Supplemental Material for Curvature-Induced Skyrmion Mass

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ENERGY FUNCTIONAL

In this supplemental material we provide the full expressions of the energy potentials for a skyrmion stabilized in a curvilinear defect. To begin with, the energy functional equals,

$$\mathcal{W}_0(\Pi_c)/J = (\boldsymbol{\nabla}\Theta_c - k_1\mathbf{e}_1)^2 + (\sin\Theta_c r'/r + \cos\Theta_c k_2)^2 + 1 - \cos^2\Theta_c + d(2\boldsymbol{\nabla}\Theta_c\cdot\mathbf{e}_1\sin^2\Theta_c - (k_1+k_2)\cos^2\Theta_c)\,, \tag{1}$$

while the operator $\mathcal{L}_\xi$ equals,

$$\mathcal{L}_\xi/J = (\cos^2\Theta_c - 1)(k_1^2 - k_2^2) + \Theta_c' k_1 + \sin\Theta_c\cos\Theta_c k_2 r'/r + \sin\Theta_c\cos\Theta_c k_1\partial_s - \boldsymbol{\nabla}\sin\Theta_c\cdot\boldsymbol{\nabla} - \sin\Theta_c\boldsymbol{\nabla}^2 - d\sin^2\Theta_c\Theta_c\,. \tag{2}$$

EFFECTIVE MASS

In this Supplemental Note we consider the mass renormalization terms if we take into account both the fluctuations in the fields Π_c and Φ_c. The starting point of the analysis is the imaginary time Euclidean action

$$\mathcal{S}_E = \frac{L}{\alpha}\int_0^\beta d\tau\int d\mathcal{A}[\frac{iSl^2}{\alpha^2}\dot{\Phi}(1-\Pi) + \mathcal{W}(\Phi_c,\Pi_c)]\,, \tag{3}$$

where Π,Φ are the fields using a decomposition in cartesian coordinates, $\mathbf{m} = \sin\Theta\cos\Phi\mathbf{x} + \sin\Theta\sin\Phi\mathbf{y} + \cos\Theta\mathbf{z}$, while Π_c and Φ_c are the magnetization components in the local orthonormal basis $\mathbf{m} = \sin\Theta_c\cos\Phi_c\mathbf{e}_1 + \sin\Theta_c\sin\Phi_c\mathbf{e}_2 + \cos\Theta_c\mathbf{n}$. For the particular geometry considered here, $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{n}\}$ is defined by unit vectors expressed in the cartesian basis as $\mathbf{e}_1 = \{e_1^x, e_1^y, e_1^z\} = g(r)\{\cos\phi, \sin\phi, z'(r)\}$, $\mathbf{e}_2 = \{e_2^x, e_2^y, e_2^z\} = \{-\sin\phi,\cos\phi,0\}$, and $\mathbf{n} = \{n^x,n^y,n^z\} = g(r)\{-z'(r)\cos\phi, -z'(r)\sin\phi, 1\}$. We find then find that $\Pi = G(\Pi_c,\Phi_c)$ and $\Phi = K(\Pi_c,\Phi_c)$, where $G(\Pi_c,\Phi_c) = \cos^{-1}[\sin\Theta_c\cos\Phi_c e_1^z + \sin\Theta_c\sin\Phi_c e_2^z + \cos\Theta_c n^z]$ and $K(\Pi_c,\Phi_c) = \tan^{-1}[(\sin\Theta_c\cos\Phi_c e_1^y + \sin\Theta_c\sin\Phi_c e_2^y + \cos\Theta_c n^y)/(\sin\Theta_c\cos\Phi_c e_1^x + \sin\Theta_c\sin\Phi_c e_2^x + \cos\Theta_c n^x)]$. We can rewrite the action of Eq. (3) in terms of fields Π_c and Φ_c as

$$\mathcal{S}_E = \frac{L}{\alpha}\int_0^\beta d\tau\int d\mathcal{A}\left[\frac{iSl^2}{\alpha^2}\left(\dot{\Theta}_c\frac{\delta G}{\delta\Theta_c} + \dot{\Phi}_c\frac{\delta G}{\delta\Phi_c}\right)(1-K) + \mathcal{W}(\Phi_c,\Pi_c)\right]\,. \tag{4}$$

