B Mode Locking in Homogeneously Broadened Laser Systems

The analysis of mode locking in inhomogeneous laser systems in Section 6.6 assumed that the role of internal modulation was that of locking together the phases of modes that, in the absence of modulation, oscillate with random phases. In the case of homogeneous broadening, only one mode can normally oscillate. Experiments, however, reveal that mode locking leads to short pulses in a manner quite similar to that described in Section 6.6. One way to reconcile the two points of view and the experiments is to realize that *in the presence of internal modulation,* power is transferred continuously from the high gain mode to those of lower gain (that is, those which would not normally oscillate). This power can be viewed simply as that of the sidebands at $(\omega_0 \pm n\omega)$ of the mode at ω_0 created by a modulation at ω. Armed with this understanding we see that the physical phenomenon is not one of mode locking but one of mode generation. The net result, however, is that of a large number of oscillating modes with equal frequency spacing and fixed phases, as in the inhomogeneous case, leading to ultrashort pulses.

The analytical solution to this case [1, 2] follows an approach used originally to analyze short pulses in traveling wave microwave oscillators [3].

Referring to Figure B-1, we consider an optical resonator with mirror reflectivites R_1 and R_2 that contains, in addition to the gain medium, a periodically modulated loss cell. The method of solution is to follow one pulse through a complete round trip through the resonator and to require that the pulse reproduce itself. The temporal pulse shape at each stage is assumed to be Gaussian.

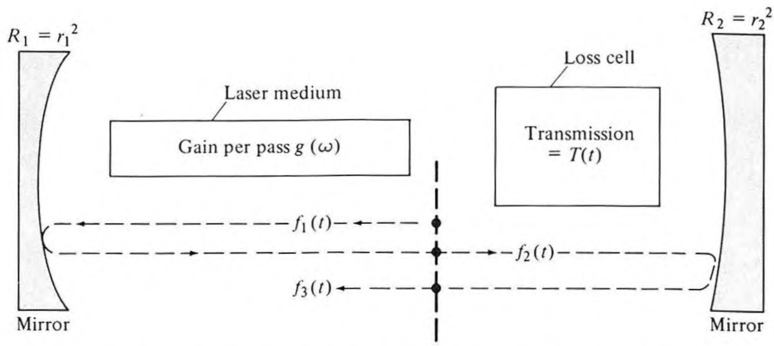

Figure B-1 The experimental arrangement assumed in the theoretical analysis of mode locking in homogeneously broadened lasers.

Before proceeding, we need to characterize the effect of the gain medium and the loss cell on a traveling Gaussian pulse.

TRANSFER FUNCTION OF THE GAIN MEDIUM

Assume that an optical pulse with a field $E_{\text{in}}(t)$ is incident on an amplifying optical medium of length l. Taking the Fourier transform of $E_{\text{in}}(t)$ as $E_{\text{in}}(\omega)$, the amplifier can be characterized by a transfer function $g(\omega)$ where

$$E_{\text{out}}(\omega) = E_{\text{in}}(\omega)g(\omega) \tag{B-1}$$

is the Fourier transform of the output field. Equation (B-1) is a linear relationship and applies only in the limit of negligible saturation.

Using (6.1-1), (5.5-1), and (5.5-2) we have

$$\begin{aligned} g(\omega) &= \exp\left\{-ikl\left[1 + \frac{1}{2n^2}(\chi' - i\chi'')\right]\right\} \\ &= \exp\left\{-ikl - \left(\frac{kl}{2n^2}\right)\frac{\Delta N\lambda^3 T_2}{8\pi^2 n t_{\text{spont}}}\left[\frac{1}{1 + i(\omega - \omega_0)T_2}\right]\right\} \\ &\simeq \exp\left\{-ikl + \frac{\gamma_{\max} l}{2}[1 - i(\omega - \omega_0)T_2 - (\omega - \omega_0)^2 T_2^2]\right\} \end{aligned}$$

where $k = \omega n/c$ and l is the length of the amplifying medium and we define $T_2 = (\pi\,\Delta\upsilon)^{-1}$. The approximation is good for $(\omega - \omega_0)T_2 \ll 1$. We recall that $\Delta N_0 < 0$ for gain. Since the pulse is making two passes through the cell, we take

$$\frac{E_{\text{out}}(\omega)}{E_{\text{in}}(\omega)} = [g(\omega)]^2 = \exp\{-i2kl + \gamma_{\max} l[1 - i(\omega - \omega_0)T_2 - (\omega - \omega_0)^2 T_2^2]\}$$

The imaginary terms in the exponent correspond to a time delay (due to the finite group velocity of the pulse) of

$$\tau_d = \frac{2l}{c} + l\gamma_{\max} T_2$$

We are considering here only the effect on the pulse shape so that, ignoring the imaginary term,[1] we obtain

$$[g(\omega)^2] = e^{\gamma_{\max} l[1-(\omega-\omega_0)^2 T_2^2]} \tag{B-2}$$

TRANSFER FUNCTION OF THE LOSS CELL

Here we need to express the effect of the cell on the pulse in the time domain.

