14 Two Laser Applications

14.1 DESIGN CONSIDERATIONS INVOLVING AN OPTICAL COMMUNICATION SYSTEM

One potential area of application for lasers is that of communication between satellites. One reason is that in this case the problem of atmospheric absorption and distortion of laser beams is of no concern. In addition, the high directionality available with laser beams can be utilized effectively.

To be specific, we will consider a communication link between a system of three synchronous (24-h orbit) satellites, as shown in Figure 14-1. Each satellite should be able to transmit and receive simultaneously.

Specifically, we agree on the following operating conditions:

1. The operating wavelength is $\lambda = 0.53$ μm. This wavelength, which can be obtained by doubling the output of a Nd^{3+}:YAG laser (see Section 7.3), is chosen because of the high quantum efficiency of photomultiplier tubes at this wavelength (see Figure 11-4).
2. The detection will be performed by a photomultiplier tube operating in the video (that is, no local oscillator) mode, as described in Section 11.3.
3. The modulation signal will be impressed on the optical beam by an electrooptic modulator. The modulation signal will consist of a microwave subcarrier with a center frequency of $\nu_m = 3 \times 10^9$ Hz and sidebands[1]

[1]The information signal $f(t)$, which may consist of, as an example, the video output of a vidicon television-camera tube, is impressed as modulation on the microwave signal. This modulation can take the form of AM, FM, PCM, or other types of modulation. The modulated microwave carrier is then applied to the electrooptic crystal to modulate the optical beam in one of the ways discussed in Chapter 9.

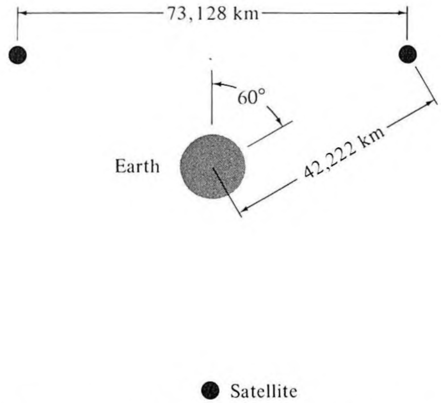

Figure 14-1 The disposition of three earth satellites with synchronous (24-h) orbits.

(caused by the information modulation) between $\nu_{\min} = 2.5 \times 10^9$ Hz and $\nu_{\max} = 3.5 \times 10^9$ Hz. The information bandwidth is thus $\Delta\nu = 10^9$ Hz.

4. The electrooptic crystal modulator will be used in the transverse mode (see Section 9.5) and will have an electrooptic coefficient of $r \simeq 4 \times 10^{-11}$ MKS and a microwave dielectric constant of $\varepsilon = 55\varepsilon_0$. The peak modulation index is $\Gamma_m = \pi/3$.
5. The collimating lens and the receiving lens will have radii of 10 cm.
6. The signal-to-noise power ratio at the output of the amplifier following the receiver photomultiplier tube should be 10^3.

Our main concern is that of calculating the total primary (dc) power that the satellite must supply in order to meet the foregoing performance specifications. We will calculate first the optical power level of the transmitted beam and then the modulation power needed to meet these performance criteria.

Synchronous Satellites

A synchronous satellite has an orbiting period of 24 h, so its position relative to the earth is fixed. To find the distance from the earth to the satellite, we equate the centrifugal force caused by the satellite's rotation to the gravitational attraction force

$$m\,\frac{v^2}{R_{E-S}} = mg\,\frac{(R_{\text{earth}})^2}{(R_{E-S})^2} \tag{14.1-1}$$

where v is the satellite's velocity, m its mass, g the gravitational acceleration at the earth's surface, R_{E-S} the distance from the center of the earth to the satellite, and R_{earth} the earth's radius. The synchronous orbit constraint (that is, a 24-hour period) is

$$\frac{v}{R_{E-S}} = \frac{2\pi}{24 \times 60 \times 60}$$

We use it in (14.1-1) to solve for R_{E-S}, obtaining $R_{E-S} = 42{,}222$ km.

