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Observing The de Sitter Space Propagator*

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Abstract

The primordial fluctuations in the mass density may have arisen from quantum fluctuations in a scalar field that occurred during an inflationary era. Fluctuations which arose in this way can be highly non-Gaussian. Also the bad infrared properties of the propagator for a massless scalar field in de Sitter space can translate itself into a power spectrum, for the two-point spatial correlation of objects that do not trace the mass, which behaves like k^{-3}, at small wavenumbers k.

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1. INTRODUCTION

In this talk I will discuss the possibility of deriving evidence that the universe went through a de Sitter phase from observations of the large scale distribution of astronomical objects like rich clusters of galaxies. At first glance this seems like a perfectly reasonable possibility. In a de Sitter epoch the correlations of massless fields have very peculiar infrared properties. The two-point correlation of a free massless scalar field for example, diverges like k^{-3} as the comoving wavenumber k goes to zero. One might expect that, in a scenario where the primordial fluctuations in the mass density arose from quantum fluctuations in a scalar field during the inflationary phase[1,2], the bad infrared behavior of correlations in de Sitter space would translate itself into unusual behavior for the primordial mass density fluctuations.

In the next section I will give a simple naturalness argument that restricts the form of the primordial mass density fluctuations. This naturalness principle of scale invariance determines the two-point correlation to have a Harrison–Zeldovich power spectrum[3]. If the primordial fluctuations are Gaussian their form is then almost completely determined by the principle of scale invariance. In this case it is not possible for the de Sitter epoch to have left an unusual imprint on the primordial mass density fluctuations. However, the principle of scale invariance does not completely determine the higher correlations of the mass density fluctuations. In this talk I will summarize the work presented in Ref. (4). There it was shown that it is possible for primordial mass density fluctuations which arose from de Sitter quantum fluctuations of a scalar field to be highly non-Gaussian and that the bad infrared behavior of the correlations of massless fields in de Sitter space can result in very unusual behavior for the two-point spatial correlation of objects that do not trace the mass.

2. THE PRINCIPLE OF SCALE INVARIANCE

The large-scale structure of the Universe probably arose from small fluctuations in the mass density that grew due to a gravitational instability. Since the mass density fluctuations were once small it is useful to analyze their growth using linear perturbation theory. In linear perturbation theory it is convenient to Fourier transform the mass density fluctuations since modes of different wavevectors evolve independently in time,

$$\frac{\delta\rho(\vec{x},t)}{\langle\rho\rangle} = \int d\vec{k}\ \frac{\delta\rho(\vec{k},t)}{\langle\rho\rangle}\, e^{i\vec{k}\cdot\vec{x}} \quad . \tag{1}$$

Here $\vec{x}$ is the comoving coordinate and $\vec{k}$ is the comoving wavevector. The physical wavenumber is $q = k/R$, where R is the Robertson–Walker scale factor. The Robertson–Walker scale factor grows like $t^{1/2}$ in the radiation dominated era and like $t^{2/3}$ in the matter-dominated era. The Hubble constant H is defined by

$$H = \dot{R}/R \quad . \tag{2}$$

The present value of the Hubble constant is $100h$ km/s-Mpc with h between 0.5 and 1. The Hubble constant falls linearly with time during the radiation and matter dominated eras. I define the horizon length to be $1/H$. Note that this is not necessarily the size of a region that has been in causal contact. In fact, in the inflationary cosmology, the size of a region that has been in causal contact is much larger than the horizon length $1/H$. Fluctuations with physical wavelengths less than the horizon length today had wavelengths greater than the horizon length at earlier times. It is this fact that makes it difficult to come up with reasonable ways to generate the primordial mass density fluctuations. In linear perturbation theory fluctuations with wavelengths less than the horizon length do not grow in the radiation-dominated era and grow proportional to $t^{2/3}$ in the matter-dominated era. The description of the evolution of fluctuations with wavelengths greater than the horizon length is gauge-dependent. I shall work in a gauge where they are constant.

