Erratum: Nonasymptotic critical behavior from field theory at $d=3$. II. The ordered-phase case [Phys. Rev. B 35, 3585 (1987)]

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In Tables I and II we present the corrected Table I of the original paper.[1] They display the values of the Feynman integrals of the $O(1)$ scalar theory contributing to the free energy up to five loops and their symmetry factors. The values differ from those Feynman integrals previously calculated in Ref. 2 due to the necessity of introducing a "soft" mass parameter instead of the usual renormalized (at zero-momentum) mass (see the text of the original paper[1] and Ref. 3 for details). Consequently, many of the estimates of Feynman integrals presented in Table I of Ref. 1 have been extracted from Ref. 2 by accounting for a harmless 3-d renormalization in order to get a soft-mass parameter (characterizing a minimal subtraction scheme similar to that introduced in Ref. 4). Since the five-loop contributions to the free energy involve φ^3 vertices mixed to φ^4 vertices, Feynman integrals which are different or cannot be obtained from those considered in Ref. 2 have been estimated in three dimensions for the occasion.[5] These are the following:

(i) The four-loop integrals with Heap's numbers 13–17 (column h in the following Tables I and II, see Table IV of Ref. 6). They involve only φ^3 vertices. They have been estimated and successfully compared to similar calculations extracted from Ref. 7.

(ii) The five-loop integrals with Heap's numbers 80–102 (see Table V of Ref. 6). They involve φ^3 vertices mixed with a single φ^4 vertex. They have been calculated exclusively for this work.

(iii) The five-loop integrals with Heap's numbers 103–118 (see Table V of Ref. 6). They involve only φ^3 vertices. They have been calculated for this work and successfully compared to similar calculations extracted from Ref. 8.

Tables I and II (corrected Table I of Ref. 1) account for two corrections compared to the original paper:[1]

(i) For $b=5$, $h=15$, $l=0$, $m=1$, in the column "Value at $d=3$," one reads 6.24797746 instead of 4.21825152.

(ii) Equation (A13), which gives the contributions $b=5$, $h=49$, $l=2$, $m=1$ and $b=5$, $h=50$, $l=2$, $m=2$ in Table I, should read:

$$4I_1^{(5,2)}(1)+2I_2^{(5,2)}(1)=-0.1202442510 -1.558293723\ln(r_0'/g_0^2). \qquad (A13)$$

As consequences, in Table II of Ref. 1, the values of F_{500} and F_{510} should read:

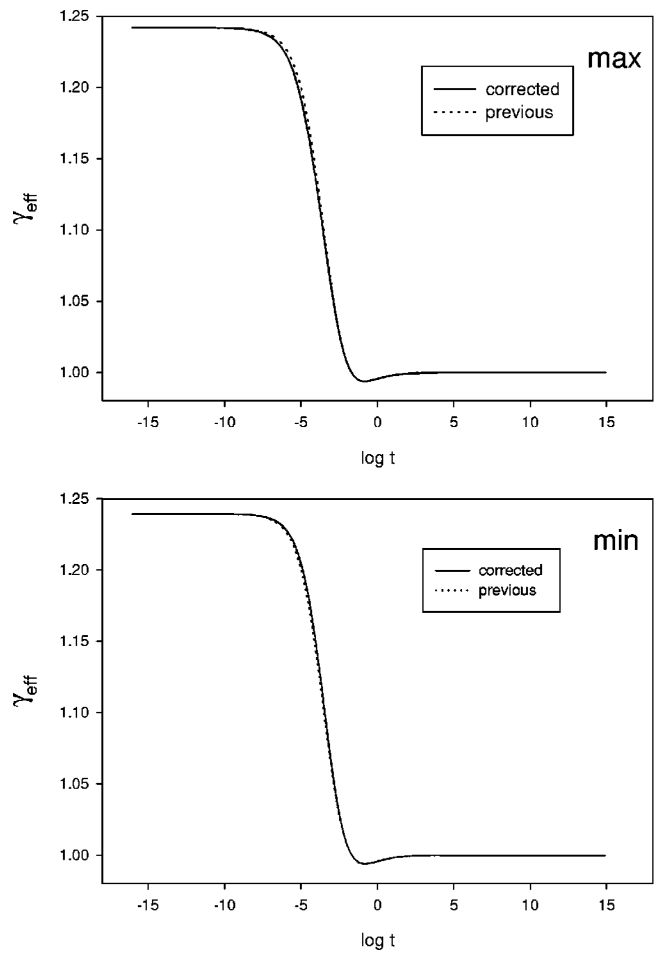

FIG. 1. Illustrations on the effective exponent $\gamma_{eff}(t)$ of the (small) effect of the corrections of the errors mentioned in the text. The two uncertainty bounds "max" and "min" corresponding to the resummation criteria of Ref. 1 are displayed. These curves have been obtained using the results of Ref. 12.