We employ three satellites so as to obtain coverage of the earth's surface, as shown in Figure 14-1. The distance between two satellites is $R =$ 73,128 km.

Calculation of the Transmitted Power

First we will derive an expression relating the received power to the transmitted power as a function of the transmitted beam diameter, the receiving aperture diameter, and the distance R between the transmitter and the receiver.

If the transmitted power P_T is beamed into a solid angle Ω_T and if the receiving aperture subtends a solid angle Ω_R at the transmitter, the power received is

$$P_R = P_T \frac{\Omega_R}{\Omega_T} \tag{14.1-2}$$

The transmitted beam diffracts with a half-apex angle θ_{beam}, as shown in Figure 14-2. This angle is related to the minimum beam radius ω_0 (spot size) by (2.5-18):

$$\theta_{\text{beam}} = \frac{\lambda}{\pi\omega_0} \tag{14.1-3}$$

The corresponding solid angle is $\Omega_T = \pi(\theta_{\text{beam}})^2$. If we choose ω_0 to be equal to the radius a_T of the transmitting lens, we obtain

$$\Omega_T = \frac{\lambda^2}{\pi a_T^2} \tag{14.1-4}$$

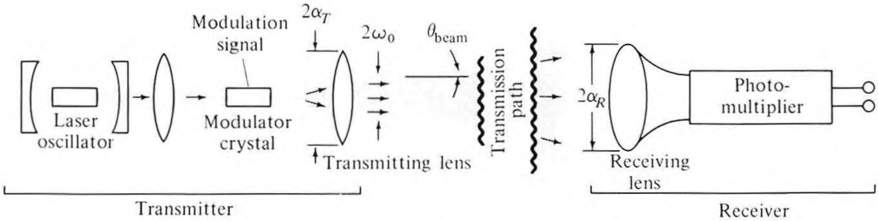

Figure 14-2 An optical communication link consisting of a laser oscillator, an electrooptic modulator, a collimating (transmitting) lens, a transmission medium, a receiving lens, and a receiver using a photomultiplier.

The receiving solid angle is

$$\Omega_R = \frac{\pi a_R^2}{R^2} \tag{14.1-5}$$

where a_R is the radius of the receiving lens and R is the distance between transmitter and receiver. Using (14.1-4) and (14.1-5) in (14.1-2) leads to

$$P_T = P_R \frac{\lambda^2 R^2}{\pi^2 a_T^2 a_R^2} \tag{14.1-6}$$

According to (11.3-5) and (11.3-9), the signal-to-noise power ratio at the output of a photomultiplier operating in the quantum-limited region (that is, the main noise contribution is the shot noise generated by the signal itself) is

$$\frac{S}{N} = \frac{2(P_R e\eta/h\nu)^2 G^2}{2G^2 e\bar{i}_e \Delta\nu} = \frac{P_R \eta}{h\nu\Delta\nu} \tag{14.1-7}$$

where P_R is the optical power. Using $\lambda = 0.53\ \mu\text{m}$, $\eta = 0.2$, $\Delta\nu = 10^9$ Hz, and the required S/N value of 10^3, we obtain

$$P_R \simeq 2 \times 10^{-6} \text{ watt}$$

The required transmitted power is then calculated using (14.1-6), and $R = 7.31 \times 10^7$ meters, yielding

$$P_T \simeq 3 \text{ watts}$$

Calculation of the Modulation Power

In Section 9.6 we derived an expression for the power dissipated by an electrooptic modulator operated in the parallel RLC configuration shown in Figure 9-9. This power is given by (9.6-2)

$$P = \frac{\Gamma_m^2 \lambda^2 d^2 \varepsilon \Delta\nu}{4\pi l r^2 n^6} \tag{14.1-8}$$

