

end increased there was an increase in the mean length of lateral produced from each bud. In the first group a maximum was reached at about the 48th bud. The maximum length of lateral in the second group was attained near the 80th bud, and in the third group near the 119th bud.

The second group overlaps the margins of the adjoining groups and the calculated values of the overlapping portions must be added to approximate the observed values.

The satisfactory agreement between observed and calculated values seems to justify the conclusion that the length of each lateral shoot was a function of its position in its group and, consequently, of its position on the branch. To perhaps an even greater extent, the size of the cycle depends upon its position on the branch.

The decreasing amplitude of the three curves suggests that the successive cycles of laterals may represent damped oscillations of the growth process. The limits of the third group are, however, too poorly defined to afford satisfactory material for study. In any case, the present study adds something to the already extensive evidence which indicates that the processes of growth are characteristically cyclical.

<sup>1</sup> Paper No. 105, University of California, Graduate School of Tropical Agriculture and Citrus Experiment Station, Riverside, California.

<sup>2</sup> Cf. Reed, H. S., Table 1, PROC. NAT. ACAD. SCI., *Washington*, 6, 1920 (397-410).

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## STOKES' LAW OF FALL COMPLETELY CORRECTED

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The present problem is one of fundamental interest: first, from the standpoint of the *kinetic theory*, since its solution definitely settles moot questions as to the reflection of molecules which have been in controversy since Maxwell's time; second, from the standpoint of *electronic physics*, since no subelectron theories have any basis whatever when the complete law of motion of particles of all sizes is known, and third, from the standpoint of *cosmical physics*, since the rate of settling of cosmical dust and of condensed vapor through the atmospheres of planets can now be quite accurately predicted, and since it may have interesting bearings upon theories of atmospheric electricity.

The detailed results of the studies, both experimental and theoretical, of the author and his collaborators in this field will presently appear in a group of articles in the *Physical Review*. The main conclusions may be summarized as follows:

(1) Both at very small and at very large values of  $l/a$  (mean free path over radius) it is shown theoretically by the author that the law of motion must be of the form

$$F = \frac{6 \pi \eta a v}{1 + A'l/a} \quad (1)$$

in which  $F$  is the acting force,  $v$  the velocity produced,  $\eta$  the viscosity of the medium, and  $A'$  a constant. That this form must be correct when the gas is dense, and  $l/a$  is therefore small, it is seen from the fact that the resistance  $F$  is then due wholly to the viscosity of the ambient medium and is therefore proportional to  $a$ , as equation (1) makes it when the term  $A'l/a$  is negligible in comparison with 1. That the form of (1) must also be correct when  $l/a$  is very large and viscosity has disappeared entirely as a cause of resistance is seen from the fact that the resistance  $F$  must then become proportional simply to the number of molecules hit per second by the moving particle, that is to  $a$ , as equation (1) shows it to do when  $A'l/a$  is large compared to fig. 1, since then  $F$  reduces simply to  $6 \pi \eta a^2 v / A'l$ . The values of  $A'$  at the two extremes are not, however, at all the same.

(2) With the aid of a simple combination of hydrodynamics and the kinetic theory, it is shown that when  $l/a$  is small the theoretical, lower, limiting value of  $A'$  is .7004, when  $l$  is defined by  $\eta = .3502 \rho \bar{c} l$ ; while, when  $l/a$  is very large the theoretical value of  $A'$  must be considerably greater than  $l$ . The change in the value of  $A'$  with diminishing gas density means physically a change from a viscous resistance to one due to direct molecular impact.

(3) The oil-drop method, applied to the rates of fall under gravity and rise against gravity of minute charged particles in gases at all densities down to pressures as low as about a millimeter, permits the experimental determination of  $A'$  whenever an equation of the form of (1) holds,  $A'$  being a constant for the region of values of  $l/a$  under consideration; for  $A'$  is then simply the straight line which exhibits the necessary linear relation between  $e^2/3$  and  $l/a$ . In this way the author has experimentally determined the values of  $A'$  for a number of drop-substances and gases, and has found, for oil and air, for example, when  $l/a$  is very small,  $A' = .864$ , when  $l/a$  is very large  $A' = 1.154$ .

(4) The theoretical form of the complete law of fall is then seen at once to be  $F = \frac{6 \pi \eta a v}{1 + A'l/a}$  in which  $A'$  is no longer a constant, but now shifts from a smaller constant value  $A$  to a larger constant value  $(A + B)$  as, with decreasing density,  $l/a$  changes from very small to very large values. To adapt itself to such a change  $A'$  must obviously be given the form  $A' = A + B e^{-ca/l}$ , the exponential term being inserted merely to bring about the shift in the value of  $A'$  (the slope in the  $e^2/3$   $l/a$  graph) from its lower value  $A$ , to

its higher value ( $A + B$ ), the constant  $c$  being introduced simply to take care of the rapidity of shift. That this must be the correct form of  $A'$  is seen from the fact that for small values of  $l/a$  the expression  $e^{-ca/l}$  reduces to 0 and the slope to  $A$ ; while, for large values of  $l/a$  the expression becomes unity and  $A'$  changes to ( $A + B$ ).

