Spatial coherence of laser output far below threshold*

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The light emerging from the output aperture of a laser should be spatially coherent even when the laser is operated far below its threshold and behaves substantially as a thermal source. The spatial coherence has been demonstrated experimentally by photoelectric correlation measurements at different pairs of points in the aperture.

Index Headings: Laser; Coherence.

As is now well known,[1] and has been demonstrated experimentally,[2] a laser that is operating far below its threshold of oscillation behaves substantially as a thermal source, with gaussian amplitude distribution. However, unlike the light from most thermal sources, the light output from the laser even below threshold should be spatially coherent at the output aperture. For, the action of the Fabry–Perot cavity, combined with the distributed nature of the emitters, produces the effect of a distant source, such that the coherence area of the optical field at the output aperture equals or exceeds the aperture size.

In order to see this, we consider a laser cavity consisting of two mirrors separated by a distance L. Let $\mathbf{r}$ be an observation point just outside the exit aperture of the laser. We may represent the complex amplitude $V(\mathbf{r},t)$ of the total (supposedly polarized) optical field at $\mathbf{r}$ at time t by

$$V(\mathbf{r},t)=\sum_{j=0}^{\infty} V_j(\mathbf{r},t), \tag{1}$$

where $V_j(\mathbf{r},t)$ $(j=0, 1, 2, \ldots)$ is that portion of the total amplitude at $(\mathbf{r},t)$ contributed by light that has suffered j reflections inside the cavity. If the reflectance $\mathcal{R}$ is very high, the ratio $|V_j(\mathbf{r},t)|/|V_1(\mathbf{r},t)|$ $(j\geqslant 1)$ will be of the order of magnitude $\mathcal{R}^{j-1}/j$, so that successive contributions to the sum in Eq. (1) decrease slowly and the higher-order terms dominate the sum. It is not difficult to write explicit expressions for $V_0(\mathbf{r},t)$ and the reflected contributions $V_1(\mathbf{r},t)$, $V_2(\mathbf{r},t)$, etc., in terms of the source density, with the help of the Fresnel–Kirchhoff diffraction integral.[3] However, for our purposes it suffices to observe that, under the usual conditions, the normalized correlation functions defined by

$$\begin{aligned}\gamma_{jj}(\mathbf{r}_1,\mathbf{r}_2)&\equiv\langle V_j^*(\mathbf{r}_1,r)V_j(\mathbf{r}_2,t)\rangle/[\langle I_j(\mathbf{r}_1)\rangle\langle I_j(\mathbf{r}_2)\rangle]^{\frac{1}{2}},\\ \langle I_j(\mathbf{r})\rangle&\equiv\langle V_j^*(\mathbf{r},t)V_j(\mathbf{r},t)\rangle,\end{aligned} \tag{2}$$

are generally close to unity for any two points $\mathbf{r}_1$, $\mathbf{r}_2$ in the output aperture when j is greater than about 1 or 2. For, the coherence area at this aperture due to the field $V_j(\mathbf{r},t)$ will have linear dimensions of order[4] $(j+\frac{1}{2})L\lambda/d$, where λ is the wavelength and d the width of the active laser cavity. For a typical gas laser of beam width $d\sim 1$ mm and length $L\sim 30$ cm, the coherence area rapidly approaches and exceeds the output-aperture size as j increases beyond 1 or 2. We also note that, under the usual conditions, the normalized correlation function

$$\gamma_{ij}(\mathbf{r}_1,\mathbf{r}_2)\equiv\langle V_i^*(\mathbf{r}_1,t)V_j(\mathbf{r}_2,t)\rangle/[\langle I_i(\mathbf{r}_1)\rangle\langle I_j(\mathbf{r}_2)\rangle]^{\frac{1}{2}} \tag{3}$$

will have a very small modulus when $i\neq j$. The reason is that L is generally of the order of or greater than the coherence length of the natural emission from the atomic system. It follows from the foregoing that the total mean intensity defined by

$$\langle I(\mathbf{r})\rangle\equiv\langle V^*(\mathbf{r},t)V(\mathbf{r},t)\rangle$$

is given approximately by

$$\langle I(\mathbf{r})\rangle\simeq\sum_j\langle V_j^*(\mathbf{r},t)V_j(\mathbf{r},t)\rangle=\sum_j\langle I_j(\mathbf{r})\rangle, \tag{4}$$

and that the correlation function

$$\langle V^*(\mathbf{r}_1,t)V(\mathbf{r}_2,t)\rangle\simeq\sum_j\langle V_j^*(\mathbf{r}_1,t)V_j(\mathbf{r}_2,t)\rangle. \tag{5}$$

The normalized correlation function

$$\gamma(\mathbf{r}_1,\mathbf{r}_2)\equiv\langle V^*(\mathbf{r}_1,t)V(\mathbf{r}_2,t)\rangle/[\langle I(\mathbf{r}_1)\rangle\langle I(\mathbf{r}_2)\rangle]^{\frac{1}{2}}$$

is then given approximately by

$$\begin{aligned}\gamma(\mathbf{r}_1,\mathbf{r}_2)&\simeq\sum_j\langle V_j^*(\mathbf{r}_1,t)V_j(\mathbf{r}_2,t)\rangle/[\sum_i\langle I_i(\mathbf{r}_1)\rangle\sum_i\langle I_i(\mathbf{r}_2)\rangle]^{\frac{1}{2}}\\ &=\sum_j\gamma_{jj}(\mathbf{r}_1,\mathbf{r}_2)[\langle I_j(\mathbf{r}_1)\rangle\langle I_j(\mathbf{r}_2)\rangle/\\ &\qquad\sum_i\langle I_i(\mathbf{r}_1)\rangle\sum_i\langle I_i(\mathbf{r}_2)\rangle]^{\frac{1}{2}}.\end{aligned} \tag{6}$$

Each of the sums in Eq. (6) is dominated by the higher-order terms because of the slow convergence of the series, and $\gamma_{jj}(\mathbf{r}_1,\mathbf{r}_2)\simeq 1$ for j greater than about 2. If we now make the plausible assumption that

$$\langle I_j(\mathbf{r})\rangle=\alpha_j\langle I(\mathbf{r})\rangle\quad(0\leq\alpha_j\lesssim 1), \tag{7}$$

where α_j does not depend significantly on $\mathbf{r}$, and

$$\sum_j\alpha_j=1, \tag{8}$$

TABLE I. Coherence of light collected by separated fibers.

