\% of the \(I \leq 25\) population at \(z \sim 2\). However, if as we surmise from its X-ray properties 1603+43 is compact and
massive we can reject the Deep model at better than a 95% confidence level.

• By incorporating external constraints on the likely range of parameters for the three clusters, our preferred redshift distribution is therefore the no evolution model. We might understand how such a model occurs, notwithstanding the high number counts, if the galaxy luminosity function evolves in shape according to the empirical form described by Broadhurst et al. (1988, see their Figure 4). A possible astrophysical model of such evolution incorporating mergers of fragments whose star-formation rate is slowly declining has been described by Broadhurst et al. (1992).

• The lensing strength of our two lower redshift clusters provides a direct measure of the probable velocity dispersion ($\sigma_c$) and core radii ($r_c$). However, the estimates, particularly $\sigma_c$, are affected by systematic biases. Using detailed simulations, we correct the biases and for 1455+22 obtain $\sigma_c=1000\pm200$ km sec$^{-1}$ (compared to the spectroscopic value of $\sim 700$ km sec$^{-1}$) and, for 0016+16, $\sigma_c=1200\pm300$ km sec$^{-1}$ (compared to the spectroscopic value of 1300 km sec$^{-1}$). We attempt to derive constraints on the cluster parameters using the lensing signal to $I \leq 22$ where the field $N(z)$ is already well-understood from redshift surveys. Whilst the values derived are consistent with those above, no improved estimates are obtained due to the low surface density of galaxies available at the current spectroscopic limit. We conclude that both clusters are on the extreme tail of the predicted distribution of cluster dispersions in standard CDM (Frenk et al. 1990).

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FIGURES

Figure 1: A 1×1 arcmin test area in the 1455+22 frame. This is a 20.8 ksec I exposure with 40 objects detected above $I = 25$. Those objects in the faintest analysis sample ($I_{iso} \leq 24 - 25$) are marked.

Figure 2: Composite $V + I$ images of the clusters: (a) 1455+22 ($z = 0.26$), (b) 0016+16 ($z = 0.55$) and (c) 1603+43 ($z = 0.89$). The total exposure times are 33.0, 36.5 and 45.8 ksec respectively and the scales are in arcseconds with east to the left and north at top.

Figure 3: The $V - I$ colour-magnitude diagrams for the clusters (a) 1455+22, (b) 0016+16 and (c) 1603+43. Galaxies with colours similar to an E/S0 at the cluster redshift are shown (●), as well as the 80% and 50% completeness limits for the various catalogues, calculated from simulations. Also shown are the photometric errors as a function of magnitude for the entire samples, the line marked is the chosen magnitude limit for the total sample $I = 25$.

Figure 4: Differential number counts in $I$ of (a) 1455+22, (b) 0016+16 and (c) 1603+43 after removal of the cluster members (●) compared to the Lilly et al. (1991) field counts (○). The deficit of field galaxies in the more distant clusters at faint magnitudes arises from the cluster selection criteria.

Figure 5: (a)–(c) $V - I$ Colour distributions for the clusters (filled) and field galaxies (open) brighter than $I = 25$ in the three clusters. (d) The $V - I$ colour distribution of the combined field samples from the three clusters, split into the various magnitude slices. The bars at the top show the range of colours covered by the non-evolved morphological types as a function of redshift. The galaxy colours generally start at the left side for $z = 0$ and move right until $z \sim 0.5$ at which point they start becoming bluer again.

Figure 6: The two panels show the comparison of the raw (FOCAS) and optimally weighted ellipticities for the 1455+22 test area. The top panel illustrates the systematic offset (dotted line) introduced in the ellipticity measurement when using the optimal weighting scheme. The lower panel compares the ellipticities of objects measured on the two independent frames. It is apparent that the optimal weighting reduces the scatter in individual ellipticity measurements for $I_{iso} \in [24, 25]$ objects.

Figure 7: (a) The various normalised redshift distributions used in the analysis. The dashed curves marked WF25 and WF27 are the $B = 25$ and $B = 27$ distributions from White & Frenk 1991. The curve shown as dotted is a Bruzual $B = 27$ cumulative $N(z)$. The remaining solid curves show the no evolution differential $N(z)$ centred on the $I$ magnitude marked – these were calculated for observations in $R$ band and then converted using a fixed colour term. (b) The run of median redshift with $I$ magnitude for the three hypotheses.

Figure 8: The top panels show the likelihood contours of a test of an analytic simulation while the bottom panels show the contours for a numerical simulation with the same lens parameters. The probability contours are logarithmic and spaced every factor of 10 – starting at $10^{-10}$ below the peak probability. Details of the simulations are given in the text, the input redshift distribution was the no evolution model and the lens parameters are marked (●), the core radius ($r_c$) is in kpc and the velocity dispersion ($\sigma$, in km sec$^{-1}$).

Figure 9: Orientation histograms for 1455+22, 0016+16 and 1603+43. The histograms were constructed using all objects in the field samples with $I \in [20, 25]$ and $V - I \in [-1, 2.5]$ with optimally weighted ellipticities above a cutoff of $\epsilon \geq 0.05$. The orientations and ellipticities used for the annular bins were those quoted in the text and the optically determined lens centres were used.

Figure 10: (a) The orientation histograms for 1455+22 separated in terms of colour and magnitude. The aligned excess has a similar strength in all four samples showing that it has no strong colour or magnitude dependency. (b) As above for the 0016+16 sample. As in (a) all four samples contain an excess of tangentially aligned objects confirming that this arises from a population of objects with similar characteristics to the bulk of the faint field population.

Figure 11: The likelihood distributions for the cluster lensing models of (a) 1455+22, (b) 0016+16 and (c) 1603+43. Each plot is for one of the model redshift distributions. The units for the axes are km sec$^{-1}$ and kpc. The contours are for the smoothed probability distributions and are spaced every factor of 10 from the peak.