(5) From the difference between the lower theoretical limit of  $A'$ , namely .7004, and the observed value for air-oil it is possible to calculate the percentage of the molecules which suffer specular reflection at the surface of oil.  $A$  is experimentally found to vary, when oil-drops are immersed in different gases, from .8 to .9, and to undergo somewhat larger variations with the nature of the reflecting surface.

(6) The values of the coefficients of slip and therefore of specular reflection determined by the foregoing oil-drop experiments have now been checked in every particular by measurements on the coefficients of slip made by the rotating-cylinder method, the change in the apparent viscosity of the gas being observed through the change in the deflection of the inner torsionally suspended cylinder as the pressure of the contained gas is reduced until the deflection due to the constant speed of rotation of the outer concentric cylinder begins to show the phenomenon of slip. This change occurs in accurately measurable magnitude, in the apparatus used, at pressures somewhat below a millimeter.

(7) The experimental results by the oil-drop method and the constant deflection method show that with air-oil, for example, at ordinary temperatures there is  $10\frac{1}{2}\%$  of specular and  $89\frac{1}{2}\%$  of diffuse reflection, with hydrogen-oil  $7\frac{1}{2}\%$  of specular and  $92\frac{1}{2}\%$  of diffuse, with helium-oil 12.6% of specular and 87.4% of diffuse.

(8) The experimental constants of the complete law of fall for air-oil are contained in the following empirical equation:<sup>1</sup>

$$F = 6\pi\eta av[1 + l/a(.864 + .290e^{-1.25a/l})]^{-1} \quad (2)$$

(9) The experimental value of ( $A + B$ ) is very close to its theoretical value for a "diffusely reflecting surface," which, according to Epstein's analysis, is 1.131. When allowance is made for the fact, brought to light by the foregoing experiments on slip, that with oil-air, for example, there is  $10\frac{1}{2}\%$  of specular reflection, the theoretical value of ( $A + B$ ) is found to change to 1.164, which is within the limits of observational error of the experimental value 1.154. This result yields additional evidence for the existence of about 10% of specular reflection, though the accuracy in the determination of the percentage of specular reflection through direct measurement of ( $A + B$ ) is far less than through the measurement of  $A$  alone, i.e., of "coefficients of slip."

(10) It is shown both theoretically and experimentally that the value of the final coefficient ( $A + B$ ) is not likely to vary more than 2% or 3%, at

most, with the nature of the drop-substance or of the surrounding gases, so that equation (2) is of very general validity and may be taken as very nearly correct in the computation of the rate of settling of particles of any kind through the rarified regions of the upper air.

(11) The value of  $(A + B)$  is shown to be completely inconsistent with the theory of condensation of gas molecules on the surface of the droplet and a subsequent reëvaporation of these molecules which is uniform over the surface of the droplet.

(12) The present studies show that there has never been any theoretical or experimental basis whatever for the coefficient  $A = .815$  which has been so frequently used by preceding writers in this field. The assumptions made at the time of the first introduction of this coefficient, when correctly treated theoretically, lead to a value 1.5 instead of .815 and the present work shows these assumptions themselves to be altogether inadmissible (see 11).

(13) The extension of the experimental studies by the oil-drop method to very much larger values of  $l/a$  than have ever been used in the experiments from which conclusions as to the existence of sub-electrons have been drawn, and the obtaining of completely consistent results throughout the whole range without the slightest indication of the necessity of introducing sub-electronic charges at any point, demonstrates again the erroneousness of the work from which the existence of sub-electrons has been inferred. The error has arisen from incorrect assumptions as to densities, and incorrect use of the Brownian movement data.<sup>2</sup>

(14) Slight mechanical roughnesses in a surface of glass are called upon to bring into reconciliation the author's results on the specular reflection of molecules and Knudsen's<sup>3</sup> and Gade's<sup>4</sup> apparent confirmation of the law of diffuse reflection of molecules.

(15) It is most satisfying to find a field of study in physics which has been in confusion for more than fifty years brought under complete control through the obtaining of experimental results which conform precisely with the predictions of the kinetic theory. While the discovery of new and as yet unexplained phenomena, like those connected with the facts of "quanta" and relativity, are making physics at present intensely interesting, it is the foregoing type of successes, clearing up, as they do, former mysteries and continually widening the field of fully explained phenomena that gives us confidence in its methods and faith in its values for the future of the race.

<sup>1</sup> An equation of this general type was directly obtained empirically by the author in 1911 and published in *Brit. Ass'n Reports, Dundee*, 1912, p. 410, and indirectly by Knudsen and Weber, *Ann. Physik, Leipsig*, 36, 1911 (982).

<sup>2</sup> This is clearly pointed out in *The Electron*, University of Chicago Press, 1917, chapter 8; see also German translation Vieweg, 1922.

<sup>3</sup> *Ann. Physik, Leipsig*, 28, 1909 (105), and 35, 1911 (389).

<sup>4</sup> *Ibid.*, 41, 1913 (289).