where Γ_m is the peak electrooptic retardation, d the length of the side of the (square) crystal section, l the crystal length, r the appropriate electrooptic coefficient, $\Delta\nu$ the modulation bandwidth, and ε the dielectric constant of the crystal at the modulation frequency. To ensure frequency-independent response of the crystal, the crystal length l is limited by transit-time considerations discussed in Section 9.6 to a length

$$l < \frac{c}{4\nu_{\max} n}$$

where n is the index of refraction and $\nu_{\max}$ is the highest modulation frequency. Using $\nu_{\max} = 3.5 \times 10^9$ Hz and $n = 2.2$, we obtain $l < 1$ cm. We will consequently choose a crystal length of $l = 1$ cm. Having fixed l we may be tempted by (14.1-8) to use a crystal with a minimum thickness d. The choice of d is dictated, however, by the fact that we must be able to focus the laser beam into the crystal in such a way that its spread due to diffraction inside the crystal does not exceed the transverse dimension d. The situation is illustrated by Figure 14-3. If the beam is focused so that its waist occurs at the crystal midplane ($z = 0$), its radius at the two crystal faces ($z = -l/2$ and $l/2$) is given by (2.5-11) as

$$\omega\left(\frac{l}{2}\right) = \omega_0\left[1 + \left(\frac{l\lambda}{2\pi n}\right)^2 \omega_0^{-4}\right]^{1/2} \tag{14.1-9}$$

Our problem is one of determining the value of ω_0 for which $\omega(z = l/2)$ is a minimum. The dimension d will then be chosen to be slightly larger than the minimum value of $2\omega(z = l/2)$. Setting the derivative of (14.1-9) (with respect to ω_0) equal to zero yields

$$(\omega_0)_{\min} = \sqrt{\frac{l\lambda}{2\pi n}}$$

and

$$\omega(z = l/2)_{\min} = \sqrt{\frac{l\lambda}{\pi n}}$$

so that choosing d_{optimum} as equal to $2\omega(z = l/2)_{\min}$

$$\left(\frac{d^2_{\text{optimum}}}{l}\right) = \frac{4\lambda}{\pi n} \tag{14.1-10}$$

Substituting the last result for d^2/l in (14.1-8) yields an expression for the minimum modulation power

$$(P_{\min}) = \frac{\Gamma_m^2\lambda^3\varepsilon\Delta\nu}{\pi^2 r^2 n^7} \tag{14.1-11}$$

Using $\Gamma_m = \pi/3$, $n = 2.2$, $r = 4 \times 10^{-11}$ (MKS), $\lambda = 0.53\ \mu$m, $\varepsilon = 55\epsilon_0$,

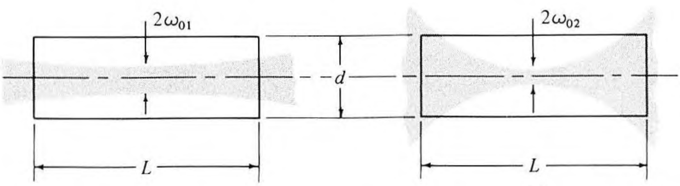

Figure 14-3 The problem of confining a fundamental Gaussian beam within a crystal of length l and height (and width) d. A decrease of the minimum beam radius from ω_{01} in (a) to ω_{02} in (b) causes the beam to expand faster and "escape" from the crystal.

and $\Delta\nu = 10^9$ yields

$$(P)_{\text{optimum}} = 0.0238 \text{ watts}$$

for the microwave modulation power under optimum focusing conditions.

We have thus determined that the laser power output should be approximately 3 watts and the modulation power around 0.0238 watt. If we assume that the efficiency of conversion of primary (dc) power to laser power is about 1 percent, each satellite will be required to supply approximately 300 watts of primary power.

14.2 HOLOGRAPHY[2]

One of the most important applications made practical by the availability of coherent laser radiation is holography, the science of producing images by wavefront reconstruction; see References [1–8]. Holography makes possible true reconstruction of three-dimensional images, magnified or reduced in size, in full color. It also makes possible the storage and retrieval of a large amount of optical information in a small volume.