We regard $\mathbf{R}$ as the dynamical variable of the skyrmion position and introduce the transformation $\Pi_c = \Pi_c^0[\mathbf{r}-\mathbf{R}(\tau)] + \eta[\mathbf{r}-\mathbf{R}(\tau),\tau]$ and $\Phi_c = \Phi_c^0[\mathbf{r}-\mathbf{R}(\tau)] + \xi[\mathbf{r}-\mathbf{R}(\tau),\tau]$, where we take into account fluctuations in both of the fields. Here Π_c^0 and Φ_c^0 is the static solution of $\delta\mathcal{S}_E = 0$ for $\mathbf{R} = 0$. Taking into account that $\Phi_c^0 = 0$ the action is expressed in terms of

$$\mathcal{S}_E = S_E^0(\Pi_c^0,\Phi_c^0) + \frac{L}{\alpha}\int_0^\beta d\tau\int d\mathcal{A}[\bar{\mathcal{J}}^T\Psi + \Psi^T\bar{\mathcal{J}} + \Psi^T(\mathcal{G}+\mathcal{K})\Psi]\,, \tag{5}$$

where we introduce the compact notation $\Psi = (\xi,\eta)^T$ and $\bar{\mathcal{J}} = (iSl^2/2\alpha^2)(-[\dot{\mathbf{R}}\cdot\nabla\Theta_c^0]\sin\Theta_c^0, 0)^T$. Here $\mathcal{G} = (iSl^2/2\alpha^2)D_0\partial_\tau + H$, with $D_0 = \begin{pmatrix} 0 & D_0^1 \\ D_0^2 & 0\end{pmatrix}$, $D_0^1 = -z'(\cos\Theta_c^0 - \sin\Theta_c^0 z')/(\sin\Theta_c^0 + z'\cos\Theta_c^0)^2$, and $D_0^2 = D_0^1 + \sin\Theta_c^0$. Moreover, H denotes the magnon Hamiltonian defined as $H = \delta_{\Psi^T}\delta_\Psi\mathcal{W}|_{\Psi=\Psi^T=0}$. Finally, we introduce the operator

$$\mathcal{K} = \begin{pmatrix} 0 & \frac{1}{2}\dot{\mathbf{R}}\cdot\nabla\Theta_c^0 K_0 + D_0^1\dot{\mathbf{R}}\cdot\nabla \\ \frac{1}{2}\dot{\mathbf{R}}\cdot\nabla\Theta_c^0 K_0 + D_0^2\dot{\mathbf{R}}\cdot\nabla & 0\end{pmatrix}\,, \tag{6}$$

where $K_0 = -z'/(\sin\Theta_c^0 + z'\cos\Theta_c^0)$. By integrating out the fluctuating fields Ψ, we then arrive at an effective action

$$\mathcal{S}_E = \mathcal{S}_E^0 + \mathcal{S}_D^0 + \tilde{\mathcal{S}}_D = \mathcal{S}_E^0 - \frac{L}{\alpha}\bar{\mathcal{J}}^T \cdot \mathcal{G}^{-1}\bar{\mathcal{J}} - \mathrm{Tr}[\frac{1}{2}(\mathcal{G}^{-1}\mathcal{K})^2]\,, \tag{7}$$

where we have performed an expansion to lowest nonvanishing order in $\dot{\mathbf{R}}$. One can demonstrate that the terms $\mathcal{S}_D^0$ and $\tilde{\mathcal{S}}_D$ appearing in Eq. (7) are nonlocal dissipative terms of the form $\int_0^\beta d\tau \int_0^\beta d\sigma \dot{R}_i(\tau)\gamma_{ij}(\tau-\sigma)\dot{R}_j(\sigma)$, with $\gamma_{ij} = \gamma_{ij}^0 + \tilde{\gamma}_{ij}$ being the nonlocal in time magnon kernel describing the skyrmion-magnon interaction, with two different contributions originating from the terms $\mathcal{S}_D^0$ and $\tilde{\mathcal{S}}_D$ respectively. Indices i, j denote the components of the vector $\mathbf{R}$ on the 2D curvilinear surface. In close analogy with the results of Ref. [1], where a similar problem is treated in the Euclidean space, we note that at asymptotic times $\tau - \sigma \gg 1$ the magnon kernel reduces to a mass term,

$$\int_0^\beta d\tau \int_0^\beta d\sigma \dot{R}_i(\tau)\gamma_{ij}(\tau-\sigma)\dot{R}_j(\sigma) \approx \int_0^\beta d\tau \frac{1}{2}\dot{R}_i(\mathcal{M}_{ij} + \tilde{\mathcal{M}}_{ij})\dot{R}_j\,. \tag{8}$$

The term $\mathcal{M}_{ij}$ corresponds to an inertia term generated by the varying curvature, while $\tilde{\mathcal{M}}_{ij}$ has an explicit temperature dependence and is a mass renormalization term for sufficiently low temperatures below the magnon gap. For the specific geometry considered here and for translations along the arc direction and under the assumptions specified in the main text, we find that the mass $\mathcal{M}$ is identical with the simplified form of Eq.(9) of the main text.

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[2] N. Papanicolaou and T. Tomaras, Nuclear Physics B **360**, 425 (1991).