Assume that the single pass amplitude transmission factor $T(t)$ of the loss cell is given by

$$E_{\text{out}}(t) = E_{\text{in}}(t)T(t) = E_{\text{in}}(t)\exp[-2\delta_l^2 \sin^2(\pi\Delta\upsilon_{\text{axial}}\, t)]$$

where $\Delta\upsilon_{\text{axial}}$, the longitudinal mode spacing, is given by

$$\Delta\upsilon_{\text{axial}} = \frac{c}{2l_c}$$

where l_c is the effective optical length of the resonator. The transmission peaks are thus separated by $2l_c/c$ sec so that a mode-locked pulse can pass through the cell on successive trips with minimum loss. Since the pulses pass through the cell centered on the point of maximum transmission, we approximate the expression for $E_{\text{out}}(t)$ by

$$E_{\text{out}}(t) = E_{\text{in}}(t)T(t) = E_{\text{in}}(t)\exp[-2\delta_l^2(\pi\Delta\upsilon_{\text{axial}}t)^2] \tag{B-3}$$

We can view the form of (B-3) as the prescribed transmission function of the cell. The form, however, is suggested by physical considerations. In the case of an electrooptic shutter with a retardation (see Section 9.3) $\Gamma(t) = \Gamma_m \sin \omega_m t$, the transmission factor is $T(t) = \cos^2(\Gamma(t)/2))$. Near the transmission peaks $\Gamma(t) \ll 1$ and $T(t)$ is given by

$$T(t) \simeq \exp[-\tfrac{1}{4}(\Gamma_m^2\omega_m^2 t^2)] = \exp[-2\delta_l^2(\pi\Delta\upsilon_{\text{axial}}t)^2]$$

where $\omega_m = \pi\Delta\upsilon_{\text{axial}}$ and $\Gamma_m = 2\sqrt{2}\delta_l$.

We now return to the main analysis. The starting pulse $f_1(t)$ in Figure B-1 is taken as

$$f_1(t) = Ae^{-\alpha_1 t^2}\, e^{i(\omega_0 t + \beta_1 t^2)} \tag{B-4}$$

corresponding to a "chirped" frequency

$$\omega(t) = \omega_0 + 2\beta_1 t \tag{B-5}$$

[1] The finite propagation delay affects the round-trip pulse propagation time that must be equal to the period of the loss modulation.

Its Fourier transform is

$$F_1(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(t)e^{-i\omega t}\,dt$$
$$= \frac{A}{2}\sqrt{\frac{1}{\pi(\alpha_1 - i\beta_1)}}\exp\left(\frac{-(\omega-\omega_0)^2}{4(\alpha_1 - i\beta_1)}\right) \tag{B-6}$$

A double pass through the amplifier and one mirror reflection (r_1) are accounted for by multiplying $F_1(\omega)$ by the transfer factor $[g(\omega)]^2 r_1$

$$F_2(\omega) = F_1(\omega)[g(\omega)]^2 r_1$$
$$= \frac{r_1 A}{2} e^{g_0}\sqrt{\frac{1}{\pi(\alpha_1 - i\beta_1)}}\exp\left\{[-(\omega-\omega_0)^2]\left(\frac{1}{4(\alpha_1 - i\beta_1)} + g_0 T_2^2\right)\right\} \tag{B-7}$$

where $g_0 \equiv \gamma_{\max} l$ and $[g(\omega)]^2$ is given by (B-2). Transforming back to the time domain

$$f_2(t) = \int_{-\infty}^{\infty} F_2(\omega)e^{i\omega t}\,d\omega$$
$$= \frac{r_1 A e^{g_0}}{2\pi}\sqrt{\frac{\pi}{\alpha_1 - i\beta_1}}\, e^{-\omega_0^2 Q}\sqrt{\frac{\pi}{Q}}\exp[-(2i\omega_0 Q - t)^2/4Q] \tag{B-8}$$

