

Proposal for teleportation of the wave function of a massive particle

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We propose a scheme for teleporting an atomic center-of-mass wave function between distant locations. The scheme uses interactions in cavity quantum electrodynamics to facilitate a coupling between the motion of an atom trapped inside a cavity and external propagating light fields. This enables the distribution of quantum entanglement and the realization of the required motional Bell-state analysis.

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In a landmark work of 1993, Bennett *et al.* [1] discovered a procedure for teleporting an unknown quantum state from one location to another. The essential ingredient in their protocol is quantum entanglement of a bipartite system shared by the sender, Alice, and receiver, Bob. This shared entanglement, in unison with suitable measurements performed by Alice and communicated via classical channels to Bob, ‘mediates’ the state transfer.

Since the work of Bennett *et al.*, a variety of possible experimental schemes for the teleportation of quantum states of *two-state* systems have been proposed, in large part by the quantum optics community (see, e.g., [2–5]). In an exciting recent development, the first experimental investigations of teleportation of such states have been performed [6,7] with, in particular, the polarization state of a photon providing the two-state system of interest.

Complementing this work on two-state systems has been research into the teleportation of states of infinite-dimensional systems [8,9], culminating last year in the experimental demonstration of quantum teleportation of *optical coherent states* [10]. This experiment was based on the specific proposal of [9], utilizing squeezed-state entanglement and balanced homodyne measurements of the light fields. Given that the experiment employed only standard optical elements and measurement techniques, it offers significant promise of further intriguing possibilities for quantum information processing with continuous quantum variables, including quantum dense coding [11], and universal quantum computation [12].

Another burgeoning field of research in quantum information science is the implementation of quantum logic with trapped atoms or ions. Inspired by the proposal of Cirac and Zoller [13] for a quantum computer based on the motional and internal degrees of freedom of a collection of trapped ions, impressive experimental progress has been made towards controlling quantum properties in such systems [14,15]. Particular advantages of trapped atom systems include long coherence times and

the exquisite control of transformations between motional and internal states.

In view of this tremendous potential of both light- and motion-based schemes for quantum information processing, it is sensible to investigate possibilities for combining the two approaches and their distinct advantages. One particular application would be to quantum networks for distributed quantum computing and communication, where, e.g., the ‘distribution’ is accomplished with light fields [16], while local processing is performed on motional states of a collection of trapped atoms. Indeed, with the protocols of [16] in mind, we have recently proposed and analyzed a cavity-QED-based system that enables the transfer of quantum states between the motion of a trapped atom and propagating light fields [17,18], which should lead to new capabilities for the synthesis and control of quantum states for both motion and light.

A particular example from [18] is the possibility of creating an EPR (Einstein-Podolsky-Rosen) state in position-momentum for distantly separated atoms. The creation of such an entangled state between remote particles suggests an avenue for achieving teleportation of an unknown wave function of a trapped particle between the EPR sites. In this letter, we propose and analyze one such protocol that enables the teleportation of an unknown one-dimensional *atomic center-of-mass wave function* and that should be attainable within the context of emerging experimental capabilities for trapping atoms in cavity QED [19,20].

Our proposed teleportation scheme is shown schematically in Fig. 1. Each of Alice (\mathcal{A}), Bob (\mathcal{B}), and Victor (\mathcal{V}) possess an atom trapped inside an optical cavity. The aim is to teleport the (x -dimension) motional state of Victor’s atom to Bob’s atom. This is achieved via the three usual stages for continuous quantum variables [9,10] – (i) preparation of quantum entanglement between Alice and Bob’s atoms, (ii) Bell-state (homodyne) measurement by Alice, and (iii) phase-space displacement [$D(\alpha^*)$] by Bob (given the classical result α of Alice’s measurement). Finally, the state of Bob’s atom may be examined by Victor for verification of the quality of the teleportation (or indeed, physically delivered to Victor). Stages (i) and (ii), as illustrated in Fig. 1, employ the cavity-mediated motion-light state transfer scheme of [17], to which we now turn our attention.