Thus for fluctuations which cross the horizon in the matter-dominated era

$$\frac{\delta\rho}{\langle\rho\rangle}(\vec{k},t) = a(\vec{k})(t/t_{h.c.})^{2/3} , \tag{3}$$

where $a(\vec{k})$ is a random variable and $t_{h.c.}$ is the time that the fluctuation with comoving wavenumber k crossed the horizon. Writing the Robertson-Walker scale factor in the matter-dominated era as $R = (t/t_0)^{2/3}$, so that at the present time t_0 physical and comoving lengths coincide, the condition

$$k/R(t_{h.c.}) = H(t_{h.c.}) \tag{4}$$

implies that

$$(t/t_{h.c.})^{2/3} = \frac{9}{4}\, k^2 t^{2/3} t_0^{\frac{4}{3}} \tag{5}$$

giving

$$\frac{\delta\rho}{\langle\rho\rangle}(\vec{k},t) = \frac{9}{4}\, a(\vec{k}) k^2 t^{2/3} t_0^{4/3} = \epsilon(\vec{k}) t^{2/3} t_0^{4/3} \quad . \tag{6}$$

The variable ϵ has the explicit k dependence that resulted from eliminating the time of horizon crossing absorbed into it. The variable ϵ has dimensions of length. The nature of the primordial mass density fluctuations are determined by the probability distribution $P[\epsilon]$, or equivalently by its moments (provided they all exist)

$$\left\langle \epsilon(\vec{k}_1)\ldots\epsilon(\vec{k}_n)\right\rangle \equiv \int [d\epsilon] P[\epsilon]\epsilon(\vec{k}_1)\ldots\epsilon(\vec{k}_n) \quad . \tag{7}$$

If the length scales associated with the physical processes that generated the primordial mass density fluctuations are very small compared to astrophysically relevant length scales, then they can be neglected in correlations of ϵ. Dimensional analysis then implies that[5)]

$$\left\langle \epsilon(\lambda\vec{k}_1)\ldots\epsilon(\lambda\vec{k}_n)\right\rangle = \lambda^{-n}\left\langle \epsilon(\vec{k}_1)\ldots\epsilon(\vec{k}_n)\right\rangle \quad . \tag{8}$$

Equation (8) is the principle of scale invariance. This plus the homogeneity and isotropy of space determines the two-point correlation to have the Harrison-Zeldovich form[3)]

$$\left\langle \epsilon(\vec{k}_1)\epsilon(\vec{k}_2)\right\rangle \propto k_1 \delta^3(\vec{k}_1 + \vec{k}_2) \quad . \tag{9}$$

Note, however, that many types of connected higher point correlations are permitted by the principle of scale invariance.

3. OBJECTS

Our goal is to use observations of the spatial distribution of astronomical objects like galaxies or rich clusters of galaxies to deduce something about the primordial mass density fluctuations. To do this, the number density of the objects $n_0(\vec{x})$ must be related to the primordial mass density fluctuations.

The simplest assumption that could be made is to assume that the objects trace the mass

$$n_0(\vec{x}) \propto \rho(\vec{x}) \quad . \tag{10}$$

It is certainly not possible to assume that all observed objects trace the mass. Different classes of objects have very different two-point correlations

$$\xi_0(|\vec{x}-\vec{y}|) \equiv \frac{\langle n_0(\vec{x})n_0(\vec{y})\rangle}{\langle n_0\rangle^2} - 1 \quad . \tag{11}$$

For example, the galaxy-galaxy two-point correlation is unity at about $5h^{-1}$ Mpc while the rich cluster-rich cluster two-point correlation is unity at about $25h^{-1}$ Mpc. Both these correlations seem to be falling like r^{-2} (although they are not measured well at large distances where the correlations are small). Even if galaxy correlations are close to mass density correlations, the rich cluster correlations are not. Clusters of galaxies come in many types. The rich clusters (or Abell clusters) are rare objects; they have many more galaxies in a small region than an average cluster would have. Because rich clusters have not had enough time for gravitational nonlinearities to cause them to move a distance comparable to their mean separation, it seems reasonable to imagine that the locations of rich clusters can be approximated by places where filtered primordial mass density fluctuations were unusually large[6]. The filtering is necessary to ensure that the fluctuations had enough mass to collapse to a rich cluster. A simple form for the number density that realizes this is[7,8]

$$n_0(\vec{x}) = C \exp T\delta_f(\vec{x}) \tag{12}$$

where

$$\delta_f(\vec{x}) = \int dy \frac{\delta\rho(\vec{y})}{\langle\rho\rangle} W(|\vec{x}-\vec{y}|) , \tag{13}$$

and W is the filter. Most of the conclusions I draw do not depend on this particular representation for the objects number density. However, it has the advantage that it is very easy to see how connected correlations of the primordial mass density fluctuations affect the two-point correlations of objects that do not trace the mass[8,9].