Figure 14-4 illustrates the experimental setup used in making a simple

[2]Chapters 17 and 18 deal with the more advanced topic of dynamic holography in nonlinear optical media. The treatment of this section is mostly kinetic. It tells us when and how things happen but does not address the magnitudes involved. It is meant as an introduction to the major concepts of holography.

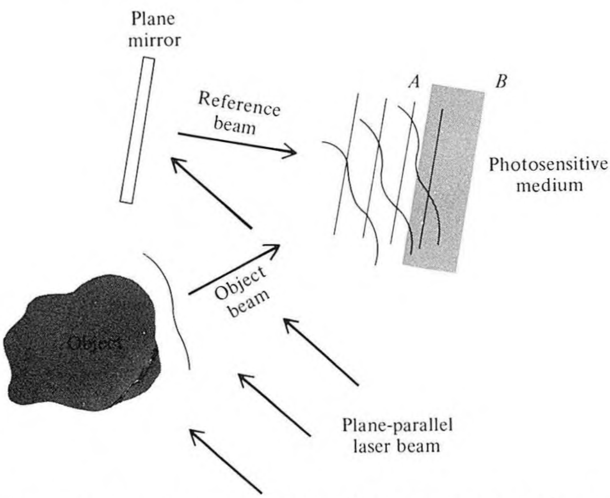

Figure 14-4 A hologram of an object can be made by exposing a photosensitive medium at the same time to coherent light, which is reflected diffusely from the object, and a plane-parallel reference beam, which is part of the same beam that is used to illuminate the object.

hologram. A plane-parallel light beam illuminates the object whose hologram is desired. Part of the same beam is reflected from a mirror (at this point we refer to it as the reference beam) and is made to interfere within the *volume* of the photosensitive medium with the beam reflected diffusely from the object (object beam). The photosensitive medium is then developed and forms the hologram.

The image reconstruction process is illustrated in Figure 14-5. It is performed by illuminating the hologram with the same wavelength laser beam and in the same relative orientation that existed between the reference beam and the photosensitive medium when the hologram was made. An observer facing the far side (B) of the hologram will now see a three-dimensional image occupying the same spatial position as the original object. The image is, ideally, indistinguishable from the direct image of the laser-illuminated object.

The Holographic Process Viewed as Bragg Diffraction

To illustrate the basic process involved in holographic wavefront reconstruction, consider the simple case in which the two beams reaching the photosensitive medium in Figure 14-4 are plane waves. The situation is depicted in Figure 14-6. We choose the z axis as the direction of the bisector of the angle formed between the two propagation directions $\mathbf{k}_1$ and $\mathbf{k}_2$ of the reference and object plane waves inside the photosensitive layer. The x axis is contained in the plane of the paper. The electric fields of the two beams are taken as

$$\begin{aligned} e_{\text{object}}(\mathbf{r}, t) &= E_1 e^{i(\mathbf{k}_1\cdot\mathbf{r}-\omega t)} \\ e_{\text{reference}}(\mathbf{r}, t) &= E_2 e^{i(\mathbf{k}_2\cdot\mathbf{r}-\omega t)} \end{aligned} \tag{14.2-1}$$

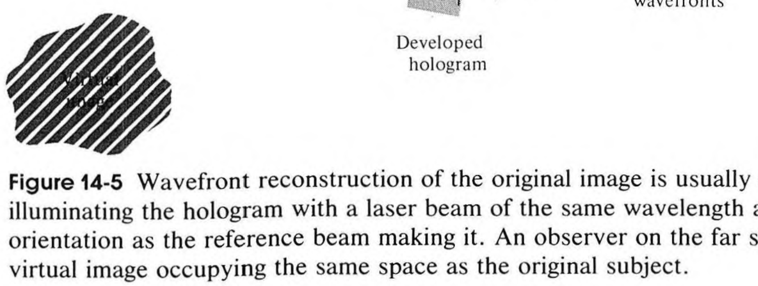

Figure 14-5 Wavefront reconstruction of the original image is usually achieved by illuminating the hologram with a laser beam of the same wavelength and relative orientation as the reference beam making it. An observer on the far side (B) sees a virtual image occupying the same space as the original subject.