Briefly, a single two-level atom (or ion) is tightly confined in a harmonic trap located inside a high-finesse optical cavity. The atomic transition of frequency ω_a is coupled to a single mode of the cavity field of frequency ω_c

and also to an external (classical) laser field of frequency ω_L and strength \mathcal{E}_L . The physical setup and excitation scheme are depicted in Fig. 2(a). The cavity is aligned along the x -axis, while the field \mathcal{E}_L is incident from a direction in the y - z plane. Both the cavity field and \mathcal{E}_L are far from resonance with the atomic transition, but their difference frequency is chosen so that they drive Raman transitions between neighboring motional number states (i.e., $\omega_c - \omega_L = \nu_x$, with ν_x the x -axis trap frequency).

A number of assumptions are made in order to achieve the desired motion-light coupling: (i) Atomic spontaneous emission is neglected and the internal atomic dynamics adiabatically eliminated. (ii) The size of the harmonic trap, located at a *node* of the cavity field, is taken to be small compared to the optical wavelength (Lamb-Dicke regime), enabling the approximations $\sin(k\hat{x}) \simeq \eta_x(\hat{b}_x + \hat{b}_x^\dagger)$ and $\mathcal{E}_L(\hat{y}, \hat{z}, t) \simeq \mathcal{E}_L(t)e^{-i\phi_L}$, where $\eta_x (\ll 1)$ is the Lamb-Dicke parameter and $\hat{x} = (\hbar/2m\nu_x)^{1/2}(\hat{b}_x + \hat{b}_x^\dagger)$. (iii) The trap frequency ν_x and cavity field decay rate κ are assumed to satisfy $\nu_x \gg \kappa \gg |(g_0\eta_x/\Delta)\mathcal{E}_L(t)|$, where g_0 is the single-photon atom-cavity mode coupling strength, and $\Delta = \omega_a - \omega_L$. The first inequality allows a rotating-wave approximation to be made with respect to the trap oscillation frequency, while the second inequality enables an adiabatic elimination of the cavity field mode.

Under these conditions, the motional mode dynamics in the x direction is well described by the simple quantum Langevin equation [17]

$$\dot{\tilde{b}}_x \simeq -\Gamma(t)\tilde{b}_x + \sqrt{2\Gamma(t)}\tilde{a}_{\text{in}}(t), \quad (1)$$

where $\tilde{b}_x = e^{i\nu_x t}\hat{b}_x$ and $\Gamma(t) = [g_0\eta_x|\mathcal{E}_L(t)|/\Delta]^2/\kappa$. The operator $\tilde{a}_{\text{in}}(t)$ obeys the commutation relation $[\tilde{a}_{\text{in}}(t), \tilde{a}_{\text{in}}^\dagger(t')] = \delta(t-t')$ and describes the quantum noise *input to the cavity field* (in a frame rotating at the cavity frequency). From the linear nature of (1), it follows that the statistics of a (continuous) light field incident upon the cavity can be ‘written onto’ the state of the atomic motion. This also means that entanglement between separate light fields can be transferred to entanglement between separate motional states, as we discuss below. From a consideration of the input-output theory of optical cavities [21], it also follows that measurements on the cavity output field amount to measurements on the motion of the atom. In particular, one can show that

$$\tilde{a}_{\text{out}}(t) \simeq -\tilde{a}_{\text{in}}(t) + \sqrt{2\Gamma(t)}\tilde{b}_x(t). \quad (2)$$

So, for a vacuum input field, homodyne measurements on the cavity output field realize position or momentum measurements (or some mixture, depending on the local oscillator phase) on the trapped atom. This enables the necessary Bell-state analysis to be performed.

To begin the teleportation procedure, Victor’s atom is prepared in a particular motional state $|\phi\rangle_{\mathcal{V}x}$ in the x dimension (e.g., by the techniques of [15]). With the motion-light coupling switched off in Victor’s cavity (i.e.,

$\mathcal{E}_{\mathcal{L}\mathcal{V}} = 0$), this state is assumed to remain unchanged until required by Alice for the Bell-state analysis.