$$1 + \xi_0(|\vec{x}_1 - \vec{x}_2|) \exp \left\{ \sum_{n=2}^{\infty} \frac{T^n}{n!} \int d\vec{y}_1 \ldots \int d\vec{y}_n \right.$$

$$\cdot \sum_{m=1}^{n-1} \binom{n}{m} \langle \delta(\vec{y}_1) \ldots \delta(\vec{y}_n) \rangle_c [W(|\vec{x}_1 - \vec{y}_1|) \ldots W(|\vec{x}_1 - \vec{y}_m|)]$$

$$\left. \cdot [W(|\vec{x}_2 - \vec{y}_{m+1}|) \ldots W(|\vec{x}_2 - \vec{y}_n|)] \right\} \tag{14}$$

where the simplified notation $\delta(\vec{y}) \equiv (\delta\rho(\vec{y})/ \langle \rho \rangle)$ has been adopted.

The connected n-point correlation of the mass density fluctuations $\langle \delta(\vec{x}_1) \ldots \delta(\vec{x}_n) \rangle_c$ depends on the locations of n points, $\vec{x}_1, \ldots, \vec{x}_n$. Suppose m of these points are kept near each other (i.e., within a distance determined by the filtering length scale) but separated a large distance r from the remaining $n-m$ points, which are also near each other. If in this limit the connected n-point correlation falls off like r^{-p}, then from eq. (14) it is evident that it gives a contribution to the spatial two-point correlation of the objects that fall off like r^{-p} at large r. A similar statement can be made in Fourier space. The Fourier transform of the spatial two-point correlation of the objects is called its power spectrum $P_0(k)$

$$\xi_0(r) \equiv \int d\vec{k} e^{i\vec{k}\cdot\vec{r}} P_0(k) \quad . \tag{15}$$

The Fourier transform of a connected n-point correlation of the mass density fluctuations depends on n wavevectors $\vec{k}_1, \ldots, \vec{k}_n$. Let k_s denote the magnitude of any partial sum of these wavevectors (i.e., $k_1, |\vec{k}_1 + \vec{k}_2|, \ldots, |\vec{k}_1 + \ldots + \vec{k}_{n-1}|$). If the connected n-point correlation of the mass density fluctuations diverges like k_s^{-p} as a partial sum of wavevectors $\vec{k}_s$ goes to zero, then it gives a contribution to the power spectrum for the two-point spatial correlation of the objects that goes like k^{-p} at small k.

If the primordial mass density fluctuations are Gaussian, then eq. (14) becomes[7)]

$$\xi_0(r) = \exp\left(T^2 \xi_f(\vec{x})\right) - 1 \tag{16}$$

where ξ_f is the filtered two-point correlation of the mass. Note that as the threshold T gets larger, the two-point correlation of the objects is enhanced more (compared with the two-point correlation of the filtered mass density fluctuations). In this model, richer clusters have larger correlations. Another interesting feature of this model is that different classes of objects, defined by differing thresholds and filters, should have cross two-point correlations that are enhanced. This is very different from what occurs in the cosmic string scenario for producing enhanced correlations. There the enhanced correlations of objects result from parent loops fragmenting into several strongly correlated daughter loops which seed the objects[10)]. Since the strings in the network move at roughly the speed of light and intercommute frequently, parent loops formed at very different times are essentially uncorrelated. The evolution of the string network does not provide any significant galaxy-rich cluster (or poor cluster–rich cluster) cross correlations[11)]. In the cosmic string scenario such cross-correlations must arise from gravitational forces.

While some features of eq. (16) are in agreement with observations, there is a potential difficulty with it. Since the Harrison–Zeldovich spectrum vanishes as k goes to zero, the two-point correlation of the filtered primordial mass density fluctuations $\xi_f(r)$ must integrate to zero. The two-point correlation of the objects in eq. (16) crosses zero at the same place as $\xi_f(r)$. If this zero crossing is not at a large enough distance, it may be difficult to explain the significant rich cluster correlations at $30h^{-1}Mpc$. Indeed with adiabatic, scale invariant, Gaussian primordial mass density fluctuations this seems to be the case. This model has difficulty explaining the significant correlations of rich clusters of galaxies at large distances. The situation is even worse if a more realistic model, where the rich clusters of galaxies are located at unusually high peaks of filtered primordial mass density fluctuations, is used[12)]. The peak condition introduces an anticorrelation (high peaks do not occur right next to each other) which reduces the amount of the enhancement and moves the zero crossing in.

