# Supplemental Information for: Effects of shear thinning and viscoelastic stresses on flagellated bacteria motility

Zijie Qu* and Kenneth S. Breuer

*Brown University, School of Engineering, 184 Hope St, Providence, RI 02912 USA*

(Dated: April 16, 2020)

## I. MODIFIED RESISTIVE FORCE THEORY FOR WOBBLING CELLS

Our approach closely follows Darnton *et al.* [1] and is repeated here for convenience. The force and torque on the cell body and flagellum are related to their translation and rotation by:

$$\begin{bmatrix} F_b \\ \tau_b \end{bmatrix} = \begin{bmatrix} A_b & 0 \\ 0 & D_b \end{bmatrix} \begin{bmatrix} v \\ \omega_b \end{bmatrix} \quad (1)$$

and:

$$\begin{bmatrix} F_f \\ \tau_f \end{bmatrix} = \begin{bmatrix} A_f & B_f \\ Bf & D_f \end{bmatrix} \begin{bmatrix} v \\ \omega_f \end{bmatrix}, \quad (2)$$

where $F_b$, $F_f$ and $\tau_b$, $\tau_f$ are the force and torque on the cell body and flagellum, $v$ is the swimming speed of the cell. $\omega_b$ and $\omega_f$ are the rotation rate of the cell body and flagellum respectively. The wobbling of the cell with an angle $\phi$ changes the drag coefficient of cell body which changes the values of the cell body resistance matrix. We assume that the cell body is a spheroid with length $2a$ and width $2b$. The coefficients $A_b$ and $D_b$ are then given as [1]:

$$A_b = -(A_1 sin^2(\phi) + A_2 cos^2(\phi)) \quad (3)$$

and

$$D_b = -((D_1 + a^2 A_1) sin^2(\phi) + D_2 cos^2(\phi)), \quad (4)$$

where

$$A_1 = 32\pi\mu a e^3/[(3e^2 - 1)E + 2e], \quad (5)$$

$$A_2 = 16\pi\mu a e^3(\phi)/[(1 + e^2)E - 2e], \quad (6)$$

$$D_1 = 32\pi\mu a b^2 e^3(2 - e^2)/3(1 - e^2)[(1 + e^2)E - 2e] \quad (7)$$

and

$$D_2 = 32\pi\mu a b^2 e^3/3[2e - (1 - e^2)E], \quad (8)$$

and $\mu$ is the fluid viscosity, $e = (a^2 - b^2)^{1/2}/a$ is the eccentricity, and $E = log[(1 + e)/1 - e]$.

The resistance matrix for the flagellum in Equation 2 given by Resistive Force Theory (RFT) [2–4]:

$$A_f = \frac{2\pi\mu L \times (8\pi^2 R^2 + p^2)}{[log(\frac{r_0}{p}) + \frac{1}{2}][4\pi^2 R^2 + p^2]}, \quad (9)$$

$$B_f = \frac{2\pi\mu L \times (-2\pi R^2 p)}{[log(\frac{r_0}{p}) + \frac{1}{2}][4\pi^2 R^2 + p^2]} \quad (10)$$

and

$$D_f = \frac{2\pi\mu L \times (4\pi R^2 + 2p^2) r_0^2}{[log(\frac{r_0}{p}) + \frac{1}{2}][4\pi^2 R^2 + p^2]}, \quad (11)$$

where $L$ is the length of the flagellar filament, $p$ is the pitch the helix, $R$ and $r$ are the radius of the helix and filament respectively. The geometry of the cell body and flagellum used in our calculations is given in Table I.

The coupled system is force-free and torque-free [5]:

$$F_f + F_b = 0 \quad (12)$$

and

$$\tau_f + \tau_b = 0\ . \quad (13)$$

The swimming speeds are then solved assuming a constant torque with varied wobbling angle.

TABLE I. Typical geometric parameters used in cell swimming calculations. Values taken from [1, 4, 6]

| Symbol | Description | Value |
|---|---|---|
| $a$ | Cell length | 2.00 $\mu$m |
| $b$ | Cell width | 0.60 $\mu$m |
| $L$ | Flagellum length | 8.00 $\mu$m |
| $p$ | Flagellum pitch | 2.00 $\mu$m |
| $R$ | Flagellum helix radius | 0.35 $\mu$m |
| $r_0$ | Flagellum filament radius | 0.03 $\mu$m |

* Current address: California Institute of Technology, The Division of Biology and Biological Engineering, 1200 East California Boulevard, Pasadena, California, 91125 USA.
Email: zijiequ@caltech.edu

[1] N. C. Darnton, L. Turner, S. Rojevsky, and H. C. Berg, On torque and tumbling in swimming *Escherichia coli*,

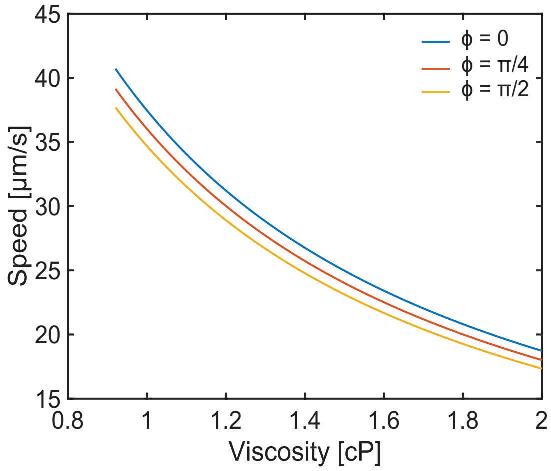


FIG. 1. Calculated swimming speed of bacteria *E.coli* with different precession angles $\phi$ using modified Resistive Force Theory [1].


Journal of bacteriology **189**, 1756 (2007).

[2] M. Holwill and R. Burge, A hydrodynamic study of the motility of flagellated bacteria, Archives of biochemistry and biophysics **101**, 249 (1963).

[3] Y. Magariyama, S. Sugiyama, K. Muramoto, I. Kawagishi, Y. Imae, and S. Kudo, Simultaneous measurement of bacterial flagellar rotation rate and swimming speed, Biophysical journal **69**, 2154 (1995).

[4] Y. Magariyama and S. Kudo, A mathematical explanation of an increase in bacterial swimming speed with viscosity in linear-polymer solutions, Biophysical journal **83**, 733 (2002).

[5] E. M. Purcell, Life at low Reynolds number, Am. J. Phys **45**, 3 (1977).

[6] V. A. Martinez, J. Schwarz-Linek, M. Reufer, L. G. Wilson, A. N. Morozov, and W. C. Poon, Flagellated bacterial motility in polymer solutions, Proceedings of the National Academy of Sciences **111**, 17771 (2014).