CALT-68-1518
DOE RESEARCH AND
DEVELOPMENT REPORT

Weak Radiative B Meson Decay*

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Abstract

Weak radiative B meson decays, $B \to \gamma X_s$ have a B meson decaying to a hard photon (i.e., $E_\gamma \gtrsim 2$ GeV) and strange hadronic final states X_s. The prediction of the standard model (with minimal particle content) for the rate for this process is reviewed. Particular attention is paid to the role of strong interaction corrections, which have a significant impact on the rate.

⋆ This work supported in part by the U.S. Department of Energy under Contract Nos. DE-AC0381-ER40050 and DE-FG03-84-ER40172

† Talk delivered by R. Springer at the XXIV International Conference on High Energy Physics at Munich, August 4–10, 1988

‡ Operated by University Research Association Inc. under contract with the United States Department of Energy

Recent experimental improvements have made the measurement of rare B meson decays feasible. Among the rare decays of B mesons are the weak radiative decays: $B \to \gamma X_s$ where X_s denotes a strange hadronic final state and γ denotes a hard photon ($E_\gamma \gtrsim 2$ GeV). Already there are limits on the branching ratio to several possible exclusive channels (e.g., $X_s = K^*(890)$, $X_s = K_2^*(1430)$, etc.) at the 10^{-4} level[1] and in the future experiments at CESR will be sensitive to branching ratios for exclusive channels at the 10^{-5} level and to the inclusive branching ratio at the 10^{-4} level.[2]

Since we are considering weak radiative decays to a hard photon we expect that the rate for the inclusive process $B \to \gamma X_s$ will be dominated by short distance physics that gives rise to the two-body quark decay $b \to s\gamma$ through a loop graph that contains a virtual top or charm quark. (An up quark in the loop is suppressed by additional small weak mixing angles.) Note that unlike rare kaon decays, for rare B meson decays a top quark loop is not suppressed by additional small weak mixing angles compared to a charm quark loop. This makes the rates for rare B meson decays very sensitive to extensions of the Higgs sector.[3] In the minimal standard model the Higgs sector consists of a single $SU(2)$ doublet and there is only one physical Higgs scalar. Since the same transformations that diagonalize the quark mass matrices diagonalize the neutral Higgs scalar couplings, there are no flavor changing couplings of the Higgs scalar at tree level. The simplest extension of the Higgs sector consists of models with two Higgs doublets. In this case there are three physical neutral spin zero particles and physical charged scalars as well. It is possible to construct models where there are no flavor changing couplings of the neutral scalars at tree level,[4] although this is less automatic than in the minimal standard model. Since the charged Higgs boson couplings to quarks q are suppressed by (m_q/M_W) in low energy physics it is processes where virtual top quarks play a prominent role that are most easily affected by virtual charged Higgs exchange, which in two Higgs doublet models occurs in addition to virtual W-boson exchange.

Neglecting strong interactions, an effective Hamiltonian for weak radiative B meson decay can be obtained by matching the calculation of some one loop diagrams to the tree level matrix element of a magnetic moment type operator: $\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$. This yields (neglecting m_c and m_b compared with m_t and M_W) in the standard model with minimal particle content

$$\mathcal{H}_{\text{eff}} = -\frac{2G_F}{\sqrt{2}}(s_3 + s_2 e^{i\delta}) A(m_t^2/M_W^2) \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \tag{1}$$

where [5]

$$A(x) = x\left[\frac{(2/3)x^2 + (5/12)x - (7/12)}{(x-1)^3} - \frac{((3/2)x^2 - x)}{(x-1)^4}\ln x\right], \tag{2}$$

and $F_{\mu\nu}$ is the electromagnetic field strength tensor. Strong interaction leading logarithmic corrections to the effective Hamiltonian density (1) can be important since $(\alpha_s(m_b)/\pi)\ln(m_t^2/m_b^2)$ is not very small. Also for $m_t^2 \ll M_W^2$ the free quark Hamiltonian vanishes like $[(m_t^2 - m_c^2)/M_W^2]$ due to the GIM mechanism.[6] However, the strong interaction corrections induce a piece that has a much weaker logarithmic GIM cancellation and vanishes only like $[\ln(m_t^2/M_W^2) - \ln(m_c^2/M_W^2)]$. For small $(m_t/M_W)^2$ the strong interaction corrections are expected to be very important.

Integrating out the top quark and W-boson (it is a good approximation to integrate the top quark and W-boson together, since for reasonable values of m_t, $\alpha_s(m_t)$ is very close to $\alpha_s(M_W)$) yields an effective five-quark theory with a Hamiltonian density for weak radiative B-meson decay that contains the following operators

$$O_1 = (\bar{s}_{L\alpha}\gamma^\mu c_{L\beta})(\bar{c}_{L\beta}\gamma_\mu b_{L\alpha}), \tag{3a}$$

$$O_2 = (\bar{s}_{L\alpha}\gamma^\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma_\mu b_{L\beta}), \tag{3b}$$

$$O_3 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\beta})], \tag{3c}$$

$$O_4 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\alpha})], \tag{3d}$$

$$O_5 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\beta})], \tag{3e}$$

$$O_6 = (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\alpha})], \tag{3f}$$

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_{L\alpha}\sigma^{\mu\nu} b_{R\alpha}) F_{\mu\nu}, \tag{3g}$$

$$O_8 = \frac{g}{16\pi^2} m_b (\bar{s}_{L\alpha} \sigma^{\mu\nu} T^a_{\alpha\beta} b_{R\beta}) G^a_{\mu\nu}. \tag{3h}$$

The first six operators are four quark operators and the last two are electromagnetic and strong interaction magnetic moment type operators. The effective Hamiltonian density for weak radiative B-meson decay is obtained by summing the operators $O_j(\mu)$ in eqs. (3) weighted with the appropriate coefficients $C_j(\mu)$, each depending on a subtraction point μ in such a way that the effective Hamiltonian density is independent of μ:

$$\mathcal{H}_{\text{eff}} = -\frac{2G_F}{\sqrt{2}}(s_3 + s_2 e^{i\delta}) \sum_{i=1}^{8} C_i(\mu) O_i(\mu). \tag{4}$$

The subtraction point independence of $\mathcal{H}_{\text{eff}}$ gives rise to renormalization group equations

$$\left(\mu \frac{d}{d\mu} - \gamma^T\right) C = 0, \tag{5}$$

where the anomalous dimension matrix γ is determined by the renormalization of the operators O_1 to O_8.