4. NON-GAUSSIAN FLUCTUATIONS FROM INFLATION

One way to generate primordial mass density fluctuations is to have them arise from quantum fluctuations in a scalar field during an inflationary era. The smallness of the primordial mass density fluctuations restricts the field that is driving the inflation to be very weakly coupled[13] so that its fluctuations (and the corresponding mass density fluctuations) are approximately Gaussian. However, there may be other fields that are essentially massless during the de Sitter era and later develop a mass for dynamical reasons (e.g., an invisible axion field). In this section I will present a model where quantum fluctuations in an invisible axion field[14] give rise to highly non-Gaussian mass density fluctuations. In the literature the central limit theorem is sometimes used to motivate Gaussian primordial fluctuations. The model presented below illustrates that, just as the central limit theorem does not imply that the interactions of elementary particles are described by free field theory, it does not imply Gaussian primordial fluctuations in the mass density.

If a Peccei–Quinn symmetry is spontaneously broken during the inflationary era, there will be an essentially massless axion field $a(\vec{x},\tau)$ whose interactions during the de Sitter phase are described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\, g^{\mu\nu}\partial_\mu a \partial_\nu a + \frac{\lambda}{4!f^4}(g^{\mu\nu}\partial_\mu a\partial_\nu a)^2 + \ldots \quad . \tag{17}$$

The de Sitter metric is taken to be

$$ds^2 = \frac{1}{H^2\tau^2}[d\tau^2 - d\vec{x}^2] \quad , \tag{18}$$

where $\tau\epsilon(-\infty,0)$. In these conformal coordinates, $k\tau = -q/H$ so astrophysically relevant length scales (which are much greater than the horizon length in the de Sitter epoch) correspond to $|k\tau| << 1$.

Treating the coupling λ as a perturbation, the connected two- and four-point correlations of the Fourier transform of the axion field that result from this Lagrangian are given at small $|k\tau|$ by

$$\left\langle \tilde{a}(\vec{k}_1,\tau)\tilde{a}(\vec{k}_2,\tau)\right\rangle = \frac{H^2}{2k_1^3}(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2) \tag{19}$$

$$\left\langle \tilde{a}(\vec{k}_1,\tau)\ldots\tilde{a}(\vec{k}_4,\tau)\right\rangle_c \simeq \frac{H^4}{4(k_1k_2k_3k_4)^{3/2}}\lambda\left(\frac{H}{f}\right)^4$$

$$\cdot g(\vec{k}_1,\ldots,\vec{k}_4)(2\pi)^3\delta^3(\vec{k}_1+\ldots+\vec{k}_4) \quad . \tag{20}$$

Here g is a computable (see Ref. (4)) homogeneous function of the comoving wavevectors $\vec{k}_1,\ldots,\vec{k}_4$ of degree minus three.

After the universe exits the de Sitter era and cools to a temperature of about 1 GeV, the non-derivative interactions of the axion become important. Thereafter, fluctuations in the axion field get converted into fluctuations in the mass density as they reenter the horizon. Linearizing these fluctuations in the axion field gives

$$\left\langle \tilde{\delta}(\vec{k}_1)\tilde{\delta}(\vec{k}_2)\right\rangle = \frac{N^2T^2(k_1)}{k_1^3}(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2) \tag{21}$$

$$\left\langle \tilde{\delta}(\vec{k}_1)\ldots\tilde{\delta}(\vec{k}_4)\right\rangle_c = \frac{N^4T(k_1)\ldots T(k_4)}{(k_1k_2k_3k_4)^{3/2}}\lambda\left(\frac{H}{f}\right)^4 g(\vec{k}_1,\ldots,\vec{k}_4)$$

$$\cdot(2\pi)^3\delta^3(\vec{k}_1+\ldots+\vec{k}_4) \quad . \tag{22}$$

Here $T(k)$ is the transfer function appropriate to isocurvature fluctuations[15)]. It goes like k^2 at small k ensuring that the fluctuations in eqs. (21) and (22) are scale invariant at small k. Since a/f is an angular variable, it is sensible to assume that the background value of the axion field is of order the axion decay constant