Next, a position-momentum EPR state of Alice and Bob’s atoms is prepared by using the motion-light coupling described in (1), with input light fields from a non-degenerate optical parametric amplifier (NOPA). This preparation, described in detail in [18], is depicted in Fig. 2(b). The two quantum-correlated output light fields from a NOPA (operating below threshold) are separated and made to impinge on Alice and Bob’s cavities, respectively. Assuming $\Gamma_{\mathcal{A}} = \Gamma_{\mathcal{B}} = \Gamma$, after a time $t \gg \Gamma^{-1}$, the following *pure* entangled motional state is prepared,

$$\begin{aligned} |\psi\rangle_{\mathcal{A}\mathcal{B}} &= S_{\mathcal{A}\mathcal{B}}(r)|0\rangle_{\mathcal{A}x}|0\rangle_{\mathcal{B}x} \\ &= [\cosh(r)]^{-1} \sum_{m=0}^{\infty} [-\tanh(r)]^m |m\rangle_{\mathcal{A}x}|m\rangle_{\mathcal{B}x}, \quad (3) \end{aligned}$$

where $|m\rangle_{\mathcal{A},\mathcal{B}x}$ are Fock states of the motional modes and $S_{\mathcal{A}\mathcal{B}}(r) = \exp[r(\tilde{b}_{\mathcal{A}x}\tilde{b}_{\mathcal{B}x} - \tilde{b}_{\mathcal{A}x}^\dagger\tilde{b}_{\mathcal{B}x}^\dagger)]$, with r the ‘entanglement’ parameter. Once this state has been prepared, the atom-cavity couplings are turned off ($\Gamma_{\mathcal{A}}, \Gamma_{\mathcal{B}} \rightarrow 0$), as is the NOPA pump field. Again, we assume that the entangled state (3) remains unchanged until the next step in the procedure.

At this stage in the protocol, the total system state is

$$|\Psi_{\text{I}}\rangle = |\phi\rangle_{\mathcal{V}x} |\psi\rangle_{\mathcal{A}\mathcal{B}}. \quad (4)$$

The Bell-state analysis performed by Alice is depicted in Fig. 3. At a predetermined time, Victor switches on his atom-cavity coupling $\Gamma_{\mathcal{V}}$ via $\mathcal{E}_{\mathcal{L}\mathcal{V}}(t)$, thus converting the state $|\phi\rangle_{\mathcal{V}x}$ to that of a freely propagating field delivered to input beam-splitter BS of Alice’s sending station. With due accounting for propagation delay, Alice has likewise switched on the coupling $\Gamma_{\mathcal{A}}$ from her cavity, where, for simplicity, $\Gamma_{\mathcal{V}} = \Gamma_{\mathcal{A}} = \Gamma$. Note that Victor and Alice’s cavities both have vacuum inputs at this stage. The two cavity output fields are combined by Alice at the 50/50 beamsplitter BS, the two outputs of which are incident on homodyne detectors D_{\pm} . Through the input-output relation (2), and through the mixing of the cavity output fields at the beamsplitter, these detectors effect homodyne measurements on the modes $\tilde{c}_{\pm} = 2^{-1/2}(\tilde{b}_{\mathcal{V}x} \pm \tilde{b}_{\mathcal{A}x})$. The effect of these measurements is to project the system state onto quadrature eigenstates of the modes \tilde{c}_{\pm} , given by $|\chi_{\pm}\rangle_{\pm} = Q_{\pm}^{\dagger}(\chi_{\pm})|0\rangle_{\pm}$, where $Q_{\pm}^{\dagger}(\chi_{\pm}) = (2\pi)^{-1/4} \exp[-(1/2)(\tilde{c}_{\pm}^{\dagger}e^{i\theta_{\pm}} - \chi_{\pm})^2 + \chi_{\pm}^2/4]$, with θ_{\pm} the local oscillator (LO) phases [22–24]. In [23,24], this projection is proved with the assumption that the local oscillator photon flux matches the temporal shape of the signal flux [which in our case is set by $\Gamma(t)$], while the variable χ is shown to be equivalent to the integrated homodyne photocurrent.