where

$$Q \equiv \frac{1}{4(\alpha_1 - i\beta_1)} + g_0 T_2^2 \tag{B-9}$$

A reflection from mirror 2 and a passage through the loss cell lead according to (B-3) to

$$f_3(t) = r_2 f_2(t) e^{-2\delta_1^2\pi^2(\Delta\upsilon_{\text{axial}})^2 t^2}$$
$$= \frac{r_1 r_2 A e^{g_0}}{2}\sqrt{\frac{1}{(\alpha_1 - i\beta_1)Q}}\, e^{i\omega_0 t}\, e^{-[2\delta_1^2(\pi\Delta\upsilon_{\text{axial}})^2 + (1/4Q)]t^2} \tag{B-10}$$

For self-consistency we require that $f_3(t)$ be a replica of $f_1(t)$. We thus equate the exponent of (B-10) to that of (B-4)

$$\alpha_1 = 2\delta_l^2(\pi\,\Delta\upsilon_{\text{axial}})^2 + \operatorname{Re}\left(\frac{1}{4Q}\right)$$
$$\beta_1 = -\operatorname{Im}\left(\frac{1}{4Q}\right) \tag{B-11}$$

Using (B-9), the second equation of (B-11) gives

$$\beta_1 = \frac{\beta_1}{(1 + 4g_0 T_2^2\alpha_1)^2 + (4g_0 T_2^2\beta_1)^2}$$

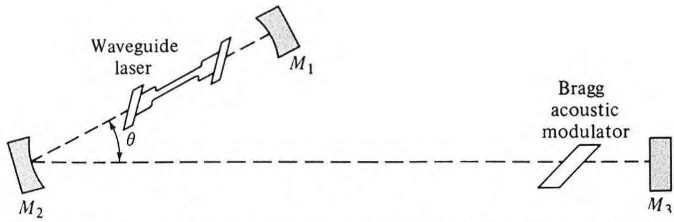

Figure B-2 A schematic drawing of the mode-locking experiment in a high pressure CO_2 laser. (After Reference [4].)

so that a self-consistent solution requires that

$$\beta_1 = 0$$

that is, no chirp. With $\beta_1 = 0$ the first of (B-11), becomes

$$2\delta_l^2(\pi\Delta v_{\text{axial}})^2 + \frac{\alpha_1}{(1 + 4g_0 T_2^2 \alpha_1)} = \alpha_1 \tag{B-12}$$

that, assuming

$$4g_0 T_2^2 \alpha_1 \ll 1 \tag{B-13}$$

results in

$$\alpha_1 = \left(\frac{\delta_l^2}{2g_0}\right)^{1/2} \frac{\pi\Delta v_{\text{axial}}}{T_2}$$

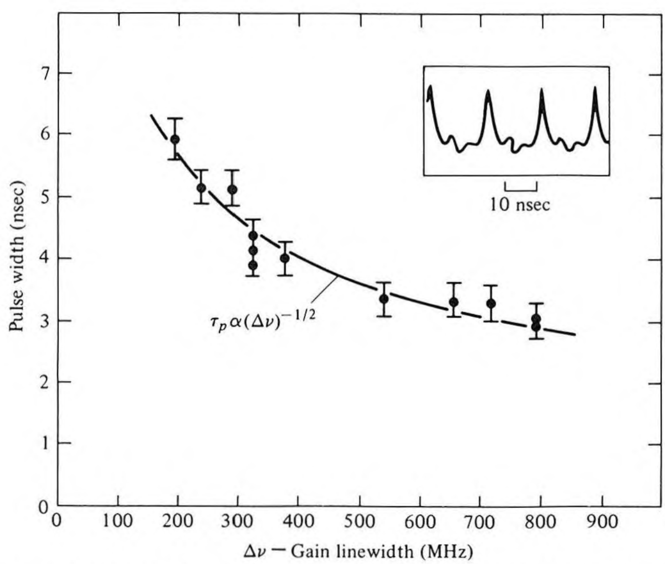

Figure B-3 The dependence of the pulse width on the gain linewidth, Δv, that is controlled by varying the pressure ($\Delta v = 8 \times 10^8$ at 150 torr). (After Reference [4].)