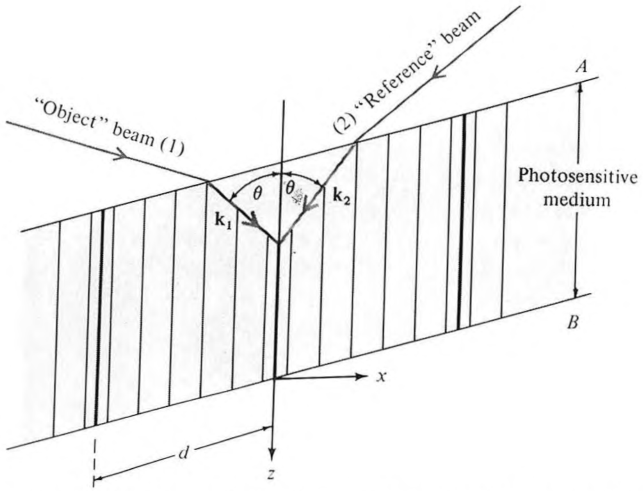

Figure 14-6 A sinusoidal "diffraction grating," produced by the interference of two plane waves inside a photographic emulsion. The density of black lines represents the exposure and hence the silver-atom density. The z direction is chosen as that of the bisector of the angle formed between the directions of propagation *inside* the photographic emulsion. It is not necessarily perpendicular to the surface of the hologram.

From Figure 14-6 and the fact that $|\mathbf{k}_1| = |\mathbf{k}_2| = k$, we have

$$\mathbf{k}_1 = \mathbf{a}_x k \sin\theta + \mathbf{a}_z k \cos\theta$$
$$\mathbf{k}_2 = -\mathbf{a}_x k \sin\theta + \mathbf{a}_z k \cos\theta \tag{14.2-2}$$

where $k = 2\pi/\lambda$, and $\mathbf{a}_x$ and $\mathbf{a}_z$ are unit vectors parallel to x and z, respectively.

The total complex field amplitude is the sum of the complex amplitudes of the two beams, which, using (14.2-1) and (14.2-2), can be written as

$$E(x, z) = E_1 e^{ik(x\sin\theta + z\cos\theta)} + E_2 e^{ik(-x\sin\theta + z\cos\theta)} \tag{14.2-3}$$

If the photosensitive medium were a photographic emulsion, the exposure to the two beams and subsequent development would result in silver atoms developed out at each point in the emulsion in direct proportion to the time average of the square of the optical field. The density of silver in the developed hologram is thus proportional to $E(x, z)E^*(x, z)$, which, using (14.2-3), becomes

$$E(x, z)E^*(x, z) = E_1^2 + E_2^2 + 2E_1E_2 \cos(2kx \sin\theta) \tag{14.2-4}$$

The hologram is thus seen to consist of a sinusoidal modulation of the silver density. The planes x = constant (that is, planes containing the bisector and normal to the plane of Figure 14-6) correspond to equidensity planes. The distance between two adjacent peaks of this spatial modulation pattern is, according to (14.2-4),

$$d = \frac{\pi}{k \sin \theta} = \frac{\lambda/n}{2 \sin \theta} \tag{14.2-5}$$

In the process of wavefront reconstruction, the hologram is illuminated with a coherent laser beam. Since the hologram consists of a three-dimensional sinusoidal diffraction grating, the situation is directly analogous to the diffraction of light from sound waves, which was analyzed in Section 12.1. Applying the results of Bragg diffraction and denoting the wavelength of the light used in reconstruction (that is, in viewing the hologram) as λ_R, a diffracted beam exists *only* when the Bragg condition (12.1-4)