For the two homodyne measurements we choose LO phases $\theta_{+} = 0$ and $\theta_{-} = \pi/2$. With these choices one can show that, in terms of the original mode operators,

$$Q_{+}(\chi_{+})Q_{-}(\chi_{-}) = (2\pi)^{-1/2} \exp(-|\alpha|^2/2)$$

$$\cdot \exp\left(-\tilde{b}_{\nu_x}\tilde{b}_{\mathcal{A}x} + \alpha\tilde{b}_{\nu_x} + \alpha^*\tilde{b}_{\mathcal{A}x}\right), \quad (5)$$

where $\alpha = (\chi_+ + i\chi_-)/\sqrt{2}$. The motional state of Bob's atom following the homodyne measurements, with results χ_{\pm} , can thus be written

$$\begin{aligned} |\varphi\rangle_{\mathcal{B}x} &\propto \nu_x \langle 0|_{\mathcal{A}x} \langle 0|Q_+(\chi_+)Q_-(\chi_-)|\Psi_I\rangle \\ &\propto \nu_x \langle \alpha^*|_{\mathcal{A}x} \langle \alpha| \exp\left(-\tilde{b}_{\nu_x}\tilde{b}_{\mathcal{A}x}\right) |\Psi_I\rangle. \end{aligned} \quad (6)$$

Using properties of the squeezing operator $S_{\mathcal{A}B}(r)$ [21], one can further reduce this to

$$|\varphi\rangle_{\mathcal{B}x} \propto \nu_x \langle \alpha^*| \exp\left[\Lambda\left(\tilde{b}_{\nu_x} - \alpha^*\right)\tilde{b}_{\mathcal{B}x}^\dagger\right] |\phi\rangle_{\nu_x}|0\rangle_{\mathcal{B}x}, \quad (7)$$

where $\Lambda = \tanh(r)$. Expanding $|\phi\rangle_{\nu_x}$ in terms of the coherent states, i.e., $|\phi\rangle_{\nu_x} = \pi^{-1} \int d^2\beta \nu_x \langle \beta|\phi\rangle_{\nu_x} |\beta\rangle_{\nu_x}$, the right-hand-side of (7) becomes

$$\begin{aligned} &\frac{1}{\pi} \int d^2\beta \nu_x \langle \beta|\phi\rangle_{\nu_x} \frac{\nu_x \langle \alpha^*|\beta\rangle_{\nu_x}}{\nu_x \langle \Lambda\alpha^*|\Lambda\beta\rangle_{\nu_x}} \\ &\cdot D_{\mathcal{B}}(-\Lambda\alpha^*)D_{\mathcal{B}}(\Lambda\beta)|0\rangle_{\mathcal{B}x}, \end{aligned} \quad (8)$$

where $D_{\mathcal{B}}(\beta) = \exp(\beta\tilde{b}_{\mathcal{B}x}^\dagger - \beta^*\tilde{b}_{\mathcal{B}x})$ is the coherent displacement operator for Bob's atom. In the limit of strong squeezing and entanglement ($\Lambda \rightarrow 1$), (8) approaches

$$D_{\mathcal{B}}(-\alpha^*) \frac{1}{\pi} \int d^2\beta \nu_x \langle \beta|\phi\rangle_{\nu_x} |\beta\rangle_{\mathcal{B}x}. \quad (9)$$

That is, $|\varphi\rangle_{\mathcal{B}x}$ approaches a state which, apart from a coherent displacement by $-\alpha^*$, is identical to the initial motional state (in the x dimension) of Victor's atom.

Given the measurement results χ_{\pm} , transmitted to Bob via a classical channel, the final step in the teleportation procedure is for Bob to apply a coherent displacement α^* (assuming $\Lambda \simeq 1$) to the motional state of his atom, i.e., $D_{\mathcal{B}}(\alpha^*)|\varphi\rangle_{\mathcal{B}x} \rightarrow |\phi\rangle_{\mathcal{B}x}$. In practice, this might be achieved by applying an electric field (in the case of a trapped ion) along the x -axis which oscillates at the trap frequency ν_x , or, alternatively, by applying off-resonant laser fields which drive stimulated Raman transitions between neighboring trap levels [15]. After this, control of Bob's atom can be passed to Victor, who is free to confirm the overall quality of the teleportation protocol, e.g., along the lines analyzed in [25].