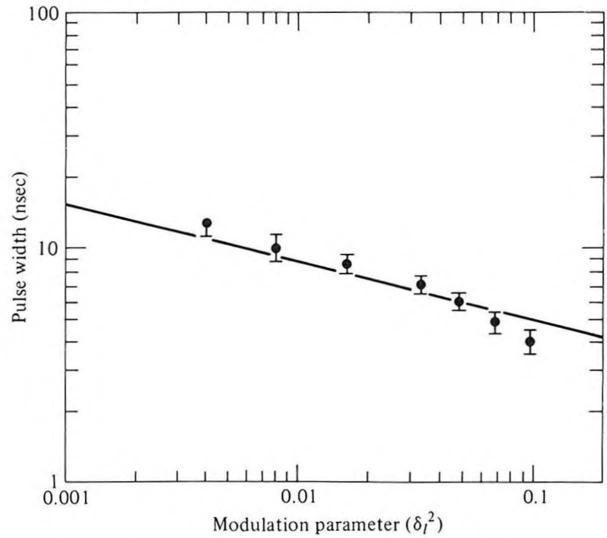

Figure B-4 The mode-locked pulse width as a function of the modulation parameter, δ_l^2. (After Reference [4].)

The pulse width at the half intensity points is from (B-4)

$$\tau_p = (2 \ln 2)^{1/2} \alpha_1^{-1/2}$$

so that the self-consistent pulse has a width

$$\tau_p = \frac{(2 \ln 2)^{1/2}}{\pi} \left(\frac{2g_0}{\delta_l^2} \right)^{1/4} \left(\frac{1}{\Delta\nu_{\text{axial}}\, \Delta\nu} \right)^{1/2} \tag{B-14}$$

where $\Delta\nu \equiv (\pi T_2)^{-1}$. The condition (B-13) can now be interpreted as requiring that $\tau_p \gg 2\sqrt{g_0} T_2$, which is true in most cases.

An experimental setup demonstrating mode locking in a pressure broadened CO_2 laser is sketched in Figure B-2. The inverse square root dependence of τ_p on $\Delta\nu$ is displayed by the data of Figure B-3, while the dependence on the modulation parameter δ_l is shown in Figure B-4.

MODE LOCKING BY PHASE MODULATION

Mode locking can be induced by internal phase, rather than loss, modulation. This is usually done by using an electrooptic crystal inside the resonator oriented in the basic manner of Figure 9-7 such that the passing wave undergoes a phase delay proportional to the instantaneous electric field across the crystal. The frequency of the modulating signal is equal, as in the loss modulation case, to the inverse of the round-trip group delay time, that is, to the longitudinal intermode frequency separation.

We employ an analysis similar to that of the homogeneous case except that the transfer function through the modulation cell is taken, instead of (B-2), as

$$E_{\text{out}}(t) = E_i(t) \exp(-i2\delta_\phi \cos 2\pi\Delta \upsilon_{\text{axial}}\, t) \qquad \text{(B-15)}$$

For pulses passing near the extrema of the phase excursion, we can approximate the last equation as

$$E_{\text{out}}(t) = E_i(t) \exp(\mp i2\delta_\phi \pm i\delta_\phi 4\pi^2\, \Delta \upsilon_{\text{axial}}^2\, t^2) \qquad \text{(B-16)}$$

An analysis identical to that leading to (B-14) yields

$$\tau_p = \frac{(2 \ln 2)^{1/2}}{\pi} \left(\frac{2g_0}{\delta_\phi}\right)^{1/4} \left(\frac{1}{\Delta \upsilon_{\text{axial}}\, \Delta \upsilon}\right)^{1/2} \qquad \text{(B-17)}$$

In this case self-consistency leads to a chirped pulse with

$$\beta = \pm\alpha = \pm\pi^2 \Delta \upsilon_{\text{axial}}\, \Delta \upsilon \sqrt{\frac{\delta_\phi}{2g_0}} \qquad \text{(B-18)}$$

The upper and lower signs in (B-16) and (B-18) correspond to two possible pulse solutions, one passing through the cell near the maximum of the phase excursion and the other near its minimum.

We note that (B-17) is similar to the loss modulation result (B-14) except that δ_ϕ appears instead of δ_l^2. The difference can be traced into a difference between (B-3) and (B-15). The choice of notation in both cases is such that δ *corresponds* to the *retardation* induced by the electrooptic crystal.

References

1. Siegman, A. E., and D. J. Kuizenga, "Simple analytic expressions for AM and FM mode locked pulses in homogeneous lasers," *Appl. Phys. Lett.* 14:181, 1969.
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4. Smith, P. W., T. J. Bridges, and E. G. Burkhardt, "Mode locked high pressure CO_2 laser," *Appl. Phys. Lett.* 21:470, 1972.