To leading order in $\alpha_s(m_b)/\pi$ the matrix elements of O_1 to O_6 and O_8 don't contribute to the process $b \to s\gamma$. Therefore it is $C_7(m_b)$ that determines the rate for inclusive weak radiative B meson decay. Although the matrix elements of O_1 to O_6 and O_8 are not relevant these operators cannot necessarily be neglected since they mix under renormalization with O_7. The mixing of O_1 to O_6 with O_8 first occurs at two loops and is of order g^2. There is also order g^2 one-loop mixing of O_8 with O_7.* Hence in the leading logarithmic approximation γ is a 8×8 matrix where all entries are order g^2. To determine $C_7(m_b)$ we need the initial values $C_j(M_W)$. One-loop diagrams determine $C_7(M_W)$ and $C_8(M_W)$ and a tree level computation gives $C_2(M_W) = -2$ and $C_1(M_W) = C_3(M_W) = \ldots = C_6(M_W) = 0$. Now O_7 contributes

⋆ In Ref. [7] we stated incorrectly that the mixing of O_8 with O_7 vanishes. This was corrected by Ref. [10].

directly to $b \to s\gamma$ while O_8 only contributes through its mixing with O_7. Because of this and because the coefficient of O_8 becomes small as μ is scaled from M_W to m_b the mixing of O_8 with O_7 can be neglected. For the four quark operators a similar situation occurs. It is O_2 that most directly affects $C_7(m_b)$ and the other four quark operators can be neglected in comparison. Thus we truncate our operator basis to O_2 and O_7. In this basis the anomalous dimension matrix becomes

$$\gamma = \frac{g^2}{8\pi^2}\begin{pmatrix} -1 & X \\ 0 & 16/3 \end{pmatrix}, \tag{6}$$

where X is determined by a two loop computation. Using

$$C_2(M_W) = -2, \qquad C_7(M_W) = A, \tag{7}$$

and eqs. (5) and (6) we find that

$$C_7(m_b) = \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{16/23}\left\{A + \frac{6X}{19}\left[\left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{19/23} - 1\right]\right\}. \tag{8}$$

Taking $[\alpha_s(m_b)/\alpha_s(M_W)] = 1.7$ the coefficient of X in the brace brackets of eq. (8) is 0.174.

To see that our approximation of truncating the operator basis is valid we add to the truncated basis, for example, the operator O_1. In the basis O_1, O_2, O_7 the anomalous dimension matrix is

$$\gamma = \frac{g^2}{8\pi^2}\begin{pmatrix} -1 & 3 & 0 \\ 3 & -1 & X \\ 0 & 0 & 16/3 \end{pmatrix}. \tag{9}$$

Now with the initial values of eq. (7) and $C_1(M_W) = 0$ we have

$$C_7(m_b) = \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)^{16/23}\left\{A + \frac{3X}{10}\left[\left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{10/23} - 1\right]\right.$$
$$\left. + \frac{3X}{28}\left[\left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{28/23} - 1\right]\right\}. \tag{10}$$

Again taking $(\alpha_s(m_b)/\alpha_s(M_W)) = 1.7$ the coefficient of X in the brace brackets of eq. (10) is $0.078 + 0.097 = 0.175$. This is within 1% of the previous result, which followed from an operator basis where O_1 is neglected.

Using dimensional regularization we found that[7)]

$$X = \frac{4}{3}\left[3Q_c - \frac{4}{9}Q_b\right] = \frac{232}{81}. \quad (11)$$

This implies a branching ratio for $B \to X_s\gamma$ which is about 10^{-4} for any reasonable value of the top quark mass.

In eq. (11) the term involving Q_c arises from two-loop graphs where the photon comes off the charm quark and the term involving Q_b arises from two-loop graphs where the photon comes off the bottom or strange quarks. The term involving Q_c has been computed previously,[8)] using a different method. In Ref. [8] the two loop graphs were computed in four dimensions and the logarithmic divergence was extracted.[9)] Our result for the term proportional to Q_c agrees with that in Ref. [8].

Recently a computation of X, using dimensional reduction, was performed.[10)] Ref. [10] finds, using dimensional reduction, that $X = 124/81$. They attribute the difference between their result and ours with the inability to maintain $\{\gamma_\mu, \gamma_5\} = 0$ in n-dimensions. In our computation we did indeed treat γ_5 as anticommuting. We moved γ_5 through the γ_μ matrices and then reduced the resulting product of γ_μ matrices using n-dimensional gamma matrix identities (that don't involve the ϵ-tensor). However, if this treatment of γ_5 is incorrect then dimensional regularization and dimensional reduction must also give different answers for the $m_b\bar{s}_\alpha\sigma^{\mu\nu}b_\alpha F_{\mu\nu}$ counterterm of the vector operator $(\bar{s}_\alpha\gamma^\mu c_\alpha)(\bar{c}_\beta\gamma_\mu b_\beta)$, which seems unreasonable.[⋆]

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