Separation of fibers in mm	Counting rates per s R_1	R_2	Counting rates per s ρ_1	ρ_2	Number of events at zero delay (extrapolated)	$n_s(R_2+\rho_2)\Delta\tau$	Degree of coherence $\lvert\gamma(\mathbf{r}_1,\mathbf{r}_2)\rvert$
0.4	304	1070	314	348	1316±24	964±5	0.99±0.04
0.65	158	696	299	325	714±21	592±5	0.93±0.08
0.90	124	438	308	340	984±24	852±5	0.98±0.08

in view of Eq. (4), we find from Eq. (6) that

$$\gamma(\mathbf{r}_1,\mathbf{r}_2)\simeq\sum_j \gamma_{jj}(\mathbf{r}_1,\mathbf{r}_2)\alpha_j. \tag{9}$$

Because the higher-order terms dominate the sum, and $\gamma_{jj}(\mathbf{r}_1,\mathbf{r}_2)\simeq 1$ for most of the terms under the sum, it follows from Eq. (9) that $\gamma(\mathbf{r}_1,\mathbf{r}_2)$ itself is close to unity. This implies that the optical field is spatially coherent at the output aperture.

We have demonstrated the spatial coherence at the output aperture of a laser below threshold by direct correlation measurement with two photodetectors, as illustrated in Fig. 1. The source was a single-mode He:Ne laser that was operated well below its threshold (corresponding to a pump parameter $a\simeq -14$) by a feedback technique, as described previously.[5] At this level the laser output was about $\frac{1}{8}$ its value at threshold. Two 0.1-mm-wide light pipes, in the form of clad-glass fibers, collected the light at two points in the exit aperture of the laser, and directed it onto two photomultiplier tubes that acted as photon counters. After amplification and pulse shaping, the pulses were fed to a digital correlator that measured the counting correlation as a function of time delay between the pulses, as previously described.[6] A counter measured the number of initiations of the measurement cycle by pulses from one detector. For the purposes of this experiment it suffices to look at zero delay, although, in practice, we made use of the information accumulated in the first 12–16 delay channels (corresponding to time intervals of 6–8 μs) to extrapolate back to zero delay.

As is well known,[7] for light of thermal statistics the degree of photoelectric correlation at zero delay is equal to the squared modulus of $\gamma(\mathbf{r}_1,\mathbf{r}_2)$, which can therefore be obtained. It can be shown that the number of events observed at zero time delay, in a correlation channel of width $\Delta\tau$, is given by[6]

$$\text{(number of events)}=n_s(R_2+\rho_2)\Delta\tau \times[1+|\gamma(\mathbf{r}_1,\mathbf{r}_2)|^2R_1R_2/(R_1+\rho_1)(R_2+\rho_2)], \tag{10}$$

where n_s is the number of times that the measurement cycle is initiated. $R_1+\rho_1$, $R_2+\rho_2$ are the mean counting rates of the two photodetectors, and R_1, R_2 are the counting rates contributed by the laser transition alone. Because of the very-low working point of the laser and the very-small field of view sampled by the glass fibers, the rates R_1, R_2 and ρ_1, ρ_2 were of the same order of magnitude. One of the experimental problems was to separate the contribution of the 6328-Å atomic line from neighboring atomic lines and from other contributions to the total counting rate. We made use of a sharply tuned, calibrated interference filter, and of the polarization produced by the Brewster windows to identify the contribution of the 6328-Å line.

Table I summarizes the experimental results for three different separations $|\mathbf{r}_1-\mathbf{r}_2|$ of the fibers. Each experimental run took 1–2 h, and the number of events observed at zero delay in a channel of width $\frac{1}{2}$ μs was of order 1000 in each case, of which about $\frac{1}{3}$ were due to genuine correlations. The counting rates were significantly lower at the greater separations $|\mathbf{r}_1-\mathbf{r}_2|$, because of the peaked beam profile. $|\gamma(\mathbf{r}_1,\mathbf{r}_2)|$ is of course bounded by unity, and the numbers listed in the table after the $|\gamma(\mathbf{r}_1,\mathbf{r}_2)|$ values are the statistical uncertainties associated with the data. It is difficult to improve on the statistical accuracy by lengthening the experimental runs, for the laser parameters tend to change after an hour or two of working time. Still, the results confirm that the light is spatially coherent across the entire 1-mm output aperture, even when the laser is operated as an effectively thermal source.

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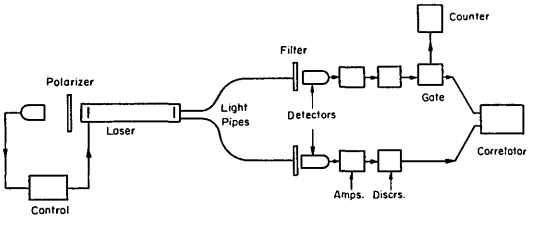

FIG. 1. Arrangement of apparatus.

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