Issues of practicality associated with the motion-light state transfer procedure central to our teleportation scheme have been discussed elsewhere [17,18]. In brief, desired conditions are of (i) strong coupling optical cavity QED, such that $g_0^2/(\kappa\gamma) \gg 1$, where γ is the atomic spontaneous decay rate, and (ii) strong confinement of the atoms with minimal motional-state decoherence. Both of these conditions have been achieved separately [14,15,19,20], and we expect that future experiments trapping single atoms inside optical cavities will be able to meet these criteria simultaneously. Note that timescales for motional-state decoherence of trapped

ions can be of the order of milliseconds [15]; the typical timescale involved in our teleportation scheme, Γ^{-1} , would likely be of the order of microseconds [17,18]. Finally, calculations in [9] suggest reasonable (nonclassical) teleportation fidelities to be possible with values of the squeezing parameter $r > 1$.

To conclude, we note that the scheme given here is just one of a number of possibilities that we have analyzed. One could, e.g., eliminate Victor's atom and cavity and, as the state to be teleported, choose the motional state of Alice's atom along an axis *orthogonal* to the x -axis. After preparing the entangled (x -dimension) motional state $|\psi\rangle_{\mathcal{A}B}$ of Alice and Bob's atoms, the orthogonal motional modes of Alice's atom could be linearly mixed within the trap itself, in the fashion of a beamsplitter, using suitable interactions with auxiliary laser fields [26,27], after which coupling to the cavity field and homodyne measurement of the output light field would again provide the Bell-state analysis. In addition, as we will discuss elsewhere [27], it is possible to eliminate the NOPA from the scheme and use only trapped atoms interacting with cavity and laser fields both to produce and distribute the quantum entanglement required for the teleportation protocol.

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FIG. 1. Schematic of proposed teleportation scheme for atomic wavepackets. Preparation of motional-state entanglement between Alice and Bob and Bell-state analysis by Alice are facilitated by cavity-mediated motion-light couplings.

FIG. 2. (a) Proposed setup and excitation scheme for coupling between the motion of a trapped atom and a quantized optical cavity mode, and thence to a freely propagating external field. The cavity is assumed to be *one-sided*, i.e., one mirror is taken to be perfectly reflecting. (b) Preparation of a position-momentum EPR state of Alice and Bob's atoms. The two output fields from a nondegenerate parametric amplifier (NOPA) impinge on Alice and Bob's cavities, respectively. Faraday isolators (F) facilitate a unidirectional coupling between the entangled light source and the atom-cavity systems.

FIG. 3. Schematic of Alice's Bell-state analysis. The output field representing Victor's unknown state is combined by Alice at a 50/50 beamsplitter (BS) with the output field from her cavity. The resulting output fields from the BS are incident on homodyne detectors D_{\pm} . The cavity output fields follow the motional modes, which decay on a timescale Γ^{-1} . The local oscillator fields (LO_{\pm}) are pulsed, with temporal profiles chosen to match that of the cavity output fields.