$$2d \sin \theta_B = \frac{\lambda_R}{n_R} \tag{14.2-6}$$

is fulfilled, where θ_B is the angle of incidence and of diffraction as shown in Figure 14-7 n_R is the index of refraction. Substituting for d its value according to (14.2-5), we obtain

$$\sin \theta_B = \left(\frac{n}{n_R}\right) \frac{\lambda_R}{\lambda} \sin \theta \tag{14.2-7}$$

In the special case when $\lambda_R = \lambda$—that is to say, when the hologram is viewed with the same laser wavelength as that used in producing it—we have

$$\theta_B = \theta$$

so that wavefront reconstruction (that is, diffraction) results only when the beam used to view the hologram is incident on the diffracting planes at the same angle as the beam used to make the hologram. The diffracted beam emerges along the same direction ($\mathbf{k}_1$) as the original "object" beam, thus constituting a reconstruction of the latter.

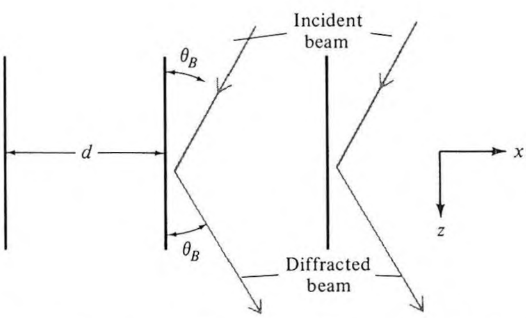

Figure 14-7 Bragg diffraction from a sinusoidal volume grating. The grating periodic distance d is the distance in which the grating structure repeats itself. In the case of a hologram we may consider the vertical lines in the figure as an edge-on view of planes of maximum silver density.

We can view the complex beam reflected from the object toward the photographic emulsion when the hologram is made, as consisting of a "bundle" of plane waves each having a slightly different direction. Each one of these waves interferes with the reference beam, creating, after development, its own diffraction grating, which is displaced slightly in angle from that of the other gratings. During reconstruction the illuminating laser beam is chosen so as to nearly satisfy the Bragg condition (14.2-6) for these gratings. Each grating gives rise to a diffracted beam along the same direction as that of the object plane wave that produced it, so the total field on the far side of the hologram (B) is identical to that of the object field.

Basic Holography Formalism

The point of view introduced above, according to which a hologram may be viewed as a volume diffraction grating, is extremely useful in demonstrating the basic physical principles. A slightly different approach is to take the total field incident on the photosensitive medium as

$$A(\mathbf{r}) = A_1(\mathbf{r}) + A_2(\mathbf{r}) \tag{14.2-8}$$

where $A_1(\mathbf{r})$ may represent the complex amplitude of the diffusely reflected wave from the object while $A_2(\mathbf{r})$ is the complex amplitude of the reference beam. $A_2(\mathbf{r})$ is not necessarily limited to plane waves and may correspond to more complex wavefronts.

The intensity of the total radiation field can be taken, as in (14.2-4), to be proportional to

$$AA^* = A_1A_1^* + A_2A_2^* + A_1A_2^* + A_1^*A_2 \tag{14.2-9}$$

The first term $A_1A_1^*$ is the intensity I_1 of the light arriving from the object. If the object is a diffuse reflector, its unfocused intensity I_1 can be regarded as essentially uniform over the hologram's volume. $A_2A_2^*$ is the intensity I_2 of the reference beam. The change in the amplitude transmittance of the hologram ΔT can be taken as proportional to the exposure density so that

$$\Delta T \propto I_1 + I_2 + A_1A_2^* + A_1^*A_2$$

The reconstruction is performed by illuminating the hologram with the reference beam A_2 in the *same* relative orientation as that used during the exposure. Limiting ourselves to the portion of the transmitted wave modified by the exposure, we have

$$R = A_2\Delta T \propto (I_1 + I_2)A_2 + A_1^*A_2A_2 + I_2A_1 \tag{14.2-10}$$

The first term corresponds to a wavefront proportional to the reference beam. The second term, not being proportional to A_1, may be regarded as undesirable "noise." Since I_2 is a constant, the third term I_2A_1 corresponds to a transmitted wave that is proportional to A_1 and is thus a reconstruction of the object wavefront.