$$\langle a\rangle \sim f \quad . \tag{23}$$

The normalization factor N is chosen so that

$$\left\langle \left(\int_{\substack{horizon\\volume}} dx \delta(\vec{x}) \right)^2 \right\rangle^{1/2} \sim (H\Omega_a/f) \tag{24}$$

where Ω_a is the fraction of critical density that is in axions. Consistency with the high degree of isotropy of the microwave background radiation and the nonlinearity of the fluctuations today demands that

$$H\Omega_a/f \sim 10^{-5} \quad . \tag{25}$$

If axions comprise the dark matter of the universe, then $\Omega_a = 1$ and the small value of H/f implies that the fluctuations are approximately Gaussian (i.e., the disconnected part of the four-point correlation dominates over the connected part). However, the axion was invented to solve the strong CP puzzle and there is no reason for us to demand that axions be the dark matter. Some other particle (e.g., the photino) may dominate the mass density of the universe.

If Ω_a is of order 10^{-5}, then $H/f \sim 1$ and the mass density fluctuations are highly non-Gaussian. This occurs for an axion decay constant of order 10^8 GeV[16]. With such a low f there may be an observable flux of axions emitted from the sun[17]. In order to have the Peccei–Quinn symmetry broken during the inflationary era, it is necessary to have the curvature of the potential for the scalar fields, whose vacuum expectation values break the Peccei–Quinn symmetry, be as large as the Hubble constant during the inflationary phase. Also, the rehating, as the de Sitter epoch is exited, must be somewhat inefficient so that the universe only reheats to a temperature of order the axion decay constant, insuring that the Peccei–Quinn symmetry is not restored upon reheating. The low reheating temperature may necessitate unconventional methods of baryon number generation[18].

This model has highly non-Gaussian scale invariant mass density fluctuations. They do not, however, have unusual behavior at small wavevectors because of the derivative axion couplings. (These fluctuations give a power spectrum $P_0(k)$, for objects that do not trace the mass, that goes to a constant at small wave numbers.) It is possible to get more dramatic behavior by imagining that there is another field χ which is not a Goldstone boson but has a mass that is small compared with the Hubble constant during the inflationary phase. Since χ is not a Goldstone boson, it can have non-derivative couplings to a. For example, a term of the form

$$\mathcal{L} = (\lambda'/f)\chi g^{\mu\nu}\partial_\mu a \partial_\nu a \tag{26}$$

can occur in the Lagrangian density. Now tree level χ exchange gives a contribution to the connected axion four-point correlation that diverges like k_s^{-3} as a partial sum $\vec{k}_s$ of two wavevectors goes to zero. This results in non-Gaussian mass density fluctuations that violate cluster decomposition and give rise to a power spectrum, for objects that do not trace the mass, which at small wavenumbers has the form

$$P_0(k) \propto k^{-3} \quad . \tag{27}$$

Also, using the methods developed in ref. (19), it can be shown that the violation of cluster decomposition in the primordial mass density fluctuations allows gravitational nonlinearities to change the normalization of the power spectrum for the two-point correlation of the mass density fluctuations at small wave numbers.

The k^{-3} behavior in eq. (27) is a direct reflection of similar behavior in the de Sitter propagator for χ. Note that the mass m of the χ field cuts this infrared divergent behavior off at a physical wavenumber (in the de Sitter era) given roughly by

$$q = H \exp\left(-\frac{3}{4}\frac{H^2}{m^2}\right) \quad . \tag{28}$$

For m's significantly smaller than H this corresponds to a length scale much larger than any relevant for astrophysics. The size of the region that has been in causal contact will also cut off the infrared divergence.

Even if the normalization of the k^{-3} tail in the power spectrum $P_0(k)$ is small (recall it depends on the undetermined parameter λ') it could have a dramatic impact on the large-scale structure of the universe. The power spectrum in (27) implies that the object's two-point correlation function becomes roughly constant at large distances. Misidentifying the mean number of objects in a sample could, however, remove this constant tail. Eq. (27) gives fluctuations in the number of objects in a large volume V (with a fuzzy boundary), divided by the mean number of objects, that do not tend to zero as V tends to infinity. Observing such peculiar behavior would be indirect evidence that the universe had passed through an inflationary era.

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