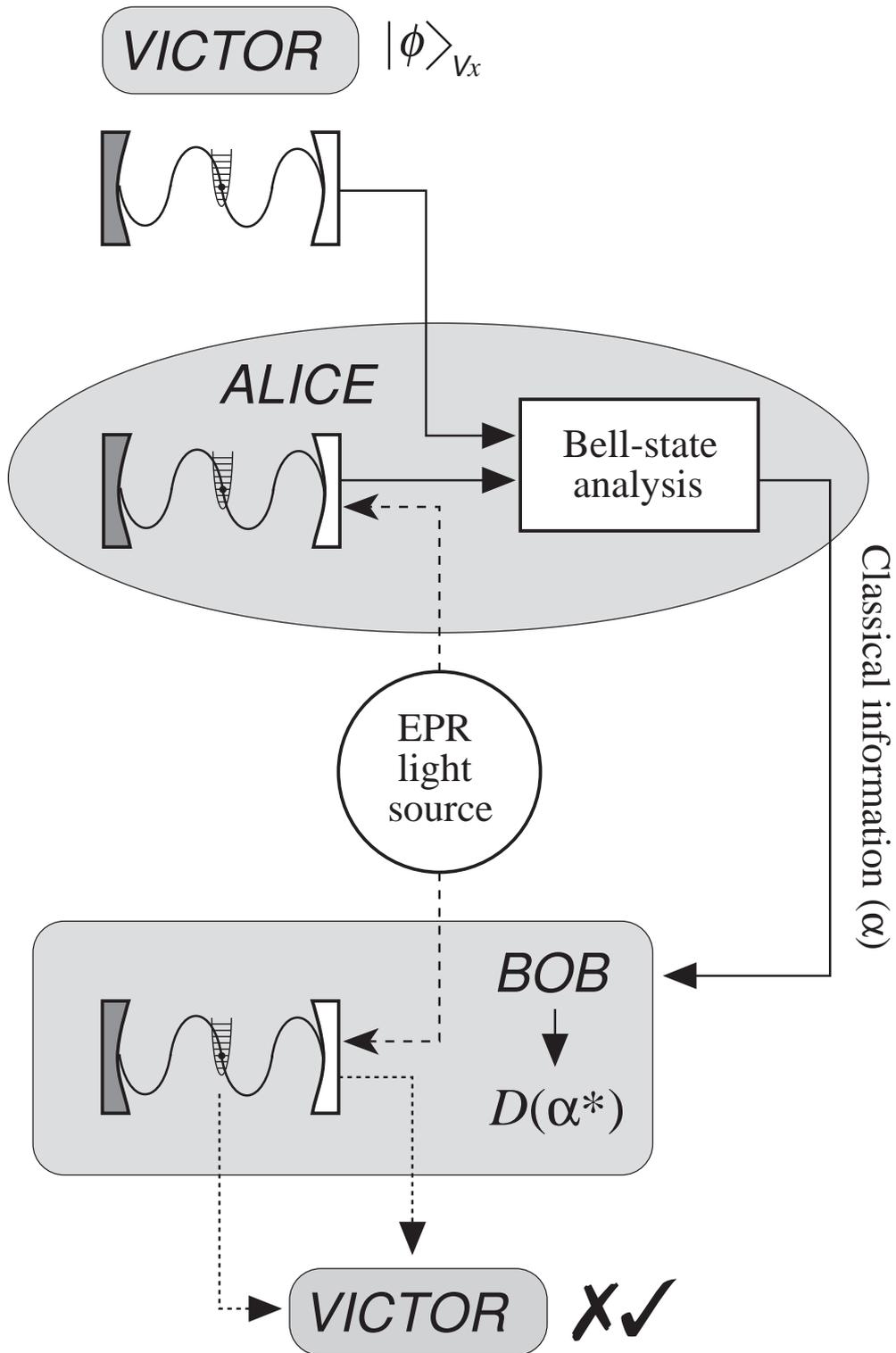


Figure 1