Some additional aspects of holography, which follow straightforwardly from the formalism introduced above, are treated in the problems.

Holographic Storage

The use of holography for the storage of a large number of images and for their retrieval can be best understood using the Bragg diffraction point of view.

We consider, for the sake of simplicity, the problem of recording (storing) holographically and then reconstructing two objects. The storage of a larger number of images is then accomplished by repeating the procedure used in recording the two images.

An exposure of the photosensitive medium is performed using the beam reflected from the first object and the reference beam is illustrated in Figure 14-4. Next, the first object is replaced by the second one, the photosensitive plate is rotated by a small angle $\Delta\theta$, and another exposure is taken. The plate is now developed and forms the hologram.

During exposure each object gave rise to a "diffraction grating" pattern. The two sets of diffraction planes are not parallel to each other, since the plate was rotated between the two exposures. A reconstruction of the first object is obtained when the hologram is illuminated with a laser beam in such a direction as to satisfy the Bragg condition with respect to first diffraction grating. If the same wavelength is used in making the hologram and in the image reconstruction, the image of the first object is reconstructed when the laser beam is incident on the hologram at the same angle as that of the reference beam (during exposure). A rotation $\Delta\theta$ of the hologram will cause the second set of diffraction planes (that due to the second object) to satisfy the Bragg condition with respect to the incident laser beam, thus giving rise to a reconstructed image of the second object.

Problems

14.1 Show that if a hologram is made using a wavelength λ but is reconstructed with a wavelength λ_R, the reconstructed image is magnified by a factor of λ_R/λ with respect to the original object. [*Hint*: Consider the process of forming a real image by placing a lens on the output side (B) of the illuminated hologram and then determining the linear scale of the image in view of Equation (14.2-7).]

14.2 By considering a complex waveform as a superposition of plane waves, show that a complex wave $A^*(r)$ is that obtained from A by making it retrace its path; that is, the wavefronts are identical but their direction of propagation is reversed. A^* is called the conjugate (waveform) of A.

14.3

a. Show that if the hologram is illuminated with a plane wave A_2^* instead of A_2 the reconstructed image is A_1^* instead of A_1.
b. Show that the reconstructed image A_1^* is real—that is, A_1^* actually converges to an image. [*Hint*: Consider what happens to a bundle of rays originally emanating from a point on the object.]
c. Show that the reconstructed image A_1 observed when the hologram is illuminated by A_2 is virtual; that is, rays corresponding to a given image point do not cross unless imaged by a lens.

14.4 Consider the problem of making a hologram in which the reference and object beam are incident on the emulsion from two opposite sides. Draw the equidensity planes for the case where the beams are nearly antiparallel. Show that the viewing (reconstructing) of this beam is performed in the reflection mode: that is, the viewer faces the side of the emulsion that is illuminated by the beam.

14.5 Show that in an infinitely thin hologram both virtual and real images can be reconstructed simultaneously. [*Hint*: Consider the problems of light scattering from a surface grating (as opposed to a volume grating).]

14.6 Calculate the reconstruction angle sensitivity $d\theta_B/d\lambda_R$ for transmission holograms (as described in the text) and in reflection holograms (as described in Problem 14.4). θ_B is the Bragg angle, and λ_R is the wavelength used in reconstruction. Show that $d\theta_B/d\lambda_R$ is much larger in the case of the transmission hologram. Which hologram will yield better results when illuminated by white light?

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