TABLE I. Corrected Table I. See the caption in the original paper Ref. 1.

b	h	l	m	P_m^{-1}	Value at $d=3$	b	h	l	m	P_m^{-1}	Value at $d=3$
1	—	0	1	2	$-4/3$						
2	1	1	1	12	$-2\ln(y)$	5	49	2	1	48	
							50		2	24	See Eq. (A13)
3	1	0	1	48	-22.79417368		51		3	24	-0.007497124
					$+16\ln(y)$						$-0.086950305\ln(y)$
	2	1	1	8	4.107471254		52		4	32	0.36927278680
	3	2	1	16	0.5194312413		53		5	64	0.36927278680
	4		2	24	0.17390061070		54		6	8	0.22602937610
							55		7	8	0.26394187370
4	3	0	1	48	$-19.73920880\ln(y)$		56		8	8	$0.97910169300\times10^{-1}$
	5	1	1	24	-0.2964527240		57		9	32	0.29097562780
					$-(4/3)\ln(y)$		58		10	8	0.26394187370
	6		2	16	2.0657193571		59		11	16	0.22602937610
	7		3	8	1.7234905497		60		12	16	0.22294544960
	8		4	8	1.2405960978		61		13	4	0.16404726530
							62		14	16	0.19621789690
	9	2	1	16	0.43051311360		63		15	8	$0.94895087300\times10^{-1}$
	10		2	4	0.31160313040		64		16	8	0.18956812860
	11		3	4	0.12578653970		65		17	8	$0.93486460600\times10^{-1}$
	12		4	8	$0.79516908900\times10^{-1}$		66		18	4	$0.94895087300\times10^{-1}$
							67		19	16	$0.97910169300\times10^{-1}$
	13	3	1	48	0.18361624610		68		20	8	0.18956812860
	14		2	16	$0.80721242900\times10^{-1}$		69		21	4	$0.66591760000\times10^{-1}$
	15		3	8	$0.37859728700\times10^{-1}$		70		22	8	$0.53216192900\times10^{-1}$
	16		4	12	$0.14620245800\times10^{-1}$		71		23	16	$0.49947540000\times10^{-1}$
	17		5	72	$0.12244670000\times10^{-1}$		72		24	4	0.17156245110
							73		25	4	$0.72746695800\times10^{-1}$
5	15	0	1	144	6.24797746		74		26	2	$0.59161184100\times10^{-1}$
					$+4.602913152\ln(y)$		75		27	4	$0.56505190000\times10^{-1}$
					$+4\ln^2(y)$		76		28	8	$0.89556123600\times10^{-1}$
	16		2	128	22.90931839		77		29	12	$0.32950940000\times10^{-1}$
	17		3	32	16.60229522		78		30	2	$0.34817630000\times10^{-1}$
							79		31	16	$0.34355390400\times10^{-1}$
	31	1	1	48	-0.142393552						
					$-1.038862482\ln(y)$		80	3	1	32	0.16930103539
	32		2	12	-0.122911141		81		2	16	$0.70750283580\times10^{-1}$
					$-0.767152192\ln(y)$		82		3	16	$0.33556467810\times10^{-1}$
	33		3	32	1.373092004		83		4	16	$0.67245000060\times10^{-1}$
	34		4	16	2.449689513		84		5	8	$0.70750283580\times10^{-1}$
	35		5	8	1.072299357		85		6	8	0.11737660585
	36		6	16	0.629580783		86		7	4	$0.30521594800\times10^{-1}$
	37		7	8	0.785125191		87		8	8	$0.33556467810\times10^{-1}$
	38		8	4	0.729050922		88		9	4	$0.50133149830\times10^{-1}$
	39		9	8	0.853563223		89		10	8	$0.29350397320\times10^{-1}$
	40		10	4	0.489725124		90		11	4	$0.30521594800\times10^{-1}$
	41		11	12	0.31815728		91		12	4	$0.23674907200\times10^{-1}$

$$F_{500}=-0.45163891229\times10^{-7},$$

$$F_{510}=0.80687809188\times10^{-7}$$

in agreement with Table 1 of Ref. 9. In addition, as indicated in Ref. 12 of Ref. 10, the five loop coefficients of $X(v)$ and $F(v)$ given in Table III of Ref. 1 should be modified but also that of $\widetilde{F}(v)$, so that it is preferable to redo Table III (see the