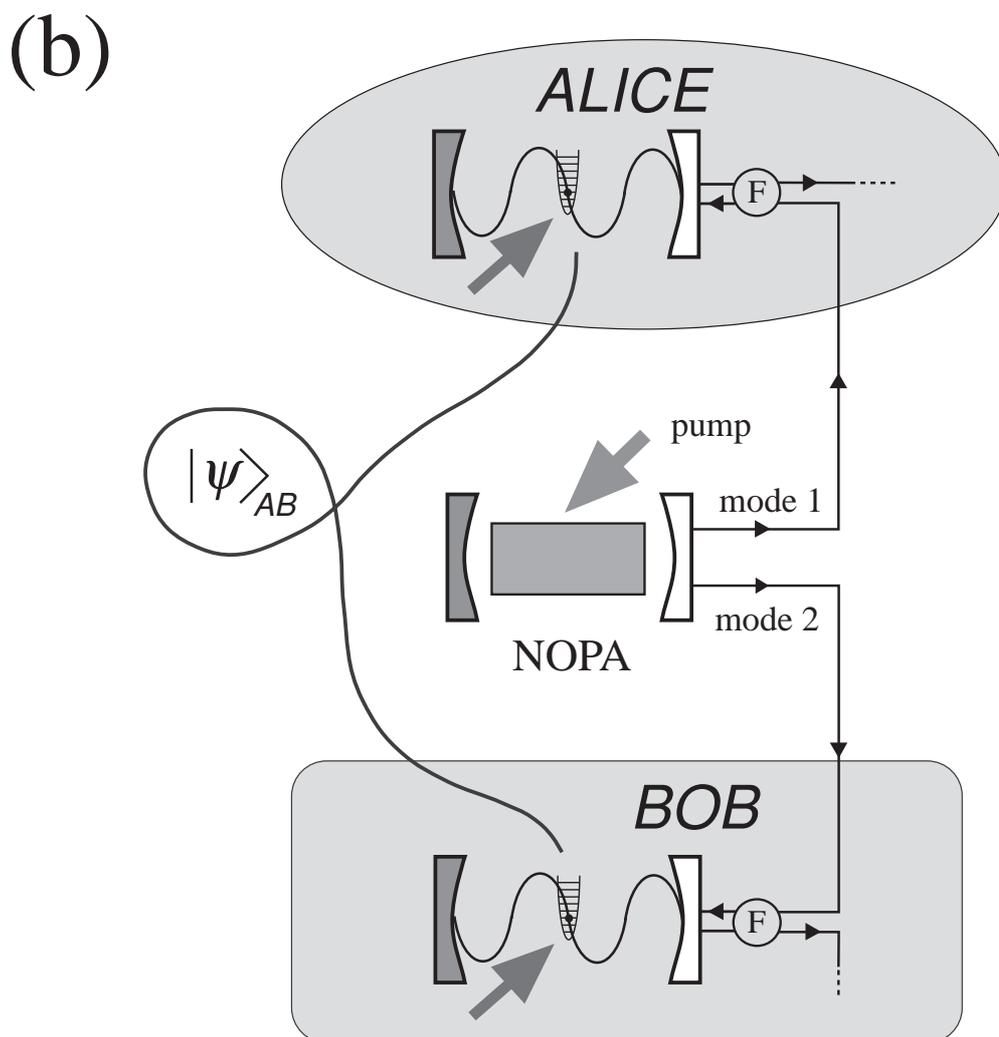
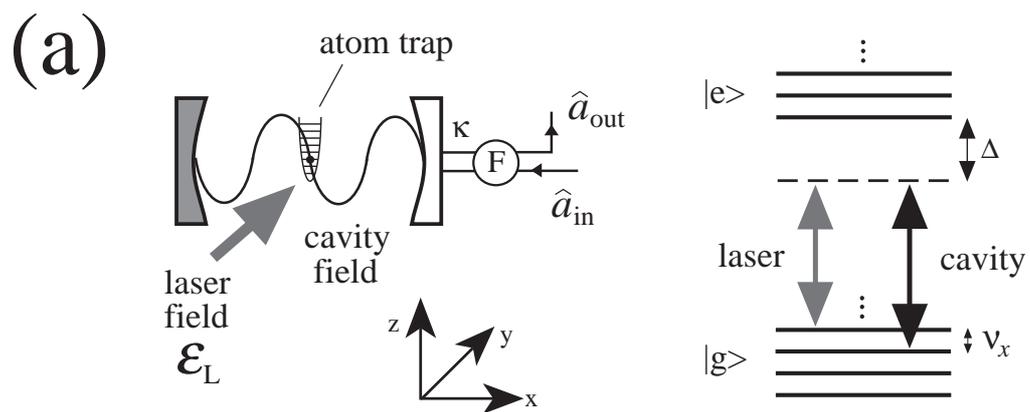


Figure 2

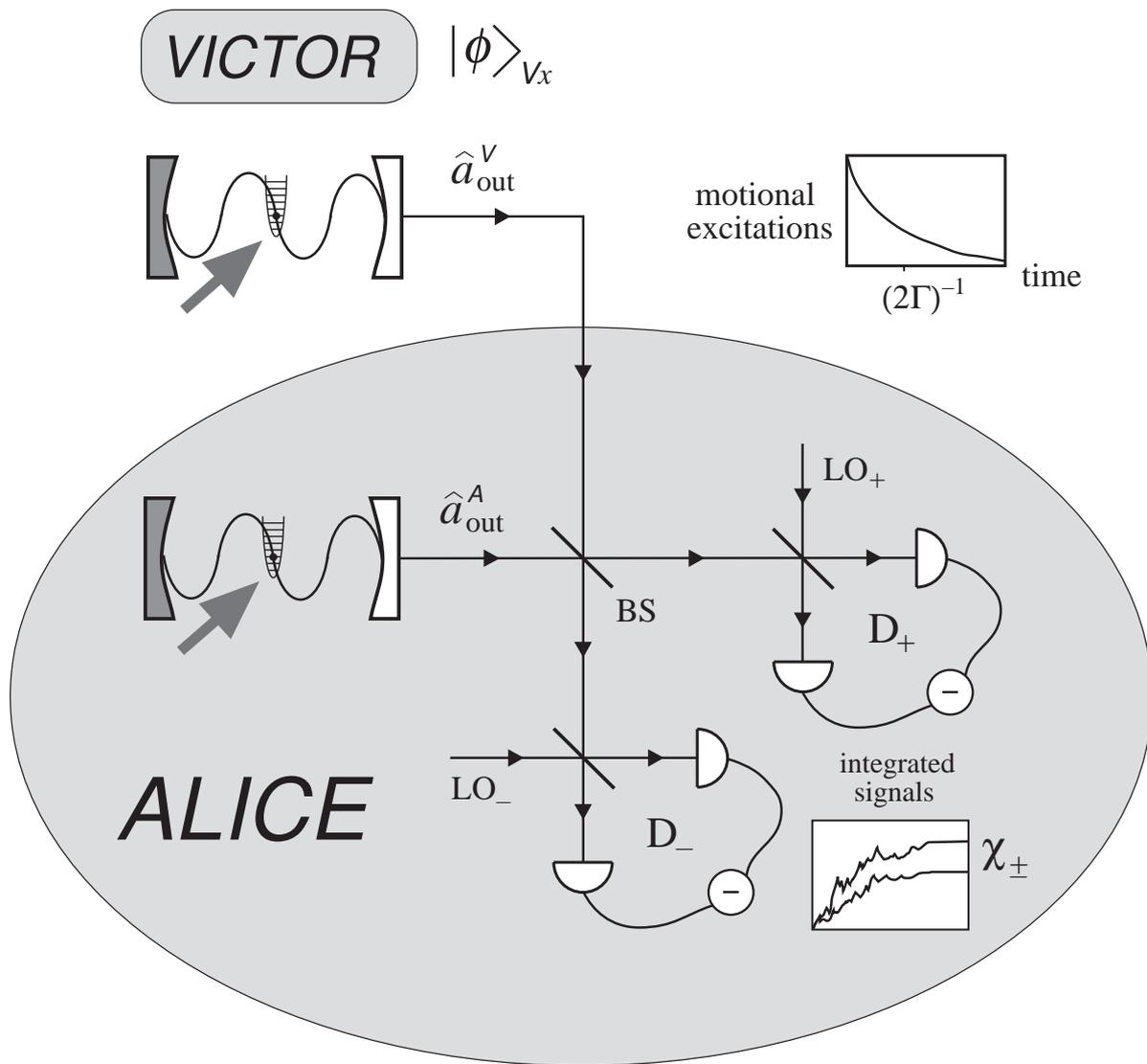


Figure 3