TABLE II. Corrected Table I continued.

b	h	l	m	P_m^{-1}	Value at $d=3$	b	h	l	m	P_m^{-1}	Value at $d=3$
5	92	2	13	4	$0.12127180970\times 10^{-1}$	5	105	4	3	96	$0.27143831355\times 10^{-1}$
	93		14	2	$0.11619507150\times 10^{-1}$		106		4	16	$0.16765892110\times 10^{-1}$
	94		15	8	$0.99110117700\times 10^{-2}$		107		5	8	$0.11447149611\times 10^{-1}$
	95		16	8	$0.38967085760\times 10^{-1}$		108		6	16	$0.13776551025\times 10^{-1}$
	96		17	4	$0.17420445300\times 10^{-1}$		109		7	32	$0.10033568785\times 10^{-1}$
	97		18	4	$0.16062044550\times 10^{-1}$		110		8	8	$0.65847074272\times 10^{-2}$
	98		19	4	$0.18506772840\times 10^{-1}$		111		9	4	$0.41535270157\times 10^{-2}$
	99		20	2	$0.68328380000\times 10^{-2}$		112		10	8	$0.47864827139\times 10^{-2}$
	100		21	8	$0.79486181900\times 10^{-2}$		113		11	16	$0.36515474403\times 10^{-2}$
	101		22	4	$0.57948451700\times 10^{-2}$		114		12	16	$0.31588757411\times 10^{-2}$
	102		23	4	$0.54193873300\times 10^{-2}$		115		13	4	$0.16563666713\times 10^{-2}$
							116		14	12	$0.13970600340\times 10^{-2}$
	103	4	1	128	$0.99256755397\times 10^{-1}$		117		15	48	$0.12151455340\times 10^{-2}$
	104		2	16	$0.34440788678\times 10^{-1}$		118		16	16	$0.11715681490\times 10^{-2}$

TABLE III. Modified Table III of Ref. 1.

X	S	$\tilde{F}$	F
1.5	1.0	4.5	-0.25
0.0	0.0	0.0	0.0
0.0	-0.16666666	0.33333333	-7.1299755×10^{-3}
2.9928535×10^{-2}	2.9018571×10^{-2}	-0.17775128	3.8703438×10^{-3}
-2.3069166×10^{-2}	1.6869642×10^{-3}	6.8988036×10^{-2}	-3.2942831×10^{-3}
2.0211486×10^{-2}	4.3368472×10^{-2}	-0.12771112	4.1253272×10^{-3}

present Table III). Moreover, the following equations were not correctly written, they should read:

$$\Gamma^{(0,0)}_{0,-}(r_0,g_0,M_0,\epsilon)=\frac{1}{2}r_0M_0^2+g_0\frac{M_0^4}{24} + \sum_{b=1}^{5}\Gamma_b(r_0,g_0,M_0,\epsilon), \quad (3.1)$$

$$H_{blk}=\left(2-\frac{b}{2}-l\right)F_{blk}+(l+1)F_{b,l+1,k}-\frac{1}{2}(k+1)F_{b,l,k+1}, \quad (3.20)$$

$$Q_{blk}=2\left(2-\frac{b}{2}-l\right)H_{b,l-1,k}+(1+2l)H_{blk} -(k+1)H_{b,l-1,k+1}. \quad (3.22)$$

To get the value of A^+/A^- given in Table VI of Ref. 1 using Eq. (4.9), the values of $\tilde{F}(v^*)$ given in Table V must be divided by v^*.

In Table VII of Ref. 1, the values of Y_3 should read:

$$-2.07825\times 10^{-3},$$

$$3.45588\times 10^{-2}.$$

All these corrections have been accounted for in Ref. 12 where an updated calculation of the complete critical-to-classical crossover is presented. In addition, we have explicitly verified (see Fig. 1) that the errors have had no important consequence on the final results as it could be clearly deduced from a comparison of our estimates of universal amplitude combinations[1] with those of Guida and Zinn-Justin[11] who